AMRITA VISHWA VIDYAPEETHAM
KOCHI CAMPUS

MATH SPACIA

ANCIENT KERALA MATH ODYSSEY

A QUEST TO ANCIENT KERALA SCHOOL OF MATHEMATICS

Department of Mathematics
School Of Physical Science


MATHSPACIA

"Progress is not possible without discipline. A nation, institution, family or individual can advance only by heeding the words of those who deserve respect and by obeying the appropriate rules and regulations. Children, obedience is not weakness. Obedience with humility leads to discipline."

Mata Amritanandamayi
Our Guiding Light

DIRECTOR'S NOTE

DR. U. KRISHNAKUMAR

Dean & HoS - Schools of Arts, Humanities, & Commerce, Kochi | Dean - School of Spiritual & Cultural Studies | Research Supervisor, Department of Computer Science and I.T., School of Arts and Sciences, Kochi

MESSAGE

This magazine brings out the embodiment of latent talents and research skills of our Mathematics Students. It really portrays the incredible long eventful journey, led by Bharat's ancestral Mathematical genius. It's a motivational treasure of knowledge for the upcoming young scholars of Mathematics. Hearty Congratulations to the whole team behind this work. Pranams and prayers to Amma to incessantly bless all of them.

HOD'S NOTE

Dr. K Sreekanth

HoD, Dept. of Mathematics, School of Arts & Sciences, Kochi

Dear Readers,
It is with great pleasure and excitement that we present before you 'Ancient Kerala Math Odyssey', a magazine dedicated to the mathematical traditions of ancient Kerala. My sincere hope is that it will provide you with a thorough introduction to the fascinating achievements of ancient Kerala mathematicians within the broader context of our rich cultural heritage.

Kerala has a long history of trade, cultural exchange and vibrant intellectual discourse and has always been a melting pot of diverse ideas. From this fertile ground emerged mathematical geniuses like Madhava, Nilakantha Somayaji, and others; their groundbreaking contributions would go on to greatly boost and direct the progress of mathematics in the wider world of Humanity as a whole. However, despite its significance and impact, their great intellectual legacy has not been adequately understood or appreciated in modern times. This magazine makes a sincere attempt to correct this shortcoming and invites mathematicians, scholars, enthusiasts and above all, the Youth to know, cherish and enhance this wonderful treasure.

From ingenious methods of calculus to astoundingly accurate astronomical calculations, each of the ancient discoveries presented here is a testament to the incredible intellect of our ancestors and more importantly, a potential source of inspiration to aspiring youngsters – to know our heritage in all its totality and to mould a future that is a worthy successor to that glorious past.

I extend my sincere gratitude to all contributors who dedicated their time and expertise to make this magazine a reality. I wish everyone an enlightening and enriching reading experience. May this Odyssey launch a thousand greater Odysseys!

Warm regards,
Head of the Department
Dr. K. Sreekanth

MESSAGE

Dr. Manohar Shinde

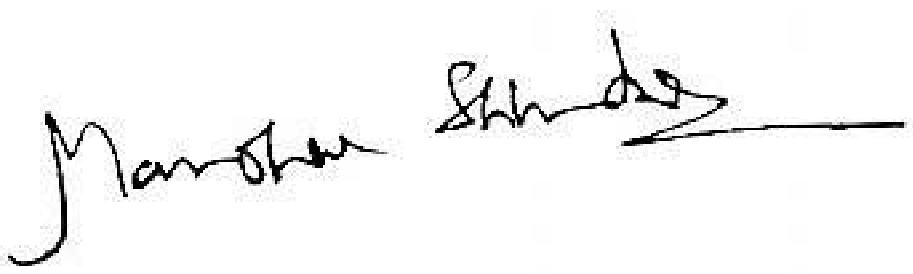
Founder of Dharma Civilization, USA

I recently had the opportunity to visit Amrita campuses in Kochi , Amritapuri and was fortunate to receive Amma's blessings and detailed guidelines on how to approach the study and research in Bharatiya (Indian) knowledge Systems.

I am delighted to learn that the faculty and researchers at Amrita are presently working on studying, and elaborating on Kerala School of Mathematics. Like many other fields, Bharath has a glorious tradition of it's phenomenal contributions in the field of Mathematics. Here, some unique and distinctive contributions came from what came to be known as "Kerala school of mathematics". Unfortunately, so little is known to most of the students even in our own country. Above work will be one way to educate and make aware our students knowledgeable about this treasure so as to develop a healthy sense of pride and an inspiration and confidence towards new knowledge production.

Let as aspire not only to be consumers of knowledge but also be producers of knowledge, as our valuable contribution to the enrichment of global pool of knowledge and wisdom. My heartfelt best wishes to the above efforts at Amrita.

Manohar Shinde ,
(Member of the National team of Pragya Pravaha,
which focus on Bharatiya knowledge systems)



EDITOR'S NOTE

Kaarthikanjana S Kumar

Editor-in-Chief MATHSPACIA
Integrated MSc Mathematics 2019

Dear Math aficionados,
It is with great excitement and pride that we present you our department magazine ANCIENT KERALA MATH ODYSSEY, This milestone marks the beginning of a new chapter in our journey together, as we celebrate our shared interest and passions.

In this edition, we endeavored to capture the essence of our club's spirit showcasing the diversity of talents and perspectives within our community. From engaging articles to captivating artwork each page reflects the collective efforts and contributions of our members.

This magazine specially contains culmination of collaborative efforts, featuring the rich history of Kerala Mathematics, great contributions insightful interviews, and thought-provoking content.

Our team has striven to complete this work to accomplish our aim of marking the beginning of a study on an area which is less explored and is a vast concept. Behind the scenes, countless hours have been invested in researching, creating, and refining the articles within these pages. We hope you find inspiration, entertainment, and perhaps a new perspective within these carefully crafted stories.

We extend our deepest gratitude to Prof. Dr . Roy Wagner , Dr. Easwaran Nambudiri T C, Dr. Venkateswara R Pai, Narayanan M Komerath , for encouraging our curious minds.

Thank you for being a part of this journey with us, and we look forward to continuing this enriching study.

Happy reading!
Kaarthikanjana S Kumar
Editor-in-Chief, MATHSPACIA

DEPARTMENT OF MATHEMATICS

MISSION

The central mission of the Mathematics Department at Amrita University is to bring synergy between education and human empowerment with the determination to inspire and educate the budding mathematicians. We are focused on inculcating the spirit of enquiry and scientific temper yet with the firm belief in the Indian wisdom from our rich cultural and civilizational ethos. We believe in the eternal beauty of mathematics and consider it to be our mission to enlarge the scope of a research driven academic space inclusive of social diversity. We inspire and focus on upgrading the quality of the students making them capable to be the leaders of the new world.

VISION

The Mathematics Department at Amrita University envision to incorporate the relentless quest on research and learning experience to the scholars and consider exploration into the depths of Mathematics as the driving force of academic engagements. Hence the beauty and utility of Mathematics is shared to society by spreading the wisdom of the subject through the well-trained scholars and faculties of the department of Mathematics. We are involved in effectively offering sustainable solutions to tackle global challenges with application of our knowledge and collective efforts centered on research, innovation catering to the practical application for social upliftment and human development.

AIMS

Empowerment of Students

We are determined to empower students by catering to their aspirations on the subject of Mathematics and strengthening the core foundation on the subject knowledge. Transforming knowledge into wisdom is the key for our university. The key focus of the department is concentrated on effective interventions to upgrade the skillsets of students for both academic excellence and personality development, ensuring integrated and holistic development. The Department is deeply committed to carving out a future generation of mathematics scholars capable of nation building, who can offer creative and effective solutions to overcome the global challenges.

Building meaningful partnership and global collaborations

We believe in the strength of togetherness in addressing the noble objective of Loka Samastah Sukhino Bhavantu and consider humankind as a single family Vasudhaiva Kutumbakam. In this regard we hope to foster deep collaboration within the department and build sustainable partnerships within and beyond South Asia, catering to the effective fulfillment of sustainable development goal on building strong partnerships to empower research in Mathematics. We believe in spreading the vistas of Mathematics beyond the department to encourage inter-disciplinary research and integration of Mathematics principles to relevant streams of knowledge generation.

Promotion of Excellence

We are committed and dedicated to maintaining and upgrading the standard of learning, teaching and research experience. Our faculty members are eminent scholars with proven academic credentials and are deeply passionate about exploring the possibilities of Mathematics in advancing knowledge and applying wisdom for the betterment of humankind.

Fostering Inclusivity

We are strong believers in compassion driven and integrated development of humanity irrespective of diversity. Unity in Diversity is a mantra which drives our commitment to empower every aspiring student and researcher in the field of Mathematics. Equal treatment and mutual respect are core values which drive the department to move ahead and create an ecosystem of universal peace. A level playing ground is set for a win-win attitude among scholars of Mathematics at Amrita University.

Community Engagement

We believe in the value of mathematics for social empowerment. Our department is committed to address the needs of communities from local to global level. This is made possible by offering outreach initiatives, sharing of expertise with community and spreading literacy with grass root level societal interventions.

Research driven Innovation

As Mathematics is a dynamic and ever evolving field of academic study we are focused on constantly updating the existing knowledge base to cater to innovations and global developments. This is reflected in methods of teaching, research outcomes and perspective building in application of wisdom derived from mathematics.

KEY FUNCTIONS AND RESPONSIBILITIES

Teaching Mathematics

The basic function of the Department is to provide quality driven instruction to students from elementary to higher levels. Thus the updated courses are incorporated with the help of innovative teaching methods thereby strengthening the core capabilities including problem solving skills and in-depth knowledge in mathematics concepts.

Curriculum Development

The department is responsible for development and maintenance of the mathematics curriculum. This is made possible by designing and creating courses defining learning objectives and ensuring the curriculum to match with the educational standards fulfilling the needs and aspirations of the changing world.

Assessment and Evaluation

Regular analysis and quality enhancement for the students is undertaken in the department of Mathematics by conducting assessments in the form of examinations and assignments at regular intervals.

Research

The department is engaged in active research with leadership provided by eminent faculty members by effective contribution via publishing scholarly works contributing to advancement of knowledge in the field of Mathematics.

Professional Development

Faculty and staff have to be prepared and stay updated with the ever-changing world of Mathematics in terms of research and innovation in teaching methodologies, learning and in-depth inquiry in Mathematics. This is ensured by fostering research driven conferences, academic workshops and opportunities to encourage the spirit of enquiry in Mathematics.

Student Support

Students who struggle with Mathematics find solace in the student support facility provided by the student support team of the department of Mathematics. This can be in the form of extramural tutoring providing additional academic resources and improvised study techniques.

Advising and Counselling

The human wellness and mental health of students and faculties are key priority areas for the Department of Mathematics at Amrita University. The department offers effective therapy and counselling sessions to students and guides them to choose suitable career pathways.

Community Outreach

Amrita University is deeply committed to empower and develop local community empowerment. The Department of Mathematics is focused on organizing workshops and educational promotional events to spread Mathematics literacy, curiosity and enthusiasm for the subject from community members and civil society.

Collaboration with multidisciplinary partnerships

Mathematics is deeply interconnected with other streams of knowledge including Physics, Engineering and Computer Science. Effective collaboration of students and teachers in these disciplines of Mathematics is the fundamental spirit of knowledge creation.

Professional Faculty Development

Realizing the potential of sharpening the skillset and knowledge base of the faculty members the Department of Mathematics at Amrita University is providing a wide range of opportunities ranging from offering scholarships, faculty development training, mentoring and constant efforts to empower the spirit of teaching and research.

Resources and Faculty Management

The Department is entrusted with the task of providing the necessary resources, text books, latest software and digital educational tools and availability and access to smart classes, computers and laboratory facilities.

Quality Assurance

The vital responsibility is ensuring the quality of Mathematics department in terms of offering education and research capabilities competent to match global standards. Resource upgradation and above initiatives ensure quality of our service.

Budget and Resource Allocation

Management, allocation of departmental finances and seeking funding opportunities for research driven educational initiatives are prominent responsibilities.

Inclusivity and Diversity

Promotion of inclusivity and strengthening synergy of the Mathematics Department is aimed at an open and welcoming learning ecosystem which respects the faculty and student needs irrespective of their background.

Policy and Governance

The Department is an active participant in institutional decision-making initiatives with the university and school leadership aiming to shape policies towards expanding the possibilities of Mathematics education.

Faculty Profile

The Mathematics Department at Amrita University is constituted by a team of dedicated and academically acclaimed faculty members. This ensures the confluence of knowledge base transforming into wisdom paving the way for a thought provoking and vibrant academic ecosystem. With the active presence of research driven and specialized academic credentials the Department of Mathematics offers innovation in terms of teaching and research outcomes that are aimed at building a strong academic center for excellence and higher learning in Mathematics.

MATHSPACIA

Welcome to pages where equations come alive and numbers dance with precision! We are delighted to introduce you to MATHSPACIA—a vibrant community where math enthusiasts, novices, and curious minds gather to explore the captivating world of mathematics. It is an immersive journey into the depths of mathematical wonders. From unraveling challenging problems to engaging in spirited discussions, we bring the joy of mathematics to life. We believe in the power of collective intelligence, where diverse minds come together to conquer mathematical challenges. We host guest speakers—mathematicians, professors, and professionals—who share their passion for numbers and illuminate the exciting paths that a love for math can lead. It will be a platform where one can explore the beauty of mathematics and science through various activities such as Gaming, Quiz, Arts and Seminars. Mathspacia has four sub clubs which are Games club, Quiz club, Seminar club and Arts club.

Games club: "Pure mathematics is the world's best game, it is more absorbing than chess, more of a gamble than poker, and lasts longer than monopoly "- Richard J Trudeau (Dots and Lines). Mathspacia presents before you The Games club which will host a plethora of events to ignite curious minds.

Art club: The world of mathematics is a form of creative art. The implementation of arts in mathematics will help you to awaken your creative mind. 'Mathspacia' would like to introduce you to The Arts club where you can learn maths through arts and also showcase your talents.

Quiz club: A little competition can inspire one to greater achievements. For those who love challenges 'Mathspacia' would like to introduce you to The Quiz club. Quiz club is a platform where you can test your skills and knowledge. We provide an engaging mathematical experience that is both competitive and educational.

Seminar club: “Uncertainty is the prerequisite to gaining knowledge and frequently the result as well” - Edith Hamilton. Knowledge opens the door to opportunity, achievements and success.' Mathspacia' presents you the seminar club where you can expand your knowledge through various activities such as debates, seminars and screening of fun math facts.

CONTENTS

A SKETCH ON KERALA'S MATHEMATICAL HISTORY

- KAARTHIKANJANA S KUMAR

Reminiscence of the social and cultural background of Kerala in the early medieval period, that is, from the 9th to 16th century, reveals the extraordinary growth of mathematics and astronomy in Kerala. in Kerala. There were Brahmin settlements called Agraharas. A new Chera dynasty with its Perumal in the capital at Mahodayapuram or Thiruvanchikulam (present Kodungallur north of Kochi) came into existence, as the Brahmins invited the prince from the land beyond the Western Ghats to rule the country with their advice and guidance. Through the gradual merger of smaller territories Cochin gained size and status. Until the dawn of the 15th century, Kodungallur was the capital of Cochin State and later Cochin City became the capital and Tripunithura the royal township. There was uninterrupted continuity of culture and education which favoured growth of scientific research.

An important factor that helped the growth of scientific tradition was the freedom from military invasions from the North and the flourishing sea trade of Kerala with its numerous natural harbours. Sandalwood, teak, and all sorts of spices, perfumes and medicinal plants were available in Kerala. These attracted traders from East and West. Continuous exposure to the cultural developments in the East and West must have widened the mental horizon and increased the intellectual capacity of the enlightened sections of people.

From early times people of Kerala held the study of Celestial luminaries - Jyotisha sastra. Jyotisha sastra comprises of two parts, a theoretical part and a practical part.

The phase of moon, solar and lunar eclipse and variations in the movements of planets, etc., are some of the events which could be calculated in advance. Descriptive details pertaining to earth and other celestial bodies, belong to the theoretical part. Forecasting future events, reckoning of auspicious moments –Muhurthams etc. fall within the scope of the prediction part.

There is evidence to show that the Aryabhatiya tradition of Astronomy was very strong in Kerala. Sankaranarayanan, the court astronomer of Ravi Kulasekhara, composed Vivarana, a commentary of Laghubhaskariya. Though the original text uses the Bhutasankhya system to suggest dates, Kerala commentators used the Kadapayadi system of letter numerals which was popular in Kerala. From the 9th century inscriptions in temples in Kerala refer to solar months, dates, weeks and Naksatras, besides other details. The Grand festival Mamankam, was celebrated once every twelve years. This is based on the Jupiter cycle. Perumals were initially allotted a twelve-year term of rule. Land records were renewed after twelve years in traditional society. The Saka era, Kali era and Kollam era were used in Kerala. Mathematics was an important and helpful tool for the study of Jyotisha sastra and fostered as such.

The result was that those mathematical topics, like properties of a circle, sphere, mensuration, etc., needed in the study of astronomy got prime attention and gained enormous development.

NUMBER SYSTEMS

Indians had separate names for the powers of ten, Eka = 1, Dasa= 10, Sata = 100, Sahasra = 1000, Ayuta = 10000, Niyuta = Laksha= 100000 Prayuta = 1000000. The Bhutha Samkhya system and Kadapayadi system were two systems which survived for long time.

Bhutha Samkhya & Katapayadi System

Bhutha Samkhya system is the older system. In this system numbers are indicated by well-known objects or concepts having as many parts or components as the numbers they represent. For example: 0 is denoted by sunya (Void) kha = sky, antariksha = atmosphere, 1 is denoted by sasi = moon, bhumi = earth, 2 is denoted by netra = eyes, bahu = hands, karna = ears, 4 is denoted by Veda - Rig, Yajur, Sama and Atharva, dik or dis = directions; east west, north and south, 5 is denoted by bhuta – Five elements (Ether, Air, Fire Water and Earth), Pandavas. Any synonym of a word denotes the same number. The same word may denote more than two different numbers and this went against universal acceptance.

Katapayadi System: The starting point of the system is in Sanskrit language. Sanskrit was the language of the scholarly section throughout India and Malayalam is the regional language of Kerala. The only difference in Malayalam and Sanskrit is in the scripts. Vowels following consonants have no special value. So each of letters k, ka, ki, ku denotes the same number – 1. The system follows decimal notation, a right to left arrangement. The first letter in the word stands for unit place digit. The second letter stands for ten's place digit and third for hundredth' place. Madhava (Ma = 5, dha = 9 ,va = 4) indicates the number 495. The mnemonic of any number is called paralperu. The flexibility in the choice of letters, consonants and vowels following them for each digit, makes it possible to frame several different expressions for the same number.

This flexibility is much better than Bhutasamkhya. The general belief is that the astronomer Vararuchi was the proponent of the Katapayadi system. From the circumstantial evidence, historians believe that Vararuchi lived in the 4th century AD. This system is believed to be one of the major contributions of Kerala to Mathematics.

Vararuchi has another feather in his cap. He composed Candra vakyam – the 248 vakyas give the positions of the rising moon on as many consecutive days. They were explained in katapayadi format. These Candra vakyas were used by scholars from 4th century onwards.

MATHEMATICIANS OF KERALA

Haridatta (650-700 AD), Govindaswami (800-850), Sankaranarayanan (825-900) and Talakkulam Govinda Bhattathiri (1237-1295) are the known Mathematicians or astronomers of Kerala of 7th to 14th Century.

Haridatta modified the Aryabhateeya methods to find planetary positions. He used Katapayadi system to simplify the methods. This simple method was known as Parahita system. This was followed to perform religious rituals. These refinements were well accepted due to their simplicity and accuracy. Sankaranarayanan enjoyed the patronage of Cera King Ravi Varma Kulasekharan of Mahodayapuram. Sankaranarayanan was in charge of the Observatory at Mahodayapuram. Govinda Bhattathiri was a bright star in the galaxy of Kerala astrologers.

Ancient astronomers of many civilizations observed that the moon and planets were never at very great angular distances from the ecliptic (apparent annual path of sun). They imagined a belt in the sky extending to eight degrees on either side of ecliptic and called it the Zodiac. Twelve groups of stars or constellations were spotted in the Zodiac. Further, they noticed the 30-day period the sun takes to travel through each constellation.

The constellations were named according to their shapes. These are known as sign of the Zodiac. The Kerala term for sign is Rasi. Kerala astronomers considered each Rasi as an arc of a circle, facing an angle of 30 degrees at the centre. The study of the circle and its chords broke the finite barrier. The manuscripts written on palm leaves were lying in palaces or illams as private collections.

HISTORY OF KERALA SCHOOL OF ASTRONAMY AND MATHEMATICS

-SARANYA MURALI

The Kerala school of astronomy and mathematics was founded by Sangamagrama Madhava (c.1380-1420) around 14th century . Sangamagrama Madhava is one of the greatest Mathematician-Astronomers of the late Middle Ages. He is always referred to as Golavid (master of spherics) by his disciples. The later members of the school started from Madahva's direct disciple Parameshwara (c.1380- 460), a major Indian mathematician and astronomer. After Madhava, the next important member of the school was Nilakantha Somyaji (c.1444–1550) of Trikantiyur, who was a disciple of Damodara, the son of Paramesvara. Another disciple of Damodara was Jyeshthadevan (c.1500–1610), the author of the celebrated work Yuktibhasha in Malayalam. The line of direct disciples of Madhava continued up to Acyuta Pisarati (c.1550–1621), a student of Jyesthadevan and the teacher of the great scholar devotee Narayana Bhattatiri.

The Kerala school is incomplete without mentioning the remarkable works of Madhava. Venvaroha and Sphutachandrapti, which are two great works of Madhava. They showcase his mathematical mastery in improving the accuracy of the ingenious vakya system of computation for the Moon. Madhava's important results on infinite series are known through his verses cited in the later works of Nilakantha, Jyesthadeva, Sankara Variar (c.1540) and others. The technique of usng end correction terms in successive approximation in order to obtain highly accurate results is also an exceptional work of Madhava. A verse of Madhava, giving the value of π to the accuracy of eleven decimal points, is cited by Nilakantha in his commentary on Aryabhatiya. Madhava is also a pioneer in discovering rapidly convergent approximations and transformations of the series of π. He also came up with an algorithm for evaluating functions accurately correct to five decimal places for an arbitrary argument. Detailed justifications for the results discovered by Madhava are presented in the famous Malayalam work Yuktibhasha (c.1530), which is also known as the first textbook of Calculus.

The indelible contributions of the Kerala school to the realm of astronomy are inevitable. Madhava's accurate sine tables, interpolation formulae etc were meant for applications in Astronomy. Madhava's disciple Parameshwara had made a series of eclipse observations over a 55-year period to verify the accuracy of the computational methods then in use. The computational scheme based on the revised set of parameters has come to be known as the Drgganita. Parameshwara had made immense contributions to the world of astronomy. At least 25 manuscripts have been discovered to be penned by Parameshwara. After Parameshwara, Nilakantha Somayaji is the one who made immense contributions in the history of astronomy. A major revision of traditional planetary theory was proposed by Nilakantha. The errors occurring in the computation of planentary longitudes was a long standing problem, Nilakntha proposed a fundamental revision of the traditional planetary theory in his treatise Tantrasangraha (c.1500). Nilakantha was also the first sage in the history of astronomy to clearly deduce from his computational scheme and the observed planetary motion that the interior planets (Mercury and Venus) go around the Sun and the period of their motion around Sun is also the period of their latitudinal motion. These are some droplets from the sea of discoveries done by the pioneers of the Kerala school. About 350 works by over 115 authors have been identified related to the Kerala school.

AN OVERVIEW OF MATHEMATICIANS OF KERALA & THEIR CONTRIBUTIONS

-KAARTHIKANJANA S KUMAR

- **Madhavan of Sangamagrama:**

The vital period in the history of Kerala mathematics started with Sangamagrama Madhavan (1340-1425). The birthplace of Madhavan is Sangamagrama, near Irinjalakuda. Venuaroha is an important work of Madhavan. Later writers refer to Madhavan as Golavid (expert in spherics). Madhavan's discoveries are known to us mainly through the statements and works of his followers. His writings can be verified in his works like Sphuta Candrapti, and the date corresponds to 1400AD. Madhavan gave the value of Pi(π) correct to eleven decimal places. He described it using the Bhuta Samkhya system. This is given as the circumference of a circle having diameter 9×10^{11} units is 2827433388233. As per this calculation the approximate value of Pi(π) is 3.1415 9265359. Sangama Gramma Madhavan also stated $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$ This series was known as the Leibniz (1646-1716) series.

The findings of sine and cosine series in trigonometry are another contribution by Madhavan which made him one of the most eminent mathematicians of the world. Madhavan's main contributions came to be known through the works of his followers like Neelakanda Somayaji, Jeyshtadevan, Sankaravaryar, etc.

- **Paramesvaran Namputiri:**

Paramesvaran Namputiri (1360-1460) belonged to Vataserri illam of Alathiyur in Malappuram. The illam stood on the northern bank of the river Nila (Bharatha Puzha) close to where it merges with the Arabian Sea. Drg Ganitha and Gola Dipika are the major works of Parameshwaran. He evolved the Drk system for astronomical calculations with more accuracy. This was the result of his astronomical studies coupled with observations of celestial bodies for more than fifty years. This system was useful to predict eclipses, etc. Parameshwaran observed and recorded several solar and lunar eclipses from 1393 AD onwards. Parameshwaran also put forward the formula for the radius of the circle circumscribing a cyclic quadrilateral in terms of its sides, in his commentary on Lilavati. If a, b, c, d are the sides of the quadrilateral, the semiperimeter, $S = (a + b + c + d)/2$ and R the radius of the circle circumscribing it $= \frac{1}{4} \sqrt{(ab+cd)(ac+bd)(ad+bc)/(s-a)(s-b)(s-c)(s-d)}$. The credit for this discovery is normally given to Simon Antoninie Jean Lhuilier, who published it in 1782 AD.

- **Vatasseri Damodaran:**

Damodaran (1410-1510) was the son and student of Paramevaran. His disciple, Nilakanda Somayaji, quoted several excerpts from Damodaran's works. Damodaran can be considered as the link connecting Parameshwaran to Nilakanda in the teacher student chain carrying the torch of celestial studies.

- **Nilakanda Somayaji:**

Nilakanda Somayaji (1443-1545) belonged to Kelallur illam of Trikkandiyur near Tirur in Malappuram. Trikkandiyur was a famous center of learning. Tantra Sangraha is his major work. It is a major work on Astronomy.

The work contains 431 verses divided into eight chapters. There are several mathematical results. Tantra Sangraha deals with Spherical triangles formed by the Sun, North Pole and zenith on the celestial sphere, which are called astronomical triangles. Tantra Sangraha also gives several methods related to calculus. The topics include the method to find sum of an arithmetic progression and the representation of series using rectangular strips. The work titled Sundararaja Prasnotharam is a collection of Neelakanda's clarifications on the doubt raised by Sundararajan, an astronomer of Tamil Nadu. Nilakanda Somayajee's other works include Gola Sara - Spherical astronomy, a primer, Siddhanta. Darpana - the mirror of the laws of astronomy, Chandrachaya Ganitha - computation of time from the shadow cast by the moon, Grahananirnaya - computation of lunar and solar eclipses, Graha Pariksakrama - principles and methods of astronomical computation from direct observation. Nilakanda Somayajee was reputed to be an excellent teacher. Jyeshta Devan and Sankara Varior were his disciples. Chithrabanu who wrote Karanamrutham in 1530, was also a disciple of Nilakanda. The Karanamrutham contains solutions for simultaneous equations in two unknowns.

- **Jyestha Devan**

Jyestha Devan Namputiri (1500- 1610) was a member of Parannottu family in Alathur Village. He was a student of Damodaran. Jyestha Devan was a younger contemporary and disciple of Nilakanda. Jyestha Devan is known as the author of Yukti Bhasha also known as Ganitha Nyaya Samgraham.This text is written in Malayalam. It explains the infinite series for Pi, Sine etc.

The text contains two parts. The first part is devoted to the logical demonstration of mathematical research and the second part to astronomical topics. The sixth and seventh chapters of Yukti Bhasha contain proofs and derivations of the methods in the text. It gives derivations of $\pi/16$, π, $\pi/8$, etc., as rapidly converging series.

- **Sankara Variyar**

Sankara Variyar (1500-1560) was a student of Neelakanda Somayaji and Damodaran. Karana Saram written in 1550 is the work of Chithrabhanu, which deals with astronomy. He also wrote several commentaries.

- **Puthumana Somayaji**

Puthumana Somayaji (1660-1740) wrote Karana Padhathi in 1732. Karana Padhadhi contains 213 verses divided into ten chapters. It deals with the results and methods explained in Yukti Bhasha and Kriya Karmakari.

- **Sankara Varma**

Sankara Varma was the younger brother of the King Udaya Varma. Sadratnamala is the only known work by Sankara Varman. It contains 211 verses in six chapters. Sadratnamala is a handbook of mathematics and astronomy. The original text was in Sanskrit and Sankara Varman also wrote a Malayalm translation. Sadratnamala gave the value of pi correct to 17 decimal places.

- **Damodara Nambudiri**

The son of Parameshvara Nambudiri is known for his work "Muhoorthaabharanam". He works extended to theoretical foundations and did notable achievements on infinite series expansions. Inspired from Madhava's Gregory series, he achieved accurate approximations of л. He initiated Nilakanda, his student into the science of astronomy and taught him basic principles in mathematical computations.

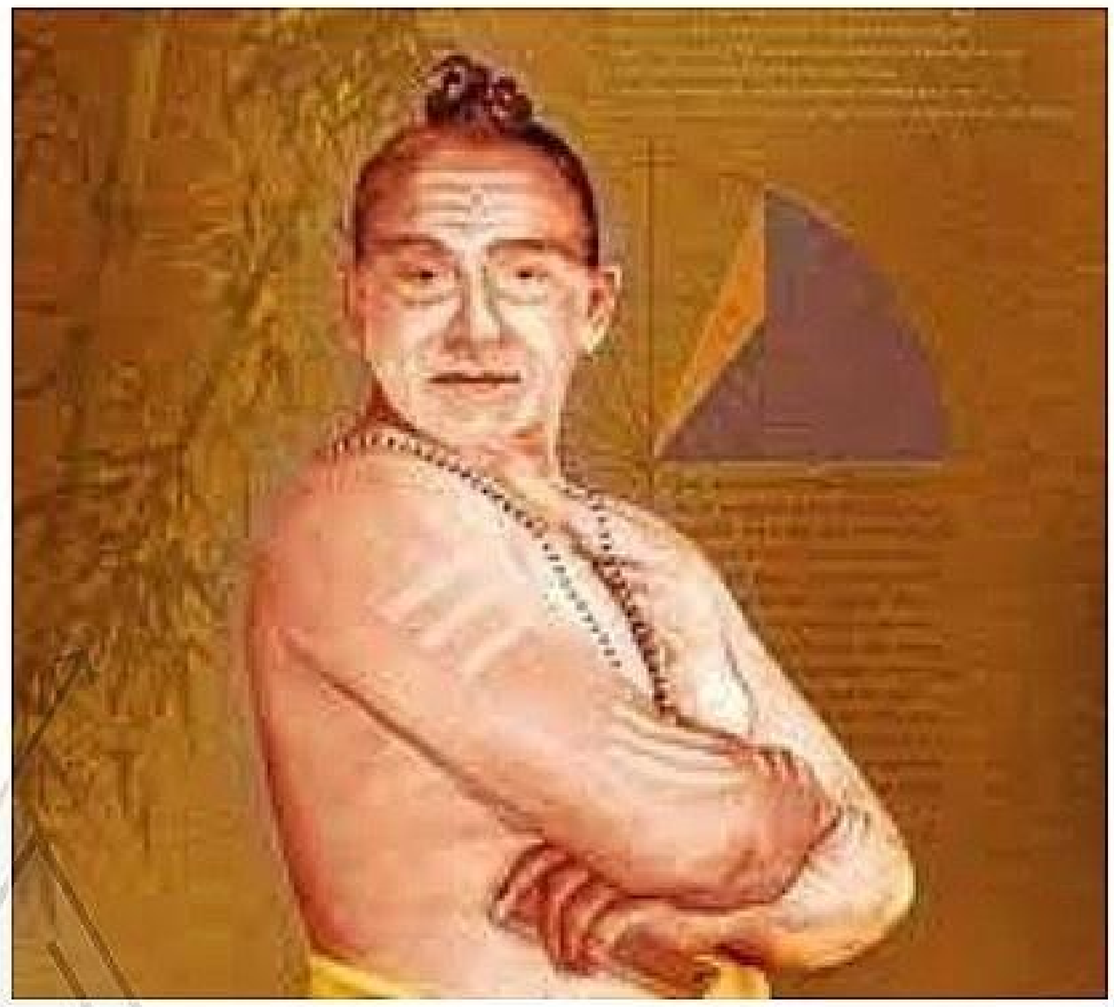
- **Kelalur Nilakanda Somayai**

A student of Damodara, he had developed series expansions for the common trigonometric functions. His best known work, written on palm leaves, is called Tantrasangraha [or Tantrasamgraha] (A Compilation of the System) was completed in 1501 CE (consists of 432 Sanskrit verses divided into 8 chapters), where he revised Aryabhata's model for the planets Mercury and Venus. In his Aryabhatiyabhasya, a commentary on Aryabhata's Aryabhatiya, he developed a computational system for a partially heliocentric planetary model in which Mercury, Venus, Mars, Jupiter and Saturn orbit the Sun, which in turn orbits the Earth, similar to the Tychonic system later proposed by Tycho Brahe in the late 16th century. His work titled Jyotir Mimamsa stressed the experimental nature of astronomical science and pointed out that the statements in ancient texts about divine intervention need not be taken literally. In his writings he refers to a Mimamsa authority, quotes extensively from Pingala's chandas-sutra, scriptures, Dharmasastras, Bhagavata and Vishnupurana also. He also wrote Candrachayaganita (Calculation of time based on sun and moon's shadow.

His other works are: Golasara, Sidhhantadarpana Sidhhantadarpana-vyakhya, Chandrachhayaganita-vyakhya, Sundaraja-prasnottara , Grahanadi-grantha , Grahapariksakrama.

THE MAN WHO INVENTED CALCULUS: A LOOK INTO THE LIFE AND WORK OF MADHAVA

In this article, we will explore the life and work of Madhava, an influential mathematician and astronomer from 14th century Kerala, India. He is considered as the founder of the Kerala school of astronomy and mathematics. Madhava made pioneering contributions to the study of Infinite series, calculus, trigonometry, geometry and algebra. He was the first to use infinite series approximations for a range of trigonometric functions, which has been called the "decisive step onward from the finite procedures of ancient mathematics to treat their limit passage to infinity". Despite his significant contributions to the field, Madhava has been largely overlooked in mainstream history. It is only very recently that the scholarly world has begun to appreciate -- still somewhat uncertainly -- the originality and depth of his mathematical achievements. We will delve into the history of his discoveries and how they predate the work of European mathematicians by centuries.



Madhava's work was first mentioned in the lecture of Charles Whish in 1832. Whish described the mathematical concepts of 'quadrature' and 'fluxion' in relation to Madhava's work but there was no mention of Madhava's name. However, it would take many years for Madhava's contributions to gain recognition and understanding in the Western world. The first mention of his name was in the form "Sangamagrama Madhavan" in print was in the Malayalam analytic commentary on the key text which describes Madhava's works, Yuktibhasa. Madhava had a great lineage of disciples like Paramesvara, Damodara, Nilakantha, Jyeshthadeva, Sankara and Achyuta and their books are our only source of information about Madhava.

Though Madhava did not write texts in mathematics he authored texts in astronomy which are:

- Golavada, Madhyamanayanaprakara
- Mahajyanayanaprakara (Method of Computing Great Sines)
- Lagnaprakarana (लग्नप्रकरण)
- Venvaroha (वेण्वारोह)
- Sphuṭacandrāpti (स्फुटचन्द्राप्ति)
- Aganita-grahacara (अगणित-ग्रहचार)
- Chandravakyani (चन्द्रवाक्यानि) (Table of Moon-mnemonics)

In subsequent writings on Indian mathematics, many of Madhava's achievements were misattributed to other mathematicians, such as Nilakantha. This led to a lack of clarity and confusion surrounding Madhava's actual contributions to the field. It wasn't until more recent times that Madhava's name and work began to be properly recognized.

Madhava's most notable contribution to mathematics is his development of infinite series expansions for trigonometric functions.

Madhava was the first to state and prove the infinite series expansions:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots.$$

Madhava's work on the value of the mathematical constant Pi is cited in the Mahajyānayana prakāra ("Methods for the great sines"). This text attributes most of the expansions to Madhava, and gives the following infinite series expansion of π, now known as the Madhava-Leibniz series.

$$\mathrm{Sin}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots.$$

And the general arctangent series

$$X = 1 - \tan x + \frac{\tan^3 x}{3} - \ldots.$$

for x in the first octant.

He also derived the formula for area and volume of the sphere:

$$A = Cd\ ,\ V = C.\frac{d^2}{6}$$

The breakthrough idea in all these is the same: calculus on the circle.

Madhava's discoveries predate those of European mathematicians, such as Newton and Leibniz, by several centuries.

His methods are inspired by Aryabhata.

Madhava has composed an accurate table of sines which is encoded in the letters of the Sanskrit alphabet using the Katapayadi system, giving entries the appearance of the verses of a poem.

Madhava's original work containing the table has not been found. The table is reproduced in the Aryabhatiyabhashya of Nilakantha Somayaji and also in the Yuktidipika/Laghuvivrti commentary of Tantrasamgraha by Sankara Variar.

The verses are:

श्रेष्ठं नाम वरिष्ठानां हिमाद्रिर्वेदभावनः ।
तपनो भानु सूक्तज्ञो मध्यमं विद्धि दोहनम् ॥ १ ॥
धिगाज्यो नाशनं कष्टं छन्नभोगाशयाम्बिका ।
मृगाहारो नरेशोयं वीरो रणजयोत्सुकः ॥ २ ॥
मूलं विशुद्धं नाळस्य गानेषु विरळा नराः ।
अशुद्धिगुप्ता चोरश्रीः शङ्कुकर्णो नगेश्वरः ॥ ३ ॥
तनुजो गर्भजो मित्रं श्रीमानत्र सुखी सखे ।
शशी रात्रौ हिमाहारौ वेगज्ञः पथि सिन्धुरः ॥ ४ ॥
छाया लयो गजो नीलो निर्मलो नास्ति सत्कुले ।
रात्रौ दर्पणमभ्राङ्गं नागस्तुङ्गनखो बली ॥ ५ ॥
धीरो युवा कथालोलः पूज्यो नारीजनैर्भगः ।
कन्यागारे नागवल्ली देवो विश्वस्थली भृगुः ॥ ६ ॥
तत्परादिकलान्तास्तु महाज्या माधवोदिताः ।
स्वस्वपूर्वविशुद्धे तु शिष्टास्तत्खण्डमौर्विकाः ॥ ७

The sine table as given by Madhava and the modern sine values are given in tabular form for the reader to compare and understand.

The quarters of the first six verses represent entries for the twenty-four angles from 3.75° to 90° in steps of 3.75° (first column). The second column contains the R-sine values encoded as Sanskrit words (in Devanagari). The third column contains the same in ISO 15919 transliterations. The fourth column contains the numbers decoded into arc minutes, arc seconds, and arc thirds in modern numerals. The modern values scaled by the traditional "radius" (21600 ÷ 2π, with the modern value of π) with two decimals in the arc thirds are given in the fifth column.

The last verse means: "These are the great R-sines as said by Madhava, comprising arc minutes, seconds and thirds. Subtracting from each the previous will give the R-sine-differences". By comparing, one can note that Madhava's values are accurate to the seventh decimal place.

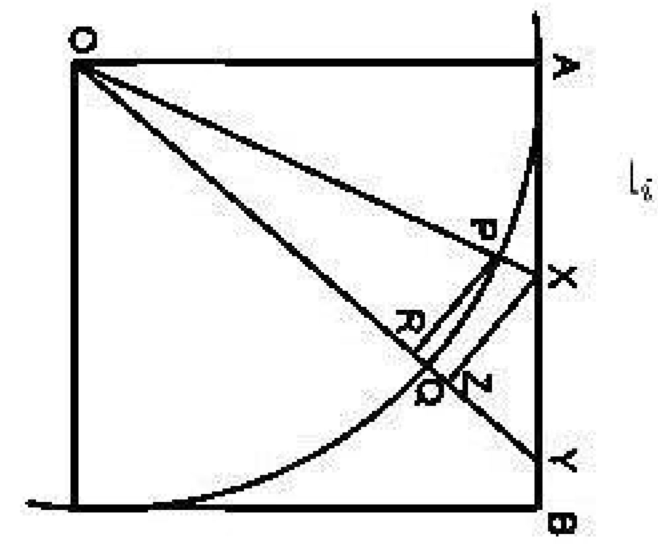
Angle A, degrees	R sin A given by Madhava			Modern sin A × (21600 ÷ 2π) to 2 decimals
	In Devanagari script	ISO 15919 transliteration	Decoded angle in minutes′ seconds″ thirds‴	
(1)	(2)	(3)	(4)	(5)
03.75	श्रेष्ठं नाम वरिष्ठानां	śreṣṭhaṁ nāma variṣṭhānāṁ	0224′50″22‴	0224′50″21.83‴
07.50	हिमाद्रिर्वेदभावनः	himādrirvēdabhāvanaḥ	0448′42″58‴	0448′42″57.58‴
11.25	तपनो भानुसूक्तज्ञो	tapanō bhānusūktajñō	0670′40″16‴	0670′40″16.05‴
15.00	मध्यमं विद्धि दोहनम्	madhyamaṁ viddhi dōhanam	0889′45″15‴	0889′45″15.61‴
18.75	धिगाज्यो नाशनं कष्टं	dhigājyō nāśanaṁ kaṣṭaṁ	1105′01″39‴	1105′01″38.94‴
22.50	छन्नभोगाशयाम्बिका	channabhōgāśayāmbikā	1315′34″07‴	1315′34″07.44‴
26.25	मृगाहारो नरेशोयं	mṛgāhārō narēśōyaṁ	1520′28″35‴	1520′28″35.46‴
30.00	वीरो रणजयोत्सुकः	vīrō raṇajayōtsukaḥ	1718′52″24‴	1718′52″24.19‴
33.75	मूलं विशुद्धं नाळस्य	mūlaṁ viśuddhaṁ nāḷasya	1909′54″35‴	1909′54″35.19‴
37.50	गानेषु विरळा नराः	gāneṣu viraḷā narāḥ	2092′46″03‴	2092′46″03.49‴
41.25	अशुद्धिगुप्ता चोरश्रीः	aśuddhiguptā cōraśrīḥ	2266′39″50‴	2266′39″50.21‴
45.00	शङ्कुकर्णो नगेश्वरः	śaṅkukarṇō nageśvaraḥ	2430′51″15‴	2430′51″14.59‴
48.75	तनुजो गर्भजो मित्रं	tanujō garbhajō mitraṃ	2584′38″06‴	2584′38″05.53‴
52.50	श्रीमानत्र सुखी सखे	śrīmānatra sukhī sakhē	2727′20″52‴	2727′20″52.38‴
56.25	शशी रात्रौ हिमाहारौ	śaśī rātrou himāhārou	2858′22″55‴	2858′22″55.11‴
60.00	वेगज्ञः पथि सिन्धुरः	vēgajñaḥ pathi sindhuraḥ	2977′10″34‴	2977′10″33.73‴
63.25	छाया लयो गजो नीलो	chāya layō gajō nīlō	3083′13″17‴	3083′13″16.94‴
67.50	निर्मलो नास्ति सत्कुले	nirmalō nāsti satkulē	3176′03″50‴	3176′03″49.97‴
71.25	रात्रौ दर्पणमभ्राङ्गं	rātrou darpaṇamabhrāṅgaṁ	3255′18″22‴	3255′18″21.58‴
75.00	नागस्तुङ्गनखो बली	nāgastuṅganakhō balī	3320′36″30‴	3320′36″30.20‴
78.75	धीरो युवा कथालोलः	dhīrō yuvā kathālōlaḥ	3371′41″29‴	3371′41″29.15‴
82.50	पूज्यो नारीजनैर्भगः	pūjyō nārījanairbhagaḥ	3408′20″11‴	3408′20″10.93‴
86.25	कन्यागारे नागवल्ली	kanyāgārē nāgavallī	3430′23″11‴	3430′23″10.65‴
90.00	देवो विश्वस्थली भृगुः	devō viśvasthalī bhṛ guḥ	3437′44″48‴	3437′44″48.37‴

The main agent of Madhava's paradigm shift was a precise idea of infinity. Numbers have no end. The infinitesimal method was invented to deal with "curvature" rigorously.

We give here the prototype problem for infinitesimal calculus for the interested reader to go through. Infinitesimal calculus was anticipated by the method of exhaustion whose prototype is the Euclid's Proportion on the area of the circle which is done by Inscribing regular polygon around the circle and to show that if it is not pi. r^2 then there exists a polygon of finite num of sides whose area inscribing the polygon will exceed the circle .When its squeezed from outside there exits a circumscribing circle whose area will be less than the circle, then the polygon with a finite number of sides will be considered, which is not calculus. This prototype problem is mainly solved by series of tangent.

Here we discuss about the circumference of a circle as a numerical multiple of diameter, more generally to describe the length of an arc as a function of its tangent.

The problem is not linear, since the relation between the half cord and the arc is not a linear relationship. Therefor we can approximate the length of the curve by a straight line segment, selectable chosen and neglecting the curvature



Geometrically,

From the figure , on considering the octant OAB .Then we get a tangent AB to determine the length of AB . Divide AB into n equal parts.

Now, let A_0 be A, A_n be B, x is A_i, y is A_{i+1} On drawing line connecting the center O with A_{i+1} ,.i.e. X and Y and on drawing perpendicular from X to OY and let it be N and the point where the circle meet at X, to OY and let it be R.

So we have the perpendicular PR and XZ. Therefore, PR is the sine of the angle POR which is the half cord of the arc and XZ is an auxiliary construction where the

$\triangle OPQ = \triangle OXZ$

$\triangle OAY = \triangle ZXY$

Then,

Sine θ_i = angle made by OY with respect to A

Sine θ_i = angle made by OX with respect to A

Then,

Sine $\delta\theta_i$ is the difference between the angles

ie, Sine $\delta\theta_i$ = PR

Since the $\triangle OPQ = \triangle OXZ$

$$PR = \frac{XY}{OX.OY}$$

$$\text{Sine } \delta\theta_i = \frac{\delta t}{d_{i-1} - d_i}$$

Where δt is the difference between AX and AY (length of XY)

d_{i-1} is the length of OX

d_i is the length of OY

On replacing, Sine $\delta\theta_i$ with $\delta\theta_i$ we get

$$\text{Sine } \delta\theta_i = \frac{\delta t}{d_{i-1} - d_i}$$
$$\delta\theta_i = \frac{\delta t}{d_{i-1} - d_i}$$

Where d_{i-1} *and* d_i are close to each other, when $\delta\theta_i$ is more so

$$\delta\theta_i = \frac{\delta t}{d_{i-1} - d_i} = \frac{\delta t}{d_i^2}$$

$$where\ d_i^2 = OA^2 + AY^2$$
$$= 1 + \frac{i^2}{n^2}$$
$$\delta\theta = \frac{\delta t}{1+t^2} = \frac{1}{n(1+\frac{i^2}{n^2})}$$

now we represent this in its finite integrate form

$$\sum_{i=1}^{n} \delta\theta_i = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{(1 + \frac{i^2}{n^2})}$$
$$= \frac{1}{n} \sum_{i=1}^{n} \left(1 - \frac{i^2}{n^2} + \frac{i^2}{n^2} - \ldots\ldots\right)$$

Where the length of the circle (1/8 th of the circle) can be written as $\frac{\pi}{4}$

$$\sum_{i=1}^{n} \delta\theta_i = \frac{\pi}{4}$$
$$= \lim_{n->\infty} \frac{1}{n} \sum_{i=1}^{n} \left(1 - \frac{i^2}{n^2} + \frac{i^2}{n^2} - \ldots\ldots\right)$$

thus,

$$\sum_{i=1}^{m} \delta\theta i = 1/n \sum_{i=1}^{m} \frac{1}{1+\frac{i2}{n2}} = \sum_{i=1}^{m} Ii$$
$$= 1/n \sum_{i=1}^{m} \left(1 - \frac{i2}{n2} + \frac{i4}{n4}\right)$$

$$\frac{1}{1+x} = 1-(1-\frac{1}{1+x})$$

$$\frac{\Pi}{4} = \lim_{n->\infty} 1/n \sum_{i=1}^{m} \left(1 - \frac{i2}{n2} + \frac{i4}{n4}\right) \quad - 1$$

$$\delta\theta = \delta t/1+t2 = \frac{1}{n\left(1+\frac{t2}{n2}\right)}$$

$$\sum_{i=1}^{m} \delta\theta i = I_n = 1/n \sum_{i=1}^{m} \frac{1}{1+\frac{i2}{n2}}$$

$I_{n,o} - I_{n,2} + I_{n,4}$

$I_{n,k} = 1/m^{k+1} \sum_{i=1}^{m} ik \qquad k=0,2,4,\ldots$

$$I = \lim_{n->\infty} 1/n \sum_{i=1}^{m} \frac{1}{1+\frac{i2}{n2}}$$

$I = \sum_{k=a+ai} \pm Jk \quad J_k = \lim_{n->\infty} 1/n^{k+1} \sum_{i=1}^{m} i_k$

$S_m{}^k = \sum_{i=1}^{m} i_k = n \sum_{i=1}^{m-1} i_k + \sum_{i=1}^{m-1} (i-n)i$

on considering the claim,

$$\sum_{i=1}^{n-1} (n-i)ik - 1 = \sum_{i=1}^{n-1} \sum_{j=1}^{i} jk - 1$$
$$\sum^{m-1} (m-i)ai = \sum_{i}^{m-1} \sum_{j}^{i} aj$$
$$\int_0^{\omega} (\omega - u) \ f(u)du = \int^{\omega} du \int^{u} f(u)du$$

now on defining,

Start with n=0 and do by induction

$S_m{}^k = nS_m{}^{k-1} - \sum_{i=1}^{n-1} S^{k-1}$

For k=1

$S^0 = i,\ \sum_{i=1}^{n-1} S^0 = \sum_{i}^{n-1} i = S_{n-1}$

$S_n{}^1 = n^2 - S_{n-1}{}^1$

${}^{2S}{}_m = n^2 \Rightarrow S_n{}^1 = 1/2\ n^2$

When units are added successively to a number , they (the results) will be higher and higher number starting with it [similarly for successive removal of units]. In this way all numbers get their indvidual identity (Swarupam, self form)

So by recalling the identities of increasing and decreasing numbers, we get the result of addition to and subtraction from each number. That will be evident if we reflect on it . If we know how to count forward and backward, we get the results of addiction and subtraction.

Calculus - Surface area of sphere :-

Madhava says that "…….. Through the middle of the sphere, imagine drawn 2 circles, one along east-west (equator) and the other along north-south (meridian). Then imagine circles, one slightly to the south and the other slightly to the north of the equator (latitudes). Their separation from the equator should be same for all parts. These two will be slightly smaller than the first one. Then imagine slightly smaller and smaller circles as described above, all of them at equal distance. Their separations along the meridian should be made equal. Now imagine that the circle shaped (annular) gap between two successive circles is cut at one place, removed and straightened. Then, of the two circles on either side of the gap, the bigger one will be the base (bhumi) and the smaller one the face (mukham) of a trapezium whose flanks will be the arc segment along the meridian of 2 successive latitudes. now cut out the part outside altitude, turn it upside down and joint it to the opposite flank. The result is a rectangle whose length is half the sum of the base and the face and whose width is the altitude. In this way think of all the gaps as rectangle. their widths are all equal. The lengths have varying measures. The widths being equal, add the lengths of all and multiply by the width. Thus will arise the area of sphere.

Now the methods to know how many gaps there are and what their lengths and widths are. The radii of the latitude circles are half chords of a circle of radius equal to the radius of the sphere ($r\sin\theta$, θ=colatitude)…………."

The area of hemisphere is

$$\int_0^{\frac{\pi}{2}} 2\,\pi\, r \sin\theta \, r \, d\theta = C.r$$

where c is circumference and r is radius

Notion and problem of infinity:-

Infinity is not greatly written in historical accounts of Indian mathematics. Recursion

played a big role in Indian mathematical thinking. recursion often leads to infinite series. infinity is used in getting to infinitesimal by dividing by large numbers and taking the limit.

Two Approaches - Making the computation of Pi efficient :-

Madhava wants to compute the value of pi. so he makes the computation of pi efficient.

$\pi/4 = 1 - 1/3 + \ldots \pm 1/j \pm 1/r(j)$: where j is a positive odd number

$= 1 - 1/3 + \ldots \pm 1/(j-2) \pm 1/r(j-2)$

= 1/r(j-2) + 1/r(j) = 1/j

Then he finds approximate rational solution of this equation by rational approximation.

About the function defined by Madhava

Each of the denominator's r(j-2) and r(j) is equal to twice the odd number j. The reason is that suppose the first denominator is double the odd number above it ie r(j-2) =2j. Then second denominator has to be double the next number ie r(j) =2(j+2). So there is no way in which both denominators can be equal to twice the odd number. This is how the approximate value of the remainder is found.

Now take r(j-2) = 2j-2 = (j-2)
So (j) = 2j+2
1/ (j-2) + 1/ (j) = 1/(j+1/j)
Let the error be ε1(j)=1/r(j-2) + 1/r(j) -1/j
= 1/ j^3 + ……

To get rid of one of the j^3 term , use recursive refining.
Write 1/r_2 (j)= 1/(2j+2+1/x(j))
Solving approximately,
1/r_2 (j) = 1/(2j+2+4/(2j+2))

Madhava gives a formula for , the next approximation which is in the form of a rational function of j written in Sanskrit verse. The great commentators makes a point that this complicated expression is actually the next term in a continued fraction expansion after these 2 terms. No book says that that the next correction should be in a continued fraction form.
So there is a tradition by which people remember that the next term is actually a continued fraction.
1/ r_3(j) = 1/(2j+2+ 4/(2j+2+16/(2j+2+…..)))
Now we can compute the third error ε3 . It will be proportional to 1/ j^7 . You will get 11 decimal places without any trouble. Madhava actually gives value of pi up to 11 decimal places .That is how the computation was done . This expression shows that pi is irrational because the expansion doesn't terminate. No book says which formula was used by Madhava.

Now let N = $a_n 10^n$+ $a_{n-1}10^{n-1}$ +….. a_0 :where ai belongs to Z/10
This is a principle which was already in rigveda . This is actually what decimal numbers are.
If you are measuring a number using 10 as standard(unit), This is exactly what a polynomial is. Instead of 10 use any j and coefficients can be zero, positive or negative integer. Then he says ,since decimal numbers are written in a certain way ,we can define a polynomial
p= | a_n |a_{n-1} |……..| a_0 |
He also says that the variable is 10 known as rashi . X takes the place of 10 in number representation and X represents the rashi . He gives the rules for cancellation in ratios in coefficient form.

Accelerating the rate of convergence of the π series:-

The rate of convergence of the basic π series is
1/r(j-2)+1/r(j)=1/j.
It can be solved in two methods . If we know the exact r value then by putting the value in the series we can simply solve it. It is the first method. And the second method

is for, If we don't know the exact r value. We have to simplify by assuming that in the limit nothing changes. For r_1 & r_2 by this series we can estimate up to 11 decimal places.

The principles of the decimal enumeration have been mustered by the time rigveda was written.

Article by - Dhruv S N
Abhirami
Vishnu Babu
Annie John
Akhila Abhilash
Meghana Chandra
Keerthana Pradeep

SLOKAS OF MADHAVACHARYA

-AKSHAY U

Madhava's Infinite Series that has been transformed into Madhava-Leibnitz Infinite Series after constant proofs provided that Madhava did this many centuries ago before Leibnitz was even born.Madhava actually devised two methods to calculate the values of any irrational number, improper fractions among other things.One of the ways is of a slowly converging function and another a fast converging one.

The Slow Method:

These two lines create a slow converging fraction that gives the value of pi to exactly 13 decimal places.

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते।

त्रिशरादिविषमसंख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात्।।

This method used by Madhava utilizes "bhoota-sankhya".

लोक: १] द्वितीयोऽध्यायः १

द्विघ्नौजसंख्या न्यूना च द्व्यधिकौ च पुरोदितौ ।
संस्कारहारौ स्यातन्ने द्विघ्नयुक्संख्यके¹ इमे ॥ २६८ ॥

सच्छेदीकरणद्विघ्नयुक्संख्योरथा कृतिर्हरः ।
चतुस्संख्यान्विताशोऽत्र द्विघ्नयुक्संख्यकोऽनयोः ॥ २६९ ॥

चतुस्संख्याभिरपवर्तनाद् युग्मार्धमंशकः ।
युग्मसंख्याकृतिः सैका च्छेदोऽप्यत्र यथोदितम् ॥ २७० ॥

[वृत्तपरिध्यानयनम्]

"व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते ।
त्रिशरादिविषमसंख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात् ॥ २७१ ॥

यत्संख्ययात्र हरणे कृते निवृत्ता हृतिस्तु जामितया ।
तस्या ऊर्ध्वगताया समसङ्ख्या तद्दलं गुणोऽन्ते स्यात् ॥ २७२ ॥

तद्वर्गो रूपयुतो हारो व्यासाब्धिघाततः प्राग्वत् ।
ताभ्यामाप्तं स्वमृणे कृते धने क्षेप एव करणीयः ॥ २७३ ॥

लब्धः परिधिः सूक्ष्मो बहुकृत्वो हरणतोऽतिसूक्ष्मः स्यात्" ॥ २७४ ॥

इति ।

Tantra-Samgraha, Adhyaya 2, Shlok 271

Bhoota-sankhya uses names of different elements to represent numbers (just like Katapayadi system uses different varna or alphabet combinations).

सागर for example represents 4 (an ocean surrounds land from all 4 sides). All synonyms (पर्यायवाची) of सागर like वारिधि, रत्नाकर etc mean the same.

Meaning of the shloka:

व्यासे (diameter) वारिधि (4) निहते *(अर्थ: संबद्ध, संलग्न* meaning multiply) रूप (1, everyone has unique form) हृते (divide) व्यास सागरा (4) भिहते।

त्रि (3) शर (5) आदि (etc) विषम (odd) संख्या भक्तम् (dividing) ऋणं (-) स्वं (+) पृथक क्रमात (alternatively) कुर्यात् (do)।

So the meaning of the first line becomes, "Four times the Diameter (in numerator) divided by one."

Mathematically represented as (4D/1)

In the second line it defines again four times the Diameter divided by a series of odd numbers starting with 3 having alternating plus and minus signs.

4D*(1/3–1/5+1/7–1/9…)

The second one is then subtracted from the first giving the value of pi.

This gives **Circumference**

The next is of the *Gitivrittam* measure:

Vyásé varidhinihité rúpa hrité vyásaságarábhihate
Trisaráddhi vishama sanchyá bhactamrinamswam prithacramát curyát
Yatsanchyayátra haraṇé critérnivrittá hritistujámitayá
Tasyá úrdhwa gatáyássamasanchyá taddalamgunóntésyát.
Jadvarggó rúpayutó háró vyásábdhighátacah prágwat.
Tabhyámáptam swamriṇé critédhané śódhananchacaraṇiyám
Súcshmah paridhissasyát bahucritwóharaṇatóti súcshmascha.

"Multiply the diameter by 4, and from it subtract and add alternately the quotients " obtained by dividing four times the diameter by the odd numbers 3, 5, 7, 9, 11, &c., " do thus to the extent required; and having fixed a limit, take half the even number " next less than the last odd divisor for a multiplier, and its square *plus* one for a divisor. " Multiply four times the diameter by the multiplier, and divide the product by the " divisor, and add it or subtract it, according to the sign of the last quote in the series, " from the sum of the series: thus the circumference of the given diameter will be " obtained very correctly."

The Fast Method:

The series however as said converges very slowly with the given method.

To ease the process and make the method fast another shloka was given.

The method is to force the series to converge at a desired point where the method becomes cumbersome.

The odd term where the process is to be stopped (due to boredom), we take the half of next even number as numerator.

That is if we stop at an odd number suppose p then we take (p+1)/2, where p+1 obviously becomes an even number.

The square of that (even number) added to unity is the denominator.

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \pm \frac{1}{p} \mp \frac{\left(\frac{p+1}{2}\right)}{(p+1)^2+1}$$

Fast converging fraction with correction

The addition in the end is like a correction term to definitely end the series on a specific point. This correction term is called अंत्य संस्कार: (meaning to end the process).

The sign of addition and subtraction in the correction term depends on the preceding term sign.

The result is quite accurate; in fact, more accurate than the one which may be obtained by continuing the division process (with a large number of terms in the series).

Yet another way to find Pi:

Another way of finding the value of pi given by Madhava is

विबुधनेत्रगजाहिहुताशनत्रिगुणवेदाभवारणबाहवः।
नवनिखर्वमितेवृतिविस्तरे परिधिमानमिदं जगदुर्बुधः।।

विबुध (33) नेत्र (2) गज (8) अहि (8) हुताशन(3) त्रि (3) गुण (3) वेदा(4) भ (नक्षत्र 27)
वारण (8) बाहु (2)।

नव (9) निखर्वम् (10^{11}) इते वृति (circle) विस्तरे परिधिमानम् (Circumference) इदं जगदुर्बुधः (as said by wise men).

The first line so stands as "2827433388233" as numbers are taken in opposite order.

येथें, स्वर् = २१ व धृति = १८ असल्यानें संवत् १८२१ निर्दिष्ट आहे आणि षट् = ६, नाग = ८, अंग = ६ व भू = १ असल्यानें शक १६८६ उल्लिखित आहे. दुसरें उदाहरण त्रिवेंद्रम् संस्कृत सीरीजमधील 'आर्यभटीयम्' या ग्रंथांतून घेतलें आहे (क्र. १०१, पृ. ४२). या ग्रंथाच्या टीकेंत वर्तुळ-परिधि व व्यास यांचें नातें सांगतांना एक श्लोक उद्धृत केला आहे तो असा—

विबुधनेत्रगजाहिहुताशनत्रिगुणवेदभवारणबाहवः ।
नवनिखर्वमिते वृतिविस्तरे परिधिमानमिदं जगदुर्बुधाः ॥

विबुध = ३३, नेत्र = २, गज = ८, अहि = ८, हुताशन = ३, त्रि = ३, गुण = ३, वेद = ४, भ = २७, वारण = ८, बाहु = २ असल्यानें पहिल्या ओळींवरून २८२७४३३३८८२३३ या संख्येचा बोध होतो.

Meaning of the shloka:
9 x 10^{11} व्यास वाले वृत्त की परिधि 2827433388233 होगी।
That is,
(2827433388233/9x10^{11}) = 3.141592653922 or value of pi.

Venuaroham

Venuaroham is one of the most important works of Madhava which describes the true positions (longitudes) of the moon in the sky. Information about the moon's correct position is needed since time for yajnas, poojas, etc. are calculated based on this knowledge. Calculations are based on the anomalistic cycle of the moon around the earth.

Anomalistic Cycle used by Sangamagrama Madhava

The fundamental lunar cycles in relation to the Earth are the Synodic cycle, which has a period of 29.5 days (New Moon to New Moon) and the Anomalistic cycle (perigee to perigee) which is 27 days 13hrs 18min 34.45s (about 27.5 days). Anomalistic cycles from a zero epoch will end respectively at cycle no., days, h, m, s as follows (for example the first cycle 1 ends at 27d , 13 h, 18min, 34.45 sec) :

1, 27,13 18 34.45
2, 55, 02,37, 08.90
3, 82, 15, 55, 43.35
4, 110, 05, 14, 17.79
5, 137, 18, 32, 52.24
6, 165, 07, 51, 36.69
7, 192, 21, 10, 01.14
8, 220, 10, 38, 35.59
9, 247, 23, 47, 10.04

Note that the difference between successive cycles is about 12 hours and alternate cycles is about 24 hours . This means that the successive durations are alternately corresponding to day-night, day-night difference.

Nine cycles constitute nearly 248 days and the difference in longitudes of successive days (delta lambda) constitute the chandravakyas developed by Madhava based on the katapayadi number system.

The series of vakyas begins from the moment when the moon is at apogee (Chandra thunga yogam) and each vakya corresponds to successive day's longitude of the moon.

Algorithm of Madhava

Position of the moon and the apogee coincide during a time called Dhruvam or dhruva kalam. The algorithm of Madhava is based on Computation of 9 dhruvas (D1, … D9) using Madhava's constant and Kalidina K. When position of the moon and the apogee coincide during a time, Dhruvam or dhruva kalam, from that point of time there will be 9 Chandra thunga yogam which can be computed as follows. For each moon revolution around the earth, the Chandra thunga yogam will get shifted by 3 degrees every day. To start with a reference point, the starting point will be when Chandra thunga yogam happens at sunrise (Suryodaya Madhya). How many days have completed when the Chandra thunga yogam takes place at Suryodaya Madhya. The number of days of this type is 188611 and the completed moon's orbiting around the earth is 6845. From this we can calculate the moon orbiting period as 188611/ 6845 = 27.5 days approximately (27 days 13h 18m 34.45s to be exact). Based on Madhava's algorithm, we can find that anomalistic cycles from a zero epoch will end respectively at Dhruvas B1 to B9 at 6460, 3411, 362, 4158, 1109, 4905, 1856, 5652, and 2603 respectively and corresponding chandravakya S1 to S9 as 0, 28, 56, 83, 111, 138, 166, 193, and 221respectively.

We can represent chandravakyas with corresponding Dhruvas as shown in the figure. Vakyas are arranged in ascending order while corresponding dhruvas are in descending order. The successive difference between chandravakyas is 28 appearing on the day side on the right and night side on the left. This is like branches of a bamboo stem (Venu) going up alternately left and right in a bamboo pole or venu pole and hence the method of Sangamagrama Madhava to evaluate position of Moon in the sky is known as VENUAROHAM.

CALCULUS AND KERALA – AN UNPOPULAR STORY

- SREELAKSHMI MURALI

The history of Parashurama Kshetram (Kerala) is not just confined to Kalaripayyattu, Ayurveda and Heritage, they hold centuries old of knowledge and heritage in lots of fields. The commercial impact Kerala had back in the times of Vasco Da Gama and even earlier was immense and beyond extraordinary which provided us strong connections with West Asia, Africa and Europe. The popularity of kerala as a commercial hub was specially mentioned in the 'Tabula Peutingeriana'. Due to this immense popularity, Kerala acted as a portal for knowledge transfer from India to the world.

Calculus, in simple words means rate of change, which further internally have two parts or subdivisions – differential and integral calculus. It got it's organic roots in Jyothisha Shastra along with Trignometry and Geometry. Talking about Calculus and Kerala, eventually leads to the topic of the great Madhava of Sangamagrama. He was the founder of Kerala school of Astronomy and Mathematics. Tantrasangraham, Yuktibhasha and Karanapadhathi were three manuscripts which was written in Malayalam font containing a lot of information regarding mathematical concepts and formulaes that dealt with various topics in Calculus. Charles M Whish, a British official who visited Kerala back in 1830s collected these manuscripts and went on to publish a paper named 'The Hindu Quadranture of the Circle', stating about the discoveries and conclusions he got from them. He translated shlokas, one of which dealt with calculating the value of 'pi' in an accurate, generalised and precise way with the help of an infinite series, which was later named as Madhava Series. In a similar way, Whish translated lots of shlokas into simpler form.

The mention of infinite series in the script led Whish undoubtedly doubt about the origin of Calculus, which was from Kerala taking the chronology of the work into consideration compared to the works of the famous Issac Newton.

Some key contributions from Kerala School of Mathematics in it's modern terminology includes the Newton Gauss Interpolation, Newton Power Series for Sine and Cosine, Taylor Series for Sine and Cosine Functions and Tycho Brahe's Reduction to Elliptic, all of which when observed under the chronological order was first mentioned in the manuscripts of Govindaswami, Madhava and Achyuta. The list also includes the L'Huilier's Formula for Circumradius of Cyclic Quardrilaterals, Leibniz Series for Inverse Tangent, Leibniz Series Series for Pi and the Approximations for value of Pi.

There exist a lot of rumours and assumptions which are being spread on how this knowledge got transferred from Kerala to other parts of the world. One maybe the knowledge going out of India through travellers which eventually got rebranded as if it is a local invention.

One of the prime example of this being the Pascal's Triangle which was discovered and mentioned multiple times even before Blaise Pascal was born. Another one being the possibility of development of knowledge parallelly and independently. One example being the reference of zero in various manuscripts from different parts of the world. Another one being the possibility where knowledge arrives from outside to India such as mentioned in 'Pancha Siddhantika' about Greek and Roman astronomy.

Going through a simple timeline from 1200 CE to 1800 CE, the time period of birth and development of Calculus. The events in this time period includes the destruction of Indian Universities, Europe's adoption of Hindu numeral system, Birth of Madhava, Black death Pandemic in Europe, Neelakanta authoring 'TANTRASANGRAHA', Vasco Da Gama's arrival, Introduction of Gregorian Calendar, Formulation of Kepler's Laws of Motion and Publication of Calculus by Newton and Leibniz independently. Presence of Jesuit Missionaries in Malabar Coast and their keen interest in local language signals the possible effect of works from Kerala in the development of Gregorian calendar which further led to the spread of other knowledge from Kerala to other parts of the world. The fair share of Calculus being organically developed in Europe is never to be ignored.

Summing up, the fact of Calculus being in India at least a few centuries before it's official publication in Europe is to be noted. Ignoring these discoveries as some national propaganda or pseudoscience which happens usually also maybe where we wrong. In the end, today's world which is mad behind the patent system needs to realise and accept that the credit should belong to the ones who deserves that.

AN EXCURSION THROUGH MATHEMATICS IN INDIA

-VISHNU BABU

Mahavirachrya on the all pervasiveness of Ganita

लौकिके वैदिके वापि तथा सामायिकेऽपि यः।
व्यापारस्तत्र सर्वत्र संख्यानमुपयुज्यते॥
कामतन्त्रेऽर्थशास्त्रे च गान्धर्वे नाटकेऽपि वा।
सूपशास्त्रे तथा वैद्ये वास्तुविद्यादिवस्तुषु॥
छन्दोऽलङ्कारकाव्येषु तर्कव्याकरणादिषु।
कलागुणेषु सर्वेषु प्रस्तुतं गणितं परम्॥
सूर्यादिग्रहचारेषु ग्रहणे ग्रहसंयुतौ।
त्रिप्रश्ने चन्द्रवृत्तौ च सर्वत्राङ्गीकृतं हि तत्॥

One of the standard codes of mathematics is started by saying that mathematics is important in all areas and concludes by saying that in the whole three worlds there is nothing that is not provided by mathematics whether it is astronomy, architecture or in conjunction granite, position, poetry , grammar i.e.
The number the diameter and the perimeter of the islands, oceans and mountains. The extensive dimensions of the row of habitats and holds belonging to the inhabitants of the world of the interfaces between the world and the world of light , the world of Gods and of the dwellers in hell and other miscellaneous measurements of all sorts, all these are understood by the help of Ganita.

In the configuration of living beings there is the length of their lives. There are eight attributes and other similar things, they say staying together dependent on Ganita.

Ganita : Indian mathematics of computation

गण्यते संख्यायते तद् गणितम्। तत्प्रतिपादकत्वेन तत्संज्ञं शास्त्रमुच्यते।

As noted by Ganesha Daivajna, in HIS commentary buddhi Vilasini (C.1540) on Lilavati (c.1150),Ganita(Indian mathematics) is the science (art) of computation. Indian mathematical text gives systematic and efficient procedures of calculation.
Here is the ancient rule of squaring as cited by Bhaskara (c.629 AD)

अन्त्यपदस्य वर्गं कृत्वा द्विगुणं तदेव चान्त्यपदम्।
शेषपदैराहन्यात् उत्सार्योत्सार्य वर्गविधौ॥

In the process of calculating the square, the square of the last digit is found (and placed over it). The rest of the digits are multiplied by twice the last digit (and the result placed over them). Then (on omitting the last digit), moving the rest by one place each the process is represented again and again.
An example: To calculate 125^2

1 5 6 2 5	
25	5^2= 25
20	5^2=4. 5.2.2 =20
4	1^2=1. 2.2.1=4 5.2.1=10
1 4 10	
1 2 5	

The ancient rule for squaring uses $\frac{n(n+1)}{2}$ multiplications for squaring and an n-digit number.

The modern word algorithm derives from the medieval word algorithm which referred to the Indian method of calculation based on the place value system. The word algorithm itself is a corruption of the name of the Central Asian mathematician al Khwarizmi (c. 825) whose,
Hisab al Hindi was the source from which the Indian methods of calculation reached the western world.

Sastras: Present Systematic Procedures

The technical rules of India provides systematic rule of procedures rather than a set of proposition, one of them is the Astradhaya by Panini.
Panini's Astadhyaya is acknowledged by the paradynamic example of canonical text in Indian tradition. All other disciplines, especially mathematics, have been deeply influenced by its ingenious symbolic and technical devices, recursive and generative formalism and the system of conventions governing rule application and rule interaction. In recent times it has a deep influence on modern linguistics too.

Panini and Euclid

In Euclid's geometry propositions are derived from axioms with the help of logical rules which are accepted as true. In Panini's grammar linguistic forms are derived from grammatical elements with the help of rules which were formed ad hoc (sutras).
Historically speaking, Panini's method has occupied a place comparable to that held by Euclid's method in western thought. Scientific developments have therefore taken different directions in India and in the West.

In India, Panini's perfection and ingenuity have rarely been matched outside the realism of linguistics. Just as Plato reserved admissions to his Academy for Geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics.

The word double quotes derived means demonetized in the case of Euclidean geometry; it means generated in the case of Panini's grammar.
Panini actually generates valid occurrence of Sanskrit without proof, whereas Euclid demonstrates the proof of theorems from a set of postulates.

Development of Indian mathematics

Ancient Period (prior to 500 BCE)

- Sulvasutras, the oldest text of geometry. They give procedures for construction and transformation of geometrical figures, sand alters using rope (rajju) and gnomon (sanku).

- The ancient astronomical siddhantas are from the period.

Early classical period

- Pervasive influence of the methodology of Panini
- Pingala's chandahsutra (c.300 BCE) and the development of binary representation and combinatorics
- Mathematical ideas in Bauddha and Jaina Texts
- The nation of zeros and the decimal place value system
- Mathematical and astronomy is the most standard procedures in arithmetic, algebra and geometric

Baudhayana-sulvasutra (prior to 800 BCE)

- Using of measurements (bhumiparimana)
- Marking directions and construction of a square of a given side (samacaturasra-karna)
- Construction of a rectangle and isosceles trapezium of given sides
- Construction of $\sqrt{2}$ (Dvikarani), $\sqrt{3}$ and ($\frac{1}{\sqrt{3}}$) times a given length
- The square of the diagonal of a rectangle is the sum of the squares of its sides (bhuja-koti-karna-nyaya-Oldest Theorem in Geometry)

दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत् पृथग्भूते कुरुतस्तदुभयं करोति।

Construction of squares which are the sum and difference of two squares.

- Transforming a square into a rectangle, isosceles trapezium, isosceles triangle and a rhombus of equal area and vice versa.
- Approximate conversion of a square of side a into a circle of radius,

$$r \approx \left(\frac{a}{3}\right)(2+\sqrt{2}). \quad [\pi \approx 3.0883]$$

- An approximation for $\sqrt{2}$ (dvikarani)

$$\sqrt{2} \approx 1 + \frac{1}{3} - \frac{1}{3.4} - \frac{1}{3.4.34} = 1.4142156$$

- Positions, relative distance and areas of altars. Shapes of different alters and their construction

Katyayana-Sulvasutra

To construct a square which is n-times a given square

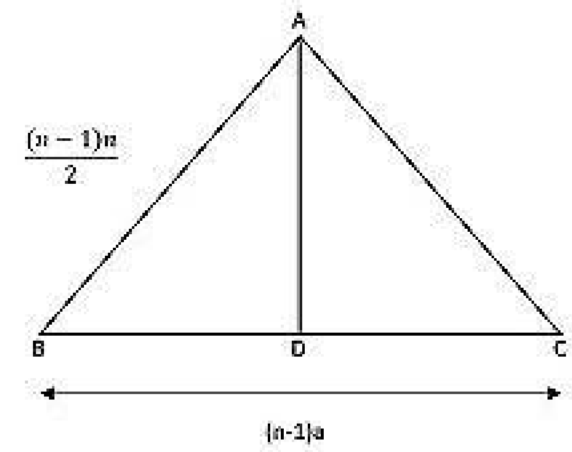
यावत्प्रमाणानि समचतुरश्राण्येकीकर्तुं चिकिर्षेत् एकोनानि तानि भवन्ति तिर्यक् द्विगुणान्येकत एकाधिकानि। त्र्यस्रिर्भवति तस्येषुस्तत्करोति। (कात्यायनशुल्वसूत्रम् ६.७)

As many squares as you wish to combine into one, the transverse line will be one less than that. Twice aside will be one more than that . That will be the triangle, it arrow (altitude) will produce that

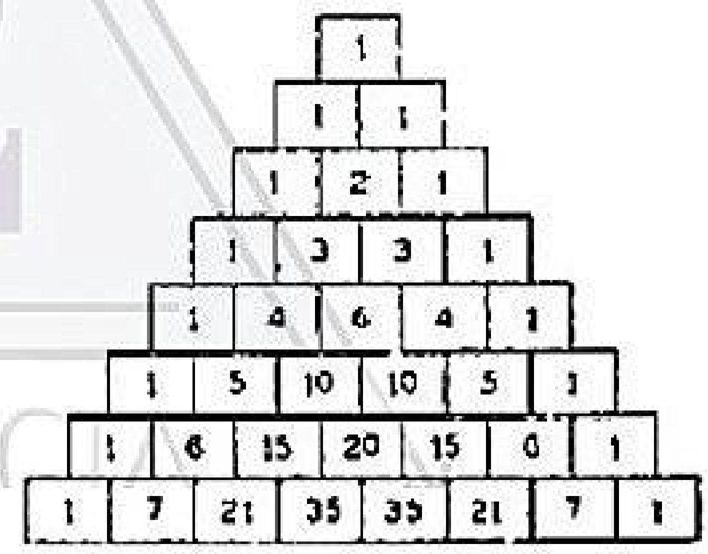
$$AD^2 = AB^2 - BD^2$$

$$= \left[\frac{(n+1)a}{2}\right]^2 - \left[\frac{(n-1)a}{2}\right]^2$$

$$= na^2$$



Varna-Mera of Pingala



The number of metrical forms with r Gurus in the prastram of metres of n-symbols in the binomial coefficient ${}^{n}C_{r}$

Decimal place value system

The Indian mathematicians developed the decimal place value system along with the notation of the zero-number.

The price value system is essentially an algebraic concept.

$5203 = 5.10^3 + 2.10^2 + 0.10 + 3$ is analogous t

to $5x^3 + 2x^2 + 0x + 3$

It is the algebraic technique of representing all numbers as polynomials of a base number, which makes all the calculations systematic and simple.

The algorithms developed in India for multiplication, division and evaluation of a square root, cube and cube root etc, have become the standard procedures, they have contributed immensely to the simplification and popularism of mathematics the world over. Sometimes, the Indian texts also discuss special techniques of calculation which are based on the algebraic formalism underlining the place value system.

Development of Decimal place value system

- The Yajurveda-Samhita talks of powers of 10 upto 10^2
- The Upanisads talk of zero and infinity.
- Panini's astadhayayn uses the idea of zero- morpheme.
- The Bauddha and Naiyayika philosophers discuss the nations of sunya and abhava

Gandhayukti of varahamihira

Chapter 76 of the great compilation Brhatsamhita of varahamihira (c.550) is devoted to a discussion of perfumery . In verse 20, Varaha mentions that there are 1820 combinations which can be formed by choosing 4 perfumes from a set of 16 basic perfumes C_4^n

षोडशके द्रव्यगणे चतुर्विकल्पेन भिद्यमानानाम्
अष्टादश जायन्ते शतानि सहितानि विंशत्या॥

16			
15	120		
14	105	560	
13	91	455	1820
12	78	364	1365
11	66	286	1001
10	55	220	715
9	45	165	495
8	36	120	330
7	28	84	210
6	21	56	126
5	15	35	70
4	10	20	35
3	6	10	15
2	3	4	5
1	1	1	1

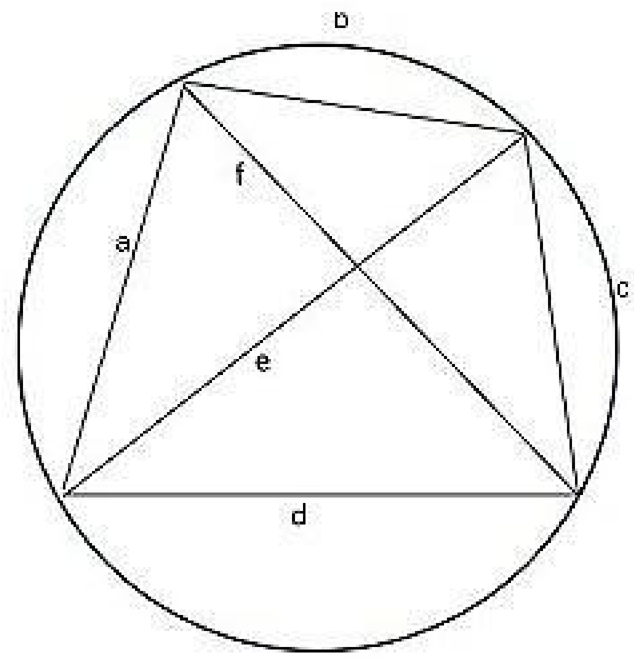
In the first column the natural numbers are written in the second column, there sums, in the third the sum of sums and so on. One row is reduced at each step the above table is based on the relation. The above meru is based on the relation

$$^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-2}C_{r-1} + \ldots\ldots + {}^{r-1}C_{r-1}$$

Ganitapada of Aryabhatiya

- Jyanayana: Computing table of Rsines
- Chaya-karma: Obtaining shadows of gnomons.
- Karnanayana: square of the hypotenuse is the sum pf the squares of the sides
- Saranayana: Arrows of intercepted arcs
- Sredhi-ganita: Summing an AP, finding the number of terms, repeated summations
- Mulaphalanayana: Interest and principal
- Trairasika: Rule of three

Brahmagupta's Formulae For Cyclic Quadrilaterals



The diagonals e,f are given in terms of the sides a,b,c,d, by the formulae

$$e=\sqrt{\frac{(ab+bc)(ac+bd)}{ab+cd}},\quad f=\sqrt{\frac{(ab+cd)(ac+bd)}{ad+bc}}$$

The area is given by

$$A=[(s-a)(s-b)(s-c)(s-d)]^{½} \qquad \text{With } s=\frac{(a+b+c+d)}{2}$$

Development of Indian Mathematics

- Ganitasarakaumudi of Thakkura Peru and other workers in regional languages such as Vyavaharaganita of Rajaditya and

Pavuluringanitamu of Pavuluri Mallana

- Ganitakaumudi and bijaganitavatamsa

Narayana Pandita (c.1350)

- Madhava, founder of the Kerala School.

Infinite series for sine and cosine functions and fast convergent approximations to them

- Works of paramesvara
- Works of Nilakantha somayaji

Narayana's Folding Method

To construct 4x4 square adding to 40
Samputikarana (folding) gives

2+15	3+10	2+0	3+5
1+0	4+5	1+15	4+10
3+15	2+10	3+0	2+5
4+0	1+5	4+15	1+10

=

17	13	2	8
1	9	16	14
18	12	3	7
4	6	19	11

Narayana also displays the other square which is obtained by interchanging the cover and covered.

This method leads to a pan diagonal magic square that is the broken dialogs also added up to the same magic sum.

History of Approximation to π

This approximation has been displayed in different form by different experts.
The table is given below.

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Rhind Papyrus - Egypt (Prior to 2000 BCE)	$\frac{256}{81} = 3.1604$	1	Geometrical
Babylon (2000 BCE)	$\frac{25}{8} = 3.125$	1	Geometrical
Śulvasūtras (Prior to 800 BCE)	3.0883	1	Geometrical
Jaina Texts (500 BCE)	$\sqrt{(10)} = 3.1623$	1	Geometrical
Archimedes (250 BCE)	$3\frac{10}{71} < \pi < 3\frac{1}{7}$	2	Polygon doubling ($6.2^4 = 96$ sides)
Ptolemy (150 CE)	$3\frac{17}{120} = 3.141666$	3	Polygon doubling ($6.2^6 = 384$ sides)
Lui Hui (263)	3.14159	5	Polygon doubling ($6.2^9 = 3072$ sides)
Tsu Chhung-Chih (480?)	$\frac{355}{113} = 3.1415929$ 3.1415927	6 7	Polygon doubling ($6.2^9 = 12288$ sides)
Āryabhaṭa (499)	$\frac{62832}{20000} = 3.1416$	4	Polygon doubling ($4.2^8 = 1024$ sides)

	Approximation to π	Accuracy (Decimal places)	Method Adopted
Mādhava (1375)	$\frac{2827433388233}{9.10^{11}}$ $= 3.141592653592\ldots$	11	Infinite series with end corrections
Al Kasi (1430)	3.1415926535897932	16	Polygon doubling (6.2^{27} sides)
Francois Viete (1579)	3.1415926536	9	Polygon doubling (6.2^{16} sides)
Romanus (1593)	$3.1415926535\ldots$	15	Polygon doubling
Ludolph Van Ceulen (1615)	$3.1415926535\ldots$	32	Polygon doubling (2^{62} sides)
Wildebrod Snell (1621)	$3.1415926535\ldots$	34	Modified Polygon doubling (2^{30} sides)
Grienberger (1630)	$3.1415926535\ldots$	39	Modified Polygon doubling
Isaac Newton (1665)	$3.1415926535\ldots$	15	Infinite series

Abraham Sharp(1699)	3.1415926535…	71	Infinite series for tan-1$\left(\frac{1}{\sqrt{3}}\right)$
John Machin(1706)	3.1415926535…	100	Infinite series relation $\frac{\pi}{4} = \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$
Ramanujan(1914), Gosper(1985)		17 Million	Modular Equation
Kondo,Yee(2010)		5 Trillion	Modular Equation

A History of Exact Result for π

Mādhava (1375)	$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \ldots$ $\pi/\sqrt{12} = 1 - 1/3.3 + 1/3^2.5 - 1/3^3.7 + \ldots$ $\pi/4 = 3/4 + 1/(3^3 - 3) - 1/(5^3 - 5) + 1/(7^3 - 7) - \ldots$ $\pi/16 = 1/(1^5 + 4.1) - 1/(3^5 + 4.3) + 1/(5^5 + 4.5) - \ldots$
Francois Viete (1593)	$\frac{2}{\pi} = \sqrt{[1/2]}\sqrt{[1/2 + 1/2\sqrt{(1/2)}]}\sqrt{[1/2 + 1/2\sqrt{(1/2 + 1/2\sqrt{(1/2)})}]}\ldots$ (infinite product)
John Wallis (1655)	$\frac{4}{\pi} = \left(\frac{3}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{4}\right)\left(\frac{5}{6}\right)\left(\frac{7}{6}\right)\left(\frac{7}{8}\right)\ldots$ (infinite product)
William Brouncker (1658)	$\frac{4}{\pi} = 1 + \frac{1^2}{2+}\frac{3^2}{2+}\frac{5^2}{2+}\ldots$ (continued fraction)
Isaac Newton (1665)	$\pi = \frac{3\sqrt{3}}{4} + 24\left[\frac{1}{12} - \frac{1}{5.32} - \frac{1}{28.128} - \frac{1}{72.512} - \ldots\right]$

James Gregory (1671)	$tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots$
Gottfried Leibniz (1674)	$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$
Abraham Sharp (1699)	$\frac{\pi}{\sqrt{12}} = 1 - \frac{1}{3.3} + \frac{1}{3^2.5} - \frac{1}{3^3.7} + \ldots$
John Machin (1706)	$\frac{\pi}{4} = 4tan^{-1}\left(\frac{1}{5}\right) - tan^{-1}\left(\frac{1}{239}\right)$

Ramanujan (1914)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4K)!(1103+26390k)}{(k!)^4 396^{4k}}$$

Ideas of calculus starter developing and they arise the context of astronomic idea for instantaneous velocity becomes important because, in order to understand the motion of moon, one need to know the rate of change of position .

Nilakantha's Formula for Instantaneous velocity

चन्द्रबाहुफलवर्गशोधितत्रिज्यकाकृतिपदेन संहरेत्।
तत्र कोटिफललिप्तिकाहतां केन्द्रभुक्तिरिह यद्य लभ्यते॥
तद्विशोध्य मृगादिके गतेः क्षिप्यतामिह तु कर्कटादिके।
तद्भवेत्स्फुटतरा गतिर्विधोः अस्य तत्समयजा रवेरपि॥

Nīlakaṇṭha gives the derivative of the second term above in the form

$$\left[\frac{\left(\left(\frac{r_0}{R}\right) R\cos(m-\alpha)\right)}{\left(R^2-\left(\frac{r_0}{R}\right)^2 R\sin^2(m-\alpha)\right)^{\frac{1}{2}}}\right]\left[\left(\frac{d}{dt}\right)(m-\alpha)\right]$$

Upapattis in Indian Mathematics

While there have been several extensive investigations on the history and achievements of the Indian Mathematics, there has not been much discussion on Indian mathematicians and philosophers understanding the nature and validation of mathematical results and procedures, their view on the nature of mathematical objects, and so on

Traditionally, such issues have been dealt in the detailed Bhasyas or commentaries, which continue to be written till recent times and played a vital role in the traditional scheme of learning. It is such commentaries that we find detailed Upapattis or proofs of the results and procedures apart from a discussion of methodological and philosophical issues.

Among the available texts of Indian mathematics a discussion of the way of validating the results (pratyayakarana) or of demonstrating them (upapatti) is found first in the Aryabhatiyabhasya of Bhaskara.

Upapatti and "proof"

The following are some of the important features of Upperpart's in Indian mathematics:

- The Indian mathematics are clear that results in mathematics, even those enunciated in authoritative texts cannot be accepted as valid unless they are supported by Yukti. It is not enough that one has merely observed the validity of a result in a large number of instances.
- Several commentaries written on major texts of Indian mathematics and astronomy present Umapati's for the results and procedures in on seated in the text.
- The Upapatti's are presented in a sequence proceeding systematically from known or established results to finally arrive at the result to be established.
- In the Indian mathematical tradition the Upapatti's mainly serve to remove doubts and obtain constant for the result among the community of mathematics.

- The Upapattits may involve observation or experimentation they also depend on the prevailing understanding of the nature of the mathematical objects involved.
- The method of "Tarka" or "proof by contradiction" is used occasionally. But there are no uppapattis which purport to establish existence of any mathematical objects merely on the basis of Tarka alone.
- The Indian mathematical tradition does not subscribe to the ideal that Upapattis should seek to provide irrefutable demonstrations establishing the absolute truth of mathematical results.
- There was no attempt made in the Indian mathematical tradition to prevent the Upapatti's in an axiomatic framework based on a set of self-evident or arbitrarily postulated axioms which are fixed at the outset.
- While Indian mathematicians made great strides in the invention and manipulation of symbols in representing mathematical results and in facilitating mathematical processes there was number attempt at formalisation of mathematics.

The Genius of Srinivasa Ramanujan (1887-1920)

In a recent article commemorating the 125th birthday of Ramanuja, Bruce Berndt was presented the following overall assessment of the results contained in his notebook (which record his work prior to leaving the England in 1914).

"Altogether, the notebook contains over 3000 claims, almost all without proof. Hartley surmised that over two-thirds of these results were rediscoveries. This estimate is much too high on the contrary, at least two third of Ramanuja's claims were new at a time that he wrote them and two-third more likely should be replaced by a large fraction. Almost all the results are correct perhaps no more than 5 of 10 are incorrect".

Ongoing work on his Notebook

The manuscript of Ramanujan discovered in the Trinity College Library by G.E Andrews in 1976 is generally referred as Ramanuja's lost notebook. This seems to pretend to work done by Ramanujan during 1919 to 1920 in India. This manuscript is about 100 pages with 138 sides of writing has around 600 results G.E Andrews and Berndt have embarked on a five-volume edition of all this material.

Ramanujan: Not a Newton but a Madhava

In 1913 Bertrand Russell had jocularly remarked about Hardy and Littlewood having discovered a "second Newton" in "Hindu clerk", if parallels are to be drawn Ramanujan may indeed be compared to the legendary Madhava.

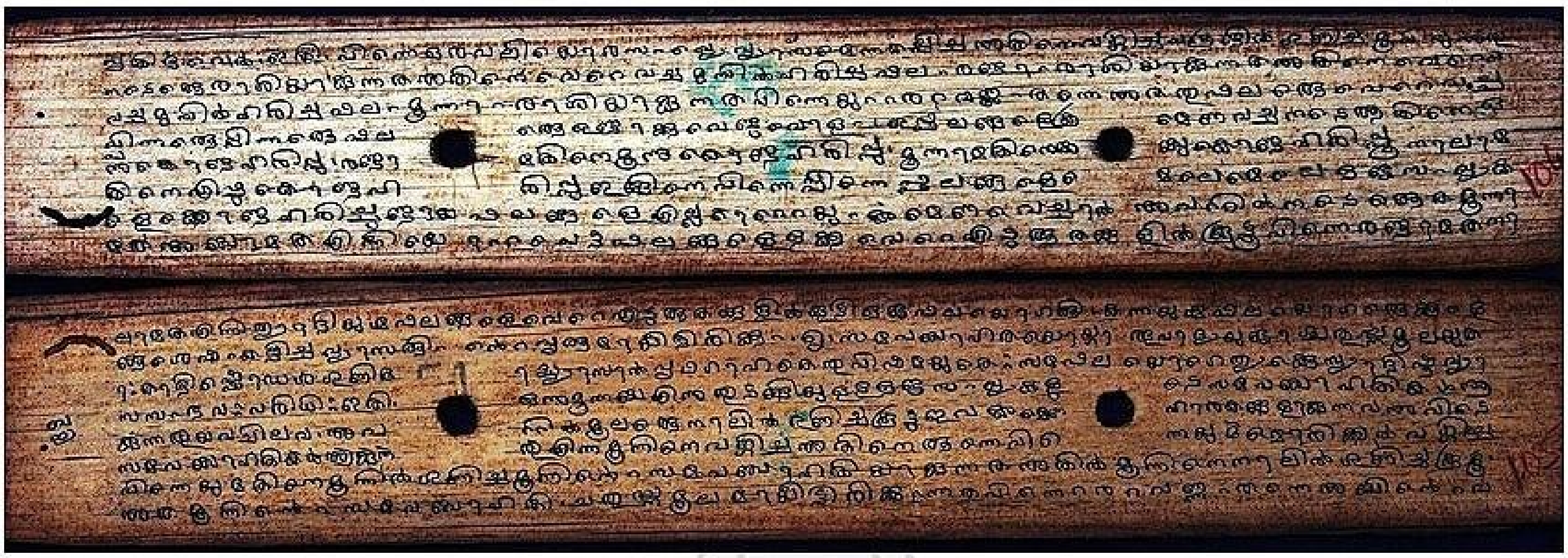
It is not merely in terms of his methodology and philosophy that Ramanujan is clearly in continuity with the earlier Indian traditions of
mathematics even in his extraordinary felicity in handling iterations, infinite series, continued fractions and transformations of them. Ramanujan is indeed a successor, a very worth one at that, of Madhava the founder of the Kerala School and a pioneer in the development of calculus.

Lessons from History

It is high time that the full story of Indian mathematics from Vedic times through 1600 become generally known . it is not minimizing the genius of the Greeks and their wonderful invention of pure mathematics, but other people have been doing math in different ways, and they have often attained the same goals independently. Rigorous mathematics in the Greek style should not be seen as the only way to gain mathematical knowledge. In India where concrete applications were never far from theory, justifications were more informal and mostly verbal rather than written. One should also recall the European Enlightenment was an ougy of correct and important but semirigorous math in which Greek ideals were forgotten. The recent episodes with deep mathematics following from quantum field theory and string theory teaches us the same lesson that the muse of mathematics can be wounded in many different ways and her secrets teached out of her .And so there were in India.

MATHEMATICAL TREASURE: THE TANTRASANGRAHA OF NILAKANTHA SOMAYAJI

-NANDANA JAYAKUMAR



The Kerala School of Mathematics and Astronomy flourished in Southern India in the fourteen and fifteenth centuries. Among its participants was the astronomer and mathematician Nilakantha Somayaji (1444-1544) who is credited with developing series expansions for the common trigonometric functions. His best known work, written on palm leaves, is called Tantrasangraha.

The treatise was completed in 1501 CE. It consists of 432 verses in Sanskrit divided into eight chapters. Tantrasamgraha had spawned a few commentaries: Tantrasamgraha-vyakhya of anonymous authorship and Yuktibhasa authored by Jyeshtadev in about 1550 CE. Tantrasangraha, together with its commentaries, bring forth the depths of the mathematical accomplishments the Kerala School of Astronomy and Mathematics , in particular the achievements of the remarkable mathematician of the school Sangamagrama Madhava. In his Tantrasangraha, Nilakantha revised Aryabhata's model for the planets Mercury and Venus. According to George C Joseph his equation of the centre for these planets remained the most accurate until the time of Johannes Kepler in the 17th century.

It was C.M. Whish, a civil servant of East India Company, who brought to the attention of the western scholarship the existence of Tantrasamgraha through a paper published in 1835. The other books mentioned by C.M. Whish in his paper were Yuktibhāṣā of Jyeshtadeva, Karanapaddhati of Puthumana Somayaji and Sadratnamala of Sankara Varman.

Nilakantha Somayaji, the author of Tantrasamgraha, was a Nanboothiri belonging to the Gargya gotra and a resident of Trikkantiyur, near Tirur in central Kerala. The name of his Illam was Kelallur. He studied under Damodara, son of Paramesvara. The first and the last verses in Tantrasamgraha contain chronograms specifying the dates, in the form Kali days, of the commencement and of the completion of book. These work out to dates in 1500-01.

Synopsis of the book

A brief account of the contents of Tantrasamgraha is presented below. A descriptive account of the contents is available in Bharatheeya Vijnana/Sastra Dhara. Full details of the contents are available in an edition of Tantrasamgraha published in the Indian Journal of History of Science.

- Chapter 1 (Madhyama-prakaranam): The purpose of the astronomical computation, civil and sidereal day measurements, lunar month, solar month, intercalary month, revolutions of the planets, theory of intercalation, planetary revolution in circular orbits, computation of kali days, mathematical operations like addition, subtraction, multiplication, division, squaring and determining square root, fractions, positive and negative numbers, computation of mean planets, correction for longitude, longitudinal time, positions of the planets at the beginning of Kali era, planetary apogees in degrees. (40 slokas)

- Chapter 2 (Sphuta-prakaranam (On true planets)): Computation of risings, and arcs, construction of a circle of diameter equal to the side of a given square, computation of the circumference without the use of square and roots, sum of series, sum of the series of natural numbers, of squares of numbers, of cubes of numbers, processes relating to R-sines and arcs, computation of the arc of a given R-sine, computation of the circumference of a circle, derivation of R-sines for given R-versed sine and arc, computation of R-sine and arcs, accurate computation of the 24 ordained R-sines, sectional R-sines and R-sine differences, sum of R-sine differences, summation of R-sine differences, computation of the arc of an R-sine according to Madhava, computation of R-sine and R-versed sine at desired point without the aid of the ordained R-sines, rules relating to triangles, rules relating to cyclic quadrilaterals, rules relating to the hypotenuse of a quadrilateral, computation of the diameter from the area of the cyclic quadrilateral, surface area of a sphere, computation of the desired R-sine, the ascensional difference, sun's daily motion in minutes of arc, application of ascensional difference to true planets, measure of day and night on applying ascensional difference, conversion of the arc of R-sine of the ascensional difference, etc. (59 slokas)

- Chapter 3 (Chhaya-prakaranam (Treatise on shadow)): Deals with various problems related with the sun's position on the celestial sphere, including the relationships of its expressions in the three systems of coordinates, namely ecliptic, equatorial and horizontal coordinates. (116 slokas)
- Chapter 4 (Chandragrahana-prakaranam (Treatise on the lunar eclipse)): Diameter

of the Earth's shadow in minutes, Moon's latitude and Moon's rate of motion, probability of an eclipse, total eclipse and rationale of the explanation given for total eclipse, half duration and first and last contacts, points of contacts and points of release in eclipse, and their method of calculation, visibility of the contact in the eclipse at sunrise and sunset, contingency of the invisibility of an eclipse, possibility of the deflection, deflection due to latitude and that due to declination. (53 slokas)

- Chapter 5 (Ravigrahana-prakaranam (Treatise on the solar eclipse)): Possibility of a solar eclipse, minutes of parallax in latitude of the sun, minutes of parallax in latitude of the moon,. maximum measure of the eclipse, middle of the eclipse, time of first contact and last contact, half duration and times of submergence and emergence, reduction to observation of computed eclipse, mid eclipse, non prediction of an eclipse. (63 slokas)

- Chapter 6 (Vyatipata-prakaranam (On vyatipata)): Deals with the complete deviation of the longitudes of the sun and the moon. (24 slokas)

- Chapter 7 (Drikkarma-prakaranam (On visibility computation)): Discusses the rising and setting of the moon and planets. (15 slokas)

- Chapter 8 (Sringonnati-prakaranam (On elevation of the lunar cusps)): Examines the size of the part of the moon which is illuminated by the sun and gives a graphical representation of it. (40 slokas)

Some noteworthy features of Tantrasamgraha

"A remarkable synthesis of Indian spherical astronomical knowledge occurs in a passage in Tantrasamgraha." In astronomy, the spherical triangle formed by the zenith, the celestial north pole and the Sun is called the astronomical triangle. Its sides and two of its angles are important astronomical quantities. The sides are 90° - φ where φ is the observer's terrestrial latitude, 90° - δ where δ is the Sun's declination and 90° - a where a is the Sun's altitude above the horizon. The important angles are the angle at the zenith which is the Sun's azimuth and the angle at the north pole which is the Sun's hour angle. The problem is to compute two of these elements when the other three elements are specified. There are precisely ten different possibilities and Tantrasamgraha contains discussions of all these possibilities with complete solutions one by one in one place.[9] "The spherical triangle is handled as systematically here as in any modern textbook."

The terrestrial latitude of an observer's position is equal to the zenith distance of the Sun at noon on the equinoctial day. The effect of solar parallax on zenith distance was known to Indian astronomers right from Aryabhata. But it was Nilakantha Somayaji who first discussed the effect of solar parallax on the observer's latitude. Tantrasamgraha gives the magnitude of this correction and also a correction due to the finite size of the Sun.

In his Aryabhatiyabhasya, a commentary on Aryabhata's Aryabhatiya, Nilakantha developed a computational system for a partially heliocentric planetary model in which Mercury, Venus, Mars, Jupiter and Saturn orbit the Sun, which in turn orbits the Earth, similar to the Tychonic system later proposed by Tycho Brahe in the late 16th century. Most astronomers of the Kerala school who followed him accepted this planetary model.

Conference on 500 years of Tantrasamgraha

A Conference to celebrate the 500th Anniversary of Tantrasangraha was organised by the Department of Theoretical Physics, University of Madras, in collaboration with the Inter-University Centre of the Indian Institute of Advanced Study, Shimla, during 11–13 March 2000, at Chennai. The Conference turned out to be an important occasion for highlighting and reviewing the recent work on the achievements in Mathematics and Astronomy of the Kerala school and the new perspectives in History of Science, which are emerging from these studies. A compilation of the important papers presented at this Conference has also been published.

SANKARA VARIAR

-ANILA LAKSHMANAN

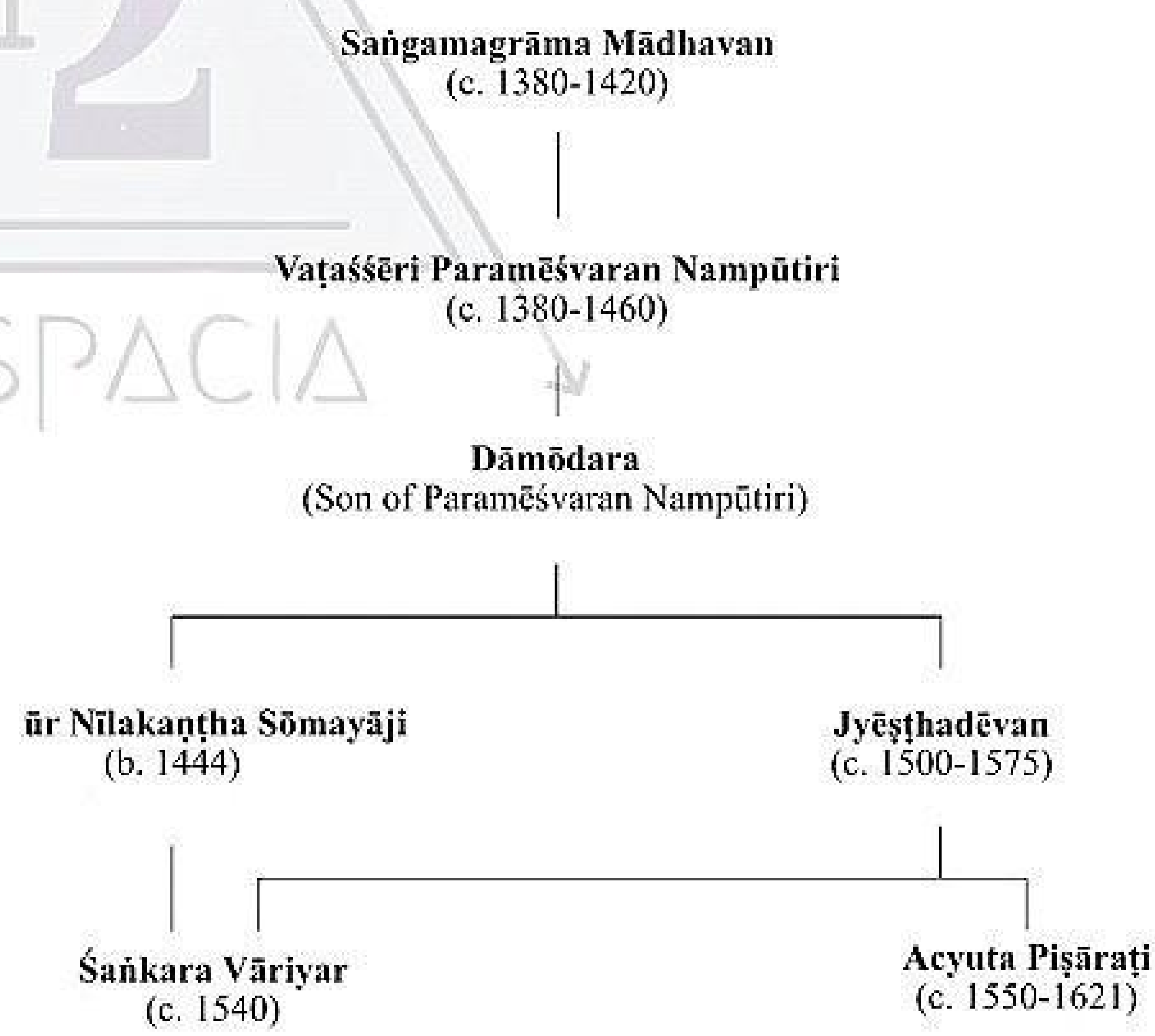
Sankara Variar (IAST: Śaṅkara Vāriyar; c. 1500 – c. 1560) was an astronomer-mathematician of the Kerala school of astronomy and mathematics. His family were employed as temple-assistants in the temple at Tṛkkuṭaveli near modern Ottapalam.He made significant contributions to the fields of mathematics and astronomy. Born in Kerala, India, in the early 20th century, Variar displayed exceptional talent from a young age.

In the realm of mathematics, Variar's work focused on number theory and algebra. His groundbreaking research delved into the intricate patterns of prime numbers, earning him recognition among his peers. Variar's innovative approaches to solving mathematical problems left an indelible mark on the academic landscape.

As an astronomer, Variar's interests extended to celestial mechanics and observational astronomy. He played a pivotal role in advancing our understanding of planetary motion and celestial bodies. His meticulous observations and calculations enhanced the precision of astronomical predictions, contributing to the overall accuracy of astronomical models.

Variar's academic journey took him to prestigious institutions, where he collaborated with leading scholars of his time. His publications in renowned mathematical and astronomical journals showcased the depth of his intellect and the relevance of his work in advancing these disciplines.

Beyond his scholarly pursuits, Variar was known for his dedication to education. He mentored numerous students, inspiring them to pursue careers in mathematics and astronomy. His passion for teaching and commitment to knowledge dissemination left an enduring legacy. In recognition of his outstanding contributions, Sankara Variar received accolades and awards, cementing his status as a luminary in the world of mathematics and astronomy. His life's work continues to inspire future generations of mathematicians and astronomers, leaving an indomitable imprint on the pursuit of knowledge.



Mathematical lineage

He was taught mainly by Nilakantha Somayaji (1444–1544), the author of the Tantrasamgraha and Jyesthadeva (1500–1575), the author of Yuktibhāṣā. Other teachers of Shankara include Netranarayana, the patron of Nilakantha Somayaji and Chitrabhanu, the author of an astronomical treaties dated to 1530 and a small work with solutions and proofs for algebraic e**quations.**

Works

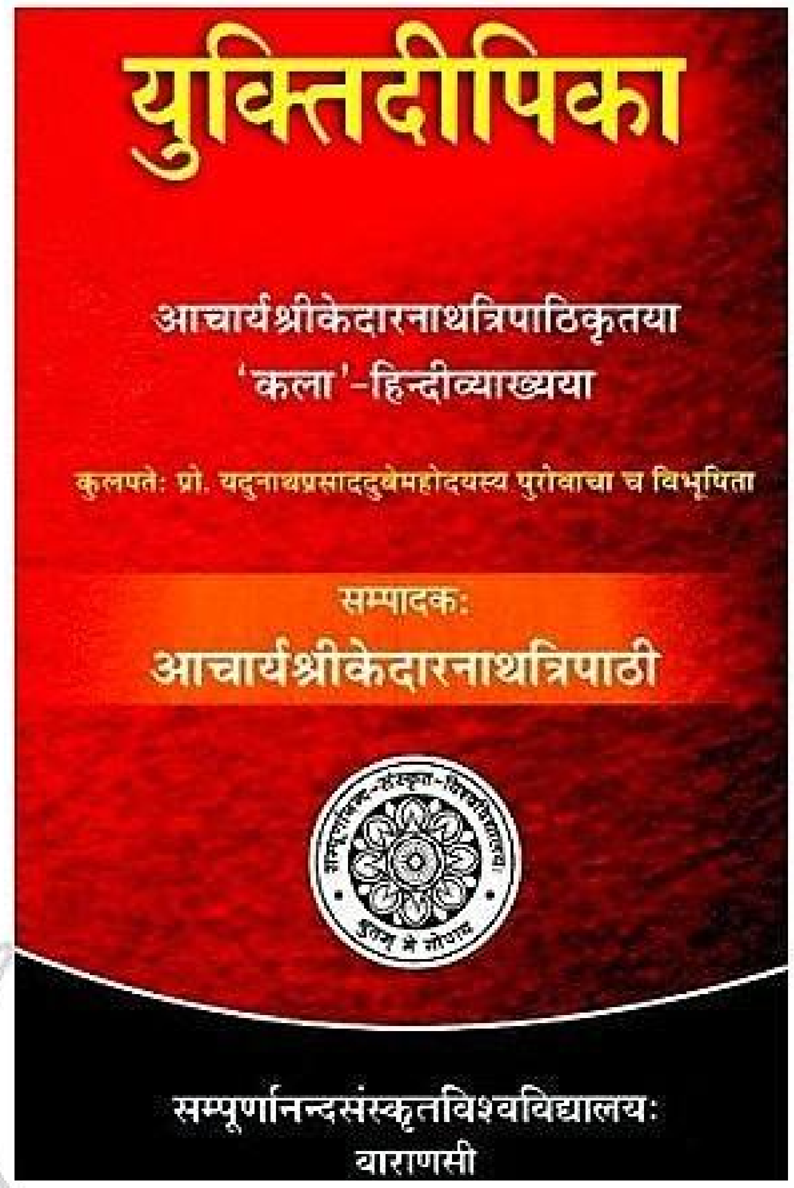
- **Yukti-dipika** - an extensive commentary in verse on Tantrasamgraha based on Yuktibhāṣā. Yukti-dipika is indeed an extensive commentary in verse on the Tantrasamgraha, a significant astronomical treatise.

The Tantrasamgraha is a major work in Indian astronomy, composed by the renowned Kerala school mathematician and astronomer Nilakantha Somayaji in the 16th century.

Yukti-dipika was written by Jyeshtadeva, who was a disciple of Nilakantha Somayaji. The commentary is based on Yuktibhāṣā, which is a set of mathematical and astronomical rules formulated by Jyeshtadeva himself. Yukti-dipika serves as an invaluable guide for understanding the complex calculations and principles presented in Tantrasamgraha.

Nilakantha Somayaji's Tantrasamgraha is divided into four parts: Golādhyāya (spherical astronomy), Grahādhyāya (planetary models), Gola-yātrā (solar and lunar eclipses), and Aṅgavidyā (mathematical techniques). Yukti-dipika extensively explains and elucidates the content of each section, providing insights into the mathematical reasoning and astronomical concepts involved.

Jyeshtadeva's commentary is highly



regarded for its clarity and systematic presentation. It plays a crucial role in preserving and disseminating the knowledge embedded in Tantrasamgraha. The Yukti-dipika not only helps scholars comprehend the intricacies of the original work but also stands as a testament to the intellectual achievements of the Kerala school of astronomy and mathematics during the medieval period in India.

The contributions of Nilakantha Somayaji and his disciples, including Jyeshtadeva, are essential in the history of Indian astronomy, and their works continue to be studied and respected by scholars and researchers interested in the development of mathematical and astronomical knowledge in the Indian subcontinent.

· **Laghu-vivrti - a short commentary in prose on Tantrasamgraha.**

Laghu-vivrti is a short commentary in prose on Tantrasamgraha, a significant Sanskrit astronomical work composed by Nilakantha

Somayaji, a mathematician and astronomer of the Kerala school in the 16th century. Nilakantha Somayaji's Tantrasamgraha is a comprehensive treatise that covers various aspects of astronomy and mathematical techniques.

Laghu-vivrti, as its name suggests, is a concise commentary that provides a simplified explanation of the content found in Tantrasamgraha. While Tantrasamgraha itself is detailed and complex, Laghu-vivrti offers a more accessible guide for those who may be beginners in the field of astronomy or mathematics. This short commentary is written in prose, making it easier for readers to follow and understand the concepts presented in the original work.

The purpose of Laghu-vivrti is to serve as an introductory text or a companion piece to Tantrasamgraha. It breaks down the intricate mathematical and astronomical principles, making them more approachable for students or scholars who may not have an advanced background in the subject matter. Through Laghu-vivrti, readers can gain a foundational understanding of the theories and calculations expounded in Tantrasamgraha.

The combined contributions of Tantrasamgraha and its commentaries, including Laghu-vivrti, have significantly enriched the field of Indian astronomy. Nilakantha Somayaji's work, along with the commentaries by subsequent scholars, reflects the mathematical sophistication and astronomical achievements of the Kerala school during a crucial period in the history of Indian science.

नीलकण्ठसोमयाजिविरचित:

तन्त्रसंग्रह:

अथ प्रथमोऽध्याय:

[मङ्गलाचरणम्]

हे विष्णो निहितं कृत्स्नं जगत् त्वय्येव कारणे।
ज्योतिषां ज्योतिषे तस्मै नमो नारायणाय ते।। १ ।।

[सावननक्षत्रदिनमानम्]

रवे: प्रत्यग्भ्रमं प्राहु: सावनाख्यं दिनं नृणाम्।
आर्क्षमृक्षभ्रमं तद्वत्, ज्योतिषां प्रेरको मरुत्।। २ ।।

भ्रमणां पूर्यते तस्य नाडीषष्ट्या मुहुर्मुहु:।
विनाडिकापि षष्ट्यंशो नाड्या गुर्वक्षरं तत:[1]।। ३ ।।

प्राणो गुर्वक्षराणां स्याद् दशकं चक्रपर्यये।
खखषड्घनतुल्यास्ते वायु: समजवो यत:।। ४ ।।

·Kriya-kramakari - a lengthy prose commentary on Lilavati of Bhaskara II.

Kriyakramakari (Kriyā-kramakarī) is an elaborate commentary in Sanskrit written by Sankara Variar and Narayana, two astronomer-mathematicians belonging to the Kerala school of astronomy and mathematics, on Bhaskara II's well-known textbook on mathematics Lilavati. Kriyakramakari ('Operational Techniques'), along with Yuktibhasa of Jyeshthadeva, is one of the main sources of information about the work and contributions of Sangamagrama Madhava, the founder of Kerala school of astronomy and mathematics.Also the quotations given in this treatise throw much light on the contributions of several mathematicians and astronomers who had flourished in an earlier era. There are several quotations ascribed to Govindasvami a 9th-century astronomer from Kerala.

Sankara Variar (c. 1500 - 1560), the first author of Kriyakramakari, was a pupil of Nilakantha Somayaji and a temple-assistant by profession. He was a prominent member of the Kerala school of astronomy and mathematics. His works include Yukti-dipika an extensive commentary on Tantrasangraha by Nilakantha Somayaji. Narayana (c. 1540–1610), the second author, was a Namputiri Brahmin belonging to the Mahishamangalam family in Puruvanagrama (Peruvanam in modern-day Thrissur District in Kerala).

Sankara Variar wrote his commentary of Lilavati up to stanza 199. Variar completed this by about 1540 when he stopped writing due to other preoccupations. Sometimes after his death, Narayana completed the commentary on the remaining stanzas in Lilavati.

· An astronomical commentary dated 1529 CE.

· An astronomical handbook completed around 1554 CE.

BOOKS

· Bashamakham

· Manikandeeyam

· Ende Kasiyathra

JYESTHADEVA

-ARSHA LEKSHMI S AND POOJASOMANADH

Jyesthadeva lived on the southwest coast of India in the district of Kerala. He belonged to the Kerala school of mathematics built on the work of Madhava, Nilakantha Somayaji, Paramesvara and others.

Jyesthadeva wrote a famous text Yuktibhasa which he wrote in Malayalam, the regional language of Kerala. The work is a survey of Kerala mathematics and, very unusually for an Indian mathematical text, it contains proofs of the theorems and gives derivations of the rules it contains. It is one of the main astronomical and mathematical texts produced by the Kerala school. The work was based mainly on the Tantrasamgraha of Nilakantha.

The Yuktibhasa is a major treatise, half on astronomy and half on mathematics, written in 1501. The Tantrasamgraha on which it is based consists of 432 Sanskrit verses divided into 8 chapters, and it covers various aspects of Indian astronomy. It is based on the epicyclic and eccentric models of planetary motion. The first two chapters deal with the motions and longitudes of the planets. The third chapter Treatise on shadow deals with various problems related with the sun's position on the celestial sphere, including the relationships of its expressions in the three systems of coordinates, namely ecliptic, equatorial nd horizontal coordinates. The fourth and fifth chapters are Treatise on the lunar eclipse and On the solar eclipse and these two chapters treata various aspects of the eclipses of the sun and the moon. The sixth chapter is On Vyatipata and deals with the complete deviation of the longitudes of the sun and the moon. The seventh chapter On visibility computation discusses the rising and setting of the moon and planets. The final chapter On elevation of the lunar cusps examines the size of the part of the moon which is illuminated by the sun and gives a graphical representation of it.

Jyeshthadeva's Works

Jyeṣṭhadeva is known to have composed only two works, namely, Yuktibhāṣā and Drkkarana. The former is commentary with rationales of Tantrasamgraha of Nilakantha Somayaji and the latter is a treatise on astronomical computations.

Three factors make Yuktibhāṣā unique in the history of the development of mathematical thinking in the Indian subcontinent:

- It is composed in the spoken language of the local people, namely, the Malayalam language. This is in contrast to the centuries-old Indian tradition of composing scholarly works in the Sanskrit language which was the language of the learned.
- The work is in prose, again in contrast to the prevailing style of writing even technical manuals in verse. All the other notable works of the Kerala school are in verse.

Most importantly, Yuktibhāṣā was composed intentionally as a manual of proofs. The very purpose of writing the book was to record in full detail the rationales of the various results discovered by mathematicians-astronomers of the Kerala school, especially of Nilakantha Somayaji. This book is proof enough to establish that the concept of proof was not unknown to Indian mathematical traditions.

ACHYUTA PISHARODI

-ARSHA LEKSHMI S AND POOJA SOMANADH

Achyuta Pisharodi (c.1550 at Thrikkandiyur (aka Kundapura), Tirur, Kerala, India – 7 July 1621 in Kerala) was a Sanskrit grammarian, astrologer, astronomer and mathematician who studied under Jyeṣṭhadeva and was a member of Madhava of Sangamagrama's Kerala school of astronomy and mathematics. He is remembered mainly for his part in the composition of his student Melpathur Narayana Bhattathiri's devotional poem, Narayaneeyam. He made significant contributions in area of spherical geometry and trigonometry.Aryabhatiyabhasa is celestial calculations and planetary motion.

Works

He discovered the techniques of 'the reduction of the ecliptic'. He authored Sphuta-nirnaya, Raasi-gola-sphuta-neeti (raasi meaning zodiac, gola meaning sphere and neeti roughly meaning rule), Karanottama (1593) and a four- chapter treatise Uparagakriyakrama on lunar and solar eclipses.

1.Praveśaka
An introduction to Sanskrit grammar.

2. Karaṇottama
Astronomical work dealing with the computation of the mean and true longitudes of the planets, with eclipses, and with the vyatūpātas of the sun and moon.

3. Uparāgakriyākrama (1593)
Treatise on lunar and solar eclipses.

4. Sphuṭanirṇaya
Astronomical text.

5. Chāyāṣṭaka
Astronomical text.

6. Uparāgaviṃśati
Manual on the computation of eclipses.

7. Rāśigolasphuṭānīti
Work concerned with the reduction of the moon's true longitude in its own orbit to the ecliptic.

8. Veṇvārohavyākhyā
Malayalam commentary on the Veṇvāroha of Mādhava of Saṅgamagrāma (ca. 1340–1425) written at the request of the Azhvanchery Thambrakkal.

9. Horāsāroccaya
An adaptation of the Jātakapaddhati of Śrīpati.

MELPATHUR NARAYANA BHATTATHIRI

-ARSHA LEKSHMI S AND POOJA SOMANADH

Melpathur Narayana Bhattathiri (1560–1666), third student of Achyuta Pisharati, was a member of Madhava of Sangamagrama's Kerala school of astronomy and mathematics. He was a mathematical linguist (vyakarana). His most important scholarly work, Prakriya-sarvasvam, sets forth an axiomatic system elaborating on the classical system of Panini. However, he is most famous for his masterpiece, Narayaneeyam, a devotional composition in praise of Guruvayurappan (Krishna) that is still sung at Guruvayur Temple.

Bhattathri was from a village named Melpathur at Kurumbathur in Athavanad Panchayat near Kadampuzha, very close to the Tirur River, as well as near to the holy town of Thirunavaya and Bharathappuzha, that was famed as the theatre of the Mamankam festival, in Malappuram district. He was born in 1560 in a pious Brahmin family, as the son of Mathrudattan Bhattathiri, a pandit himself. Bhattathiri studied from his father as a child. He learned the Rig Veda from Madhava, Tharka sastra (science of debate in Sanskrit) from Damodara and Vyakarana (Sanskrit grammar) from Achyuta Pisharati. He became a pandit by the age of 16. He married his guru Achuta Pisharati's niece and settled at Thrikkandiyur in Tirur.

He was one of the last mathematicians of the Sangamagrama school, which had been founded by Madhava in Kerala, South India.It flourished between the 14th and 16th centuries and the original discoveries of the school seems to have ended with Bhattathiri.

It seems that he had a younger brother named after his father (Mathrudattan Jr.,). One of the manuscripts of Narayaneeyam says that it was transcribed by the author's younger brother Matrdatta. The Melpathur family is now extinct and it is said that it was merged into the Maravancheri Thekkedathu family.

Narayaneeyam

Narayaneeyam is a Sanskrit devotional work in the form of a poetic hymn consisting of 1036 poems (called slokas in Sanskrit). It was written by Bhattathiri in 1586 AD and summarizes his 18,000 verses from the Bhagavata Purana. Mr. Pisharadi suffers from rheumatism. It is said that Bhattathiri, who could not watch his own pain, took on this disease himself and saved Gurudakshina through the power of yoga and Gurudakshina. Ezhutachan, a Malayalam poet and Sanskrit scholar, hinted, "Meen thotu kuotuka" (Let's start with the Fish) , a reference to the Avataras of Vishnu, to save Narayana

UNVEILING PI'S SECRETS WITH INFINITE SERIES

-LEKSHMI S

In the vibrant tapestry of mathematical history, Madhava of Sangamagrama stands tall as a pioneering astronomer and mathematician from Kerala. While his accomplishments span a vast array of fields, his contribution to the constant Pi remains a crowning achievement.

Unlike his predecessors who relied on geometrical approximations, like Archimedes polygon-based approach, Madhava dared to think differently. He embraced the power of infinite series, an endless sum of numbers converging to a specific value. Imagine dissecting a circle into infinitely many inifinitesimal slivers, each with its corresponding arc length. Madhava's stroke of genius lies in using an infinite series involving sine and cosine functions to express the arc length in terms of the circle's radius. That series is known as the Madhava- Leibniz series:

$$\frac{\Pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Each term, a fraction with an odd denominator, alternates between adding and subtracting, meticulously inching closer and closer to Pi's true value.

Madhava, employed up to 21 terms of this series, achieving an astonishing 11 decimal place accuracy for Pi, a feat unmatched for centuries. This accomplishment stands as a testament to his exceptional mathematical talent and his unwavering dedication to unlocking the secrets of the circle. But Madhava's genius didn't stop at mere calculation. He recognized the slow convergence of the series, a hurdle to reaching high levels of precision.

To overcome this he devised ingenious acceleration techniques like the 'Gregory Series'. These techniques paved the way for future mathematicians to push the boundaries of accuracy even further. Centuries later, European mathematicians like Gottfried Wilhelm Leibniz and James Gregory rediscovered his series, which formed the bedrock upon which calculus, the cornerstone of modern mathematics, was built.

So, the next time you gaze at the perfect circle of the moon, or trace the curves of a spinning wheel remember Madhava of Sangamagrama, the Indian mathematician who, with infinite ingenuity, demystified Pi and etched his name in the annals of mathematical history.

CROSSROADS OF MATHEMATICAL WISDOM: THE POSSIBLE TRANSMISSION OF KERALA MATHEMATICS TO EUROPE

-ANNIE JOHN

Nestled in the southern part of India, the Kerala school of mathematics emerged as a beacon of mathematical innovation during the medieval period. Led by luminaries like Madhava of Sangamagrama, this school made significant contributions to fields such as trigonometry, calculus, and algebra. As we explore the mathematical marvels of Kerala, we pose the question: Could these intellectual gems have traversed the vast distances to influence European mathematical thought?

The Kerala school of mathematics, known for its contributions to trigonometry and calculus, flourished in southern India during the medieval era. Here we investigates the hypothesis that the mathematical innovations of Kerala found their way to Europe, influencing the works of prominent European mathematicians. Mathematical techniques of great importance, involving elements of the calculus, were developed between the 14th and 16th centuries in Kerala, India. In this period Kerala was in continuous contact with the outside world, with China to the East and with Arabia to the West. Also after the pioneering voyage of Vasco da Gama in 1499, there was a direct conduit to Europe. The current state of the literature implies that, despite these communication routes, the Keralese calculus lay confined to Kerala.

According to the literature the general methods of the Calculus were invented independently by Newton and Leibniz in the late 17th century after exploiting the works of European pioneers such as Fermat, Roberval, Taylor, Gregory, Pascal, and Bernoulli in the preceding half century.

However, what appears to be less well known is that the fundamental elements of the calculus including numerical integration methods and infinite series derivations for π and for trigonometric functions such as sin x, cos x , (the so-called Gregory series) had already been discovered over 250 years earlier in Kerala. These developments first occurred in the works of the Kerala mathematician Madhava and were subsequently elaborated on by his followers Nilakantha Somayaji, Jyesthadeva, Sankara Variyar and others between the 14th and 16th centuries. In the latter half of the 20th century there has been some acknowledgement of these facts outside India. There are several modern European histories of mathematics which acknowledge the work of the Kerala school. However it needs to be pointed out that this acknowledgement is not necessarily universal.

The priority of Keralese developments in the Calculus over that of Newton and Leibniz is now beyond doubt. Madhava (1340-1425) is credited with the original ideas in the Keralese mathematics. These ideas led to derivation for the infinite series for p which we have illustrated earlier and to infinite series for a range of trigonometric functions. These developments in the calculus, therefore, precede the late 17th century calculus of Newton and Leibniz by at least 250 years. A communication route between the South of India and the Arabian Gulf (via the port of Basrah) had been in existence for centuries.

The arrival of the Portuguese Vasco da Gama to the Malabar coast in 1499 heralded a direct route between Kerala and Europe via Lisbon.

Thus, after 1499, despite its geographical location, which prevented easy communication routes with the rest of India, Kerala was linked with the rest of the world and, in particular, directly to Europe.

Whilst the two aspects of priority and communication routes are readily established, the existence of methodological similarities requires further discussion. Firstly there is a similarity in the approach to calculus in the Yuktibhasa and the approach to calculus adopted by Fermat, Pascal, Wallis, and others. In the Yuktibhasa the following key result is proved:

$$\lim_{n\to\infty} \frac{1}{n^{k+1}}\sum_{i=1}^{n} i^k = \frac{1}{k+1}, \quad k=1,2,3,\ldots \qquad (1)$$

This exact result was adopted by Fermat, Pascal, Wallis, and others in the 17th century to evaluate the area under the parabolas $y = x^k$, or ,equivalently, calculate $\int x^k \, dx$. At this point we remark that our conjecture of the transmission of Kerala mathematics is given credence by the fact that Wallis used reasoning similar to the ones given in the Yuktibhasa. That is, Wallis replaces the term n^2 by n(n + 1) implicitly implying that, as n tends to ∞, (n + 1) can be replaced by n so that he can use the approximation

$$n(n + 1) \approx n^2 \quad ^2.$$

Methodological similarities between the mathematics of Aryabhata school, upon which Kerala mathematics is based, and the works of the renaissance mathematicians are not infrequent.

The motivation for the import of knowledge from India to Europe arose from the need of greater accuracy in arithmetical computation [as is well attested by historian of mathematics], the calendar, and in astronomy. In astronomy we point to the attested remarkable similarities between the planetary model by the Kerala mathematician Nilakantha and the later one by Tycho Brahe and the adoption by Kepler of the 10th century Indian lunar model of the astronomer Munjala.

Jesuit missionaries played a crucial role in the transmission of mathematical knowledge from Kerala to Europe during the 16th and 17th centuries. Jesuit missionaries, particularly during the Age of Discovery, were in contact with various cultures through their global missions. In India, they encountered the advanced mathematical knowledge of the Kerala school. Recognizing the significance of the mathematical achievements in Kerala, Jesuit scholars engaged in translating Indian mathematical texts, particularly those related to trigonometry and calculus, into Latin.

The Jesuits were in communication with the Kshatriya kings of Cochin whose scholarship enabled them to be in very close proximity to the Kerala mathematics.The kings of Cochin came from the scholarly Kshatriya Varma 'Tampuran' family who were knowledgeable about the mathematical and astronomical works of medieval Kerala. This is attested bySrinivasiengar who state that the author of the Sadratnamala is Sankara Varma, the younger brother of Raja of Cadattanada near Tellicherry and further states that the Raja is a very acute mathematician.

Srinivasiengar further refers to the Malyalam History of Sanskrit Literature in Kerala which identifies the King of Cochin, Raja Varma, as being aware of the chronology of the Karana-Paddhati. Rama Varma Tampuran who, in 1948 (together with A.R. Akhileswara Iyer) had published an exposition in Malayalam on the Yuktibhasa was one of the princes of Cochin.

Ragagopal and Rangachari[1] state that Rama Varma Tampuran supplied them with the manuscript material relating to Kerala mathematics. Sarma identifies the valuable contribution to the analysis of Kerala astronomy by Rama Varma Maru Tampuran.

Mukunda Marar (who is the eldest son of the last king of Cochin) stated in a personal communication to the Aryabhata Forum that the kings of Cochin would have, at the least, been aware both of the astronomical methods for astrological prediction and of the manuscripts that contained these methods. Several were scholarly enough to publish commentaries of the mathematical and astronomical works.

Mukunda Marar, himself, worked with Rajagopal and published a work. Moreover, various authors, from Charles Whish in the 19th century to Rajagopal and Rangachari in the 20th have acknowledged that members of the royal household were helpful in supplying these manuscripts in their possession.

This suggests that the former royal family in Cochin, which was in possession of a large number of MSS, had not only a scholarly tradition, but also a tradition of helping other scholars. Thus, the royal family could itself have been a possible source of knowledge for the Jesuits.

Indeed the Jesuits working on the Malabar Coast had close relations with the kings of Cochin. Furthermore, around 1670, they were granted special privileges by King Rama Varma who, despite his misgivings about the Jesuit work in conversion, permitted members of his household to be converted to Christianity.

The close relationship between the King of Cochin and the foreigners from Portugal was cemented by King Rama Varma's appointment of a Portuguese as his tax collector[4]. Given this close relationship with the Kings of Cochin, the Jesuit desire to know about local knowledge, and the royal family's contiguity to the works on Indian astronomy, it is quite possible that the Jesuits may have acquired the key manuscripts via the royal household.

The possible transmission of Kerala mathematics to Europe emerges as a testament to the fluidity of knowledge, transcending geographical boundaries and enriching mathematical thought on a global scale. The crossroads of mathematical wisdom, where Kerala and Europe might have converged, beckon us to appreciate the universality of human curiosity and the enduring legacy of mathematical exploration.

The translated texts were integrated into the educational curriculum of Jesuit institutions in Europe. European scholars, including mathematicians and scientists, had access to these materials, exposing them to Kerala's mathematical innovations. Jesuit-educated mathematicians and scientists, such as Christopher Clavius, who was a significant figure in the Gregorian calendar reform, were exposed to the mathematical ideas from Kerala.

This exposure likely influenced their own contributions to mathematics. Jesuit missionaries maintained extensive correspondence networks, exchanging ideas and knowledge across continents.

This facilitated the dissemination of mathematical knowledge acquired in Kerala to various intellectual circles in Europe. The transmission of mathematical knowledge from Kerala contributed to the broader Scientific Revolution in Europe. The Jesuit involvement in disseminating not only mathematical but also scientific and astronomical knowledge had a lasting impact on the development of European science.

They played a pivotal role in the transmission of Kerala mathematics to Europe by actively engaging in the translation, dissemination, and integration of mathematical knowledge into the educational and intellectual landscape of Europe during the Age of Exploration and the Scientific Revolution.

ASTRONOMY AND MATHEMATICS IN ANCIENT KERALA

-KAJAL AND NADHA

Astrology (Jyothissaasthram) was popular in Kerala even in ancient times, and their deep knowledge in that branch of science is well-known. A number of great treatises (Granthhams) were written by several eminent scholars (most of them Namboothiri Braahmanans) of the area at different times. It is difficult to date some of the very ancient ones such as "Devakeralam", "Sukrakeralam" (also known as "Bhrigukeralam", "Kerala Rahasyam" or "Keraleeyam" and has 10 chapters), "Vararuchi Keralam" (or "Jaathaka Rahasyam" or "Kerala Nirnayam" - quite possibly authored by Vaakyam expert, Vararuchi), and "Keraleeya Soothram". It is said that a Kerala Braahmanan by name Achyuthan performed "Thapas" to Brihaspathi, who appeared and asked what favours he wanted.

He asked for Brihaspathi's condensed version of the 2,000 "Jaathaka Skandhham" portion of the Jyothissaasthram written by Sree Naaraayanan and comprising of four lakh obtained their 1,000 and 2,000 Granthhams respectively, and taught the Guru Matham, Sukra Matham and Saambasiva Matham to his disciples.

The 7th century (AD) witnessed tremendous development in Jyothissaasthram in Kerala. Siksha,Vyaakaranam,Niruktham,Jyothisham, Kalpam and Chhandovichithi are the six "limbs" (Shadaangams) of Vedam.

Jyothisham in those days was used for determining auspicious times for various Vaidika Karmams (religious rituals).Jyothissaasthram has three Skandhhams (branches) - Ganitham, Samhitha and Hora. In addition, it is seen to have six Angams (parts) - Jaathakam, Golam, Nimitham, Prasnam, Muhoortham and Ganitham. Of these, Golam and Ganitham are in Ganitha Skandhham, Jaathakam, Prasnam and Muhoortham in Hora Skandhham, and Nimitham in Samhitha Skandhham. Nimitham is partly covered in Hora also.

Some consider Jyothissaasthram to consist of two parts - Pramaanam and Phhalam with Ganitha Skandhham discussing the Pramaanam part and the other two Skandhhams, the Phhalam part. The former includes Soorya and Chandra Grahanams (solar and lunar eclipses), the Mouddhyam of Grahams (stars and planets), Chandra Sringonnathi (lunar cycles), and the Gathi Bhedams (changes in motion) of planets, and the methods of their prediction, and also descriptions of Bhoogola Khagolams (earth, planets and stars). Whereas, Jaathakam, Prasnam, Bhootha Sakunaadi Lakshanams (omens, etc.), Muhoorthams (auspicious days/ times), etc. are included in the Phhalam part. Of these, Jaathakam and Prasnam are extremely important.

Jaathakam (horoscope) involves predicting the good and the bad events during the entire life of a person based on the position of the planets and the stars at the precise time of his / her birth. Prasnam predicts the good and bad results for the subject again based on the planetary and star positions at the time of some special events/ tests proposed to be undertaken by the subject, usually the learned and the pious.

The connection between astronomy and mathematics has deep historical roots, dating back to ancient civilizations. The intertwining of these two disciplines has evolved over time, driven by the human pursuit to understand the celestial phenomena and navigate the skies.

Here's a brief overview of how astronomy became connected to mathematics:

1. Ancient Observations:
   - Early civilizations, such as the Babylonians, Egyptians, Greeks, and Indians, made systematic observations of the night sky. They noticed patterns and regularities in the movements of celestial bodies, prompting the development of basic mathematical tools to describe and predict these motions.

2. Geometry and Measurement:

- The ancient Greeks, particularly astronomers like Hipparchus and Ptolemy, contributed significantly to the mathematical foundation of astronomy. They developed geometric models to explain the apparent motions of the planets and introduced trigonometry to measure angles and distances in the sky.

3. Ptolemaic System:
   - Claudius Ptolemy, in the 2nd century AD, formulated a mathematical model known as the Ptolemaic system to describe the motion of celestial bodies. Although this geocentric model was later replaced by the heliocentric model proposed by Copernicus, the mathematical techniques developed by Ptolemy remained influential.

4. The Kepler's Laws:
   - In the 17th century, Johannes Kepler formulated his laws of planetary motion, which described the elliptical orbits of planets around the Sun. Kepler's laws were derived through careful analysis of observational data, and their formulation involved mathematical concepts such as geometry and algebra.

5. Newtonian Physics:
   - Isaac Newton's groundbreaking work in the late 17th century, including his laws of motion and the law of universal gravitation, provided a mathematical framework for understanding celestial mechanics. Newton's equations allowed astronomers to predict and explain the motions of planets and other celestial bodies.

6. Development of Calculus:

- Newton and Leibniz independently developed calculus in the same period, providing a powerful mathematical tool for describing changes and rates of change. Calculus became essential in formulating dynamic models of celestial bodies, allowing astronomers to study their motion and interactions.

7.Advancements in Observational Technology:

- As telescopes and other observational technologies advanced, the amount of data collected increased. Mathematics became crucial in analyzing and interpreting large datasets, leading to the development of statistical methods and data analysis techniques in astronomy.

8. Modern Astrophysics:

- The advent of 20th-century astrophysics brought a deeper understanding of the physical processes occurring in celestial objects. Mathematical models are now central to fields such as stellar astrophysics, galactic dynamics, and cosmology.

9. Predictive Power:

- The success of mathematical models in predicting celestial events, such as eclipses and planetary transits, further solidified the connection between astronomy and mathematics. The ability to predict astronomical phenomena accurately enhanced the practical applications of both disciplines.

The historical development of astronomy and its connection to mathematics reflects a mutual dependency. As astronomers sought to understand the complexities of the cosmos, they turned to mathematics to provide the language and tools needed for precise observation, modeling, and prediction. Today, the relationship between astronomy and mathematics continues to thrive, with advanced mathematical techniques playing a crucial role in our exploration of the universe.

Relationship between astronomy and mathematics

The relationship between mathematics and astronomy is highly interconnected and has been integral to the development of both fields. Here are some key aspects of their relationship:

1.Observational Precision:

- Mathematics is crucial in describing the precise positions, movements, and characteristics of celestial bodies. Using mathematical models, astronomers can predict the locations of planets, stars, and other objects in the sky with great accuracy, facilitating observational planning.

2.Celestial Mechanics:

- Mathematical principles, especially those of celestial mechanics, are fundamental to understanding the motion of celestial bodies. Newton's laws of motion and the law of universal gravitation form the basis for predicting the orbits and movements of planets, moons, and other celestial objects.

3.Coordinate Systems:

- Astronomy relies on various coordinate systems to specify the positions of celestial objects. Mathematical concepts such as spherical trigonometry and geometry are used to define coordinate systems like equatorial and ecliptic coordinates, enabling astronomers to locate objects in the sky.

4. Timekeeping and Calendars:

- Mathematical models are employed to develop accurate timekeeping systems and calendars based on astronomical events. The Earth's rotation, its orbit around the Sun, and the Moon's phases are all described mathematically to create calendars and measure time precisely.

5. Telescope Optics:

- Mathematical principles, particularly geometry and optics, are applied in the design and analysis of telescopes. Calculations involving lenses and mirrors help astronomers optimize the performance of telescopes for observing distant celestial objects.

6. Statistical Analysis:

- Astronomy often involves dealing with large datasets. Mathematical statistics is used to analyze observational data, determine the significance of results, and quantify uncertainties. Statistical methods help astronomers draw meaningful conclusions from their observations.

7. Astrophysical Modeling:

- Mathematical models are essential for understanding the physical processes occurring in space. Astrophysicists use mathematical equations to describe phenomena such as the fusion reactions in stars, the evolution of galaxies, and the behavior of matter in extreme conditions.

8. Computational Astronomy:

- Advanced mathematical algorithms and numerical simulations are employed in computational astronomy. These simulations model complex astronomical phenomena, enabling scientists to study scenarios that are difficult or impossible to observe directly.

9. General Relativity:

- Einstein's theory of general relativity, a highly mathematical framework, has provided a deeper understanding of gravity and its effects on the fabric of spacetime. This theory is crucial for explaining phenomena like gravitational lensing and understanding the behavior of massive objects in space.

In summary, mathematics is an indispensable tool in astronomy, providing the language and methods for precise observation, analysis, and theoretical understanding of celestial phenomena. The close relationship between mathematics and astronomy has allowed scientists to explore and comprehend the vastness of the universe.

List of astronomers and mathematicians of Kerala school

This is a list of astronomers and mathematicians of the Kerala school. The region surrounding the south-west coast of the Indian subcontinent, now politically organised as the Kerala State in India, has a long tradition of studies and investigations in all areas related to the branch of śāstra known as jyotiṣa. This branch of śāstra, in its broadest sense, incorporates several subdisciplines like mathematics, astronomy, astrology, horary astrology, etc.

In Indian traditional jyotiṣa scholarship, there are no clear cut boundary lines separating these subdisciplines. Hence the list presented below includes all who would be called a jyotiṣa-scholar in the Indian traditional sense.

All these persons will be, most likely, well versed in the subdisciplines of mathematics and astronomy as well. The list is an adaptation of the list of mathematicians and astronomers compiled by K. V. Sarma. Sarma has referred to all of them as astronomers. K. V. Sarma (1919–2005) was an Indian historian of science, particularly the astronomy and mathematics of the Kerala school. He was responsible for bringing to light several of the achievements of the Kerala school. He was editor of the Vishveshvaranand Indological Research Series, and published the critical edition of several source works in Sanskrit, including the Aryabhatiya of Aryabhata. He was recognised as "the greatest authority on Kerala's astronomical tradition".Additional information about the persons mentioned in the list are available in books on the history of Malayalam literature and on the history of Sanskrit literature in Kerala.

Serialnumber	Name	Period	Works	Known for	Remarks
1	Vararuci I	4th century CE		Vararuci vākyas; introduction ofKaṭapayādi system for expressing numbers;father figure in the legend ofParayi petta panthirukulam	
2	Vararuci II		Kerala-dvādasa-bhāva-vākyani,Vārarucika,Jātakarahasya		Might be identical with Vararuci I.
3	Haridatta	c.650-700	Grahacāraṇinibandhana, Mahā-mārganibabndhana	Promulgation of parahita system in 683.	
4	Govindasvāmin	c. 800 - 850	Mahābhāskarīya Bhāshya,GovindakṛitiGovindapaddhati		Court astronomer of King Ravi Varma Kerala;teacher of Śaṅkaranārāyaṇa

5	Śaṅkaranārāyaṇa	825 - 900	Laghubhāskarīya	References to the presence of an astronomical observatory at Mahodayapuram (modern day Kodungallur).	Student of Govindasvāmin
6	Udayadivākara	11th century	A commentary called Sundari onLaghubhāskarīya		Commented on a work of Jayadeva
7	Acyuta I		Devakerala (also known as Keralajyotisṣa)		
8	Keralācarya	12th century	Kerala-saṁhitā,Keralīyapraśnamārga		
9	Vyāghrapāda		Aṅkaṇasāstra		A devotee of Lord Siva enshrined at Vaikom Temple in Central Kerala.

10	Kṛṣṇa	c. 1200	Cintājñāna		
11	Kṛṣṇa-śiṣya	c. 1200	Commentary on Hora;Prśn aphalaprāptk ālanirṇaya		A disciple of Kṛṣṇa
12	Suryadeva Yajvan	1191 - c. 1250			
13	Vidyāmādhav a		Muhūrtadarś ana		A member of the Tulu Brah min family of Nīlamana; six commentaries on Muhūrtad arśana have been identified.

14	Viṣṇu (of Nīlamana)		A commentary on Muhūrtad arśana		Son of Vidyāmādha va
15	Govinda Bhaṭṭatiri (of Talakkuḷam)	1237 - 1295	Dasādhyāyi, Muhūrtaratn a, Muhūrtap adavi	Progenitor of the Pazhūr Kaniy ār family of astrologers	Belongs to Alathiyur (Malappura m) village.
16	Tāmaranallūr	14th century	Muhūrtavidh i (Tāmaranall ūr Bhāāṣā)		The work uses a mixture of Malayala m and Sanskr it languages.
17	Nityaprakāśa Yati	14th - 15th century	Commentarie s on Hora, both in Malayalam and Sanskrit		

18	Kumāra Gaṇaka	14th - 15th century	Raṇadīpikā		
19	Rudra I	c. 1325 - 1400			Teacher of Parameśvara
20	Mādhava of Saṅgama grāma	c. 1340 - 1425	Golavada, Madhyamanayan aprakara, Venvaroha, Chan dravakyani	Discoverer of the infinite series expansions for sin x and cos x	Greatest mathematician-astronomer of the Kerala School
21	Parameś vara of Vaṭaśśeri (Parame śvara I)	c. 1360 - 1455	Drigganita (1430), Bhatadipi ka – Commentary on Āryabhaṭīya of Āryabhaṭ a I, Karmadipika – Commentary on Mahabhaskariya of Bhaskara I, Paramesvari – Commentary on Laghubhaskariya of Bhaskara I, Sidhantadipika – Commentary on Mahabhaskariyabhashya of Govindasvāmi, Vivarana – Commentary on Surya Siddhanta and Lilāvati, Gol adipika – Spherical geometry and astronomy (composed in 1443 CE), Grahanamandana – Computation of eclipses (Its epoch is 15 July 1411 CE.), Grahanavyakhyadipik a – On the rationale of the theory of eclipses, Vakyakarana – Methods for the derivation of several astronomical tables	Promulgato r of the Drigganita system.	A proponent of observational astronomy in medieval India who had made a series of eclipse observations to verify the accuracy of the computational methods then in use

22	Dāmodara of Vaṭaśreṇi (Dāmodara I)	c. 1410 - 1510		Teacher of Nilakantha Somayaji	Son of Parameśvara of Vaṭaśśeri
23	Ravi Nampūtiri	c. 1425 - 1500	Ācāradīpika' (a commentary on Muhūrtadīpika)		
24	Nīlakaṇṭha Sōmayāji	1444 - 1545	Tantrasamgraha, Golasara (Description of basic astronomical elements and procedures), Sidhhanta darpana, Candrachaya ganita, Aryabhatiya-bhashya (Elaborate commentary on Aryabhatiya), Sidhhantadarpana-vyakhya, Chandrachhayaganita-vyakhya, Sundaraja-prasnottara, Grahanadi-grantha, Grahapariksa krama	Tantrasamgraha (A comprehensive treatise on astronomy)	
25	Śankara of Keḷallūr	c. 1475 - 1575			Nīlakaṇṭha Sōmayāji's younger brother. The person entrusted with task of popularizing Aryabhatiya-bhashya of Nīlakaṇṭha Sōmayāji.

26	Citrabhānu	c. 1475 - 1550	Karaṇāmṛta (a manual on astronomical computation s)		Pupil of Nīlakaṇṭha Sōmayāji
27	Citrabhānu Śiṣya	c. 1500 - 1575	Bhācintāvali (a work on astrology)		
28	Nārāyaṇa I	c. 1500 - 1575	Laghuvivṛti on Pañcabod ha IV, Uparāga kriyākrama (eclipse computation s), two commentarie s on Līlavati, one short and the other five times longer than the short, both called Kriyā kramakarī a nd Karmadī pikā		
29	Śankara Vāriyar	c. 1500 - 1560	Laghuvivṛti (commentary on Tantrasa mgraha)		Disciple of Nīlakaṇṭha Sōmayāji and protege of Āzhvāñceri Taṃprakkaḷ

30	Jyeṣṭhadeva	c. 1500 - 1610	Yuktibhāṣā (in Malayalam), Gaṣitayukti bhaṣa (in Sanskrit), Dṛ kkaraṇa (in Malayalam)	Yuktibhāṣā	The name is probably the Sanskritised form of his personal name in the local Malayalam language. Pupil of Teacher of Dāmodar a of Vaṭaśreṇi an d teacher of Acyuta Piṣāraṭi.
31	Jyeṣṭhadeva-Śiṣya	c. 1550 - 1625	A metrical commentary on Tantrasa mgraha		Disciple of Jyeṣṭhadeva
32	Māttūr Naṃpūtiri-s: Puruṣottama I and Subrahmaṇy a I	c. 1475 - 1550	Muhūrtapad vī (a condensed work in about 40 verses dealing with the prescriptions of auspicious times for functions)	The book is highly popular as attested by the availability of several commentarie s on it.	Lived in Pāññāḷ village in Thrissur district

33	Nārāyaṇa of Kaṇvavastu: Nārāyaṇa II	15th century	Muhūrtadīpika (A comprehensive treatise in about 400 verses on the auspicious times for functions)		
34	Rudra Vāariyar: Rudra II	c. 1475 - 1550	Commentary on Varāhamihira's Hora , called Nauka or Viv arana, and Aṣṭamaṅgalapr aśna		Belongs to Deśamaṅga laṃ Vāriyaṃ
35	Śaṅkara of Mahiṣamaṅgal am : Śaṅkara III	1494 - 1570	Gaṅitasāra, Candra gaṅitakramaand Ay anacalanādi- gaṅita (all books on astronomy); Jatakak rama, Jātakasāra an d Praśnamāla (books on astrology). All these are composed in simple Malayalam poetry or prose. His works on Sanskrit include Jātakasāra, commentaries on Pañcabodha and Laghubhāskarīya.	Populariz ation of astronom ical works among the masses in Kerala.	Hails from Peruvanam village near Thrissur. Spent most of his life with his teacher Parameśvar an Poṭṭi of Vāzha- māveli house in Chengannu r.
36	Madhava of Iñcakkāzhvā : Mādhava II	c. 1500 - 1575	Praśnasāra (incorpor ates several local practices)		Hails from Ramamang alam near Muvvattup uzha

37	Acyuta Piṣāraṭi : Acyuta II	c. 1550 - 1621	Spuṭanirṇaya, Raśigoḷasp uṭānīti, Karaṇottama (ast ronomical computation), Uparagakr iyākrama (eclipse computations), Chāyāṣṭa ka (shadow computations); commentaries on Veṅvāroha and Sūryas idhanta	Enunciation, for the first time in Indian astronomy, of the correction called "Reduction to the ecliptic". This was introduced in Western astronomy by Tycho Brahe at about the same time.	Teacher of the poet and grammaria n Melpattūr Narāyaṇa Bhaṭṭa
38	Nīlakaṇṭ ha II	16th - 17th centurie s	Kanakkusāraṃ couched in maṇipravāḷaṃ style which is a mixture of Malayalam and Sanskrit languages. The work deals also with practices relating to grain transactions, house building, weighing og gold and silver, land tenure, masonry, ground measurement, etc.		
39	Nārāyaṇa III	date complet ely unknow n	Laghudarśini (a short work on astrology)		
40	Dāmodar a of Maṅgalaś reṇi (Dāmoda ra II)	c. 1575 - 1675	Praśnarīti, Līlāvati-vyākhyā		Hails from Kaṇṇāṭipp arampu in Chirakkal taluk in North Malabar

41	Iṭakramañce ri Nampūtiri	c. 1625 - 1700	Bhadradīpa-gaṇita (comp osed in maṇiprav āḷaṃ style)		Pupil of Dāmodara of Maṅgalaśreṇ i
42	Maṅgalaśreṇ ivipra-Śiṣya	17th century	Jyotiṣasaṃg raha (in Malayalam)		
43	Pāṇkkāṭṭu (or, Iṭakkāṭṭu) Namputiri	c. 1625 - 1725	Praśnamārga		Praśnamārga is the most popular and authoritative work on praśna in Kerala.
44	Iṭakkāṭṭu (or Eṭakkāṭṭu) Kukkuṇiyāḷ	c. 1675 0 1750	Praśnarīti		Pupil of Pāṇkkāṭṭu Namputiri

45	Rāma-śiṣya	17th century	An explanatory rendering in Malayalam verses of the Laghuhorā of Varāahamihira		
46	Puruṣottama II	c. 1650 - 1725	Uparāgapariccheda (computation of solar and lunar eclipses)		
47	Putumana Somayāji	c. 1660 - 1740	Karaṇapaddhati (co mprehensive text on astronomical computations), Nyāy aratna, Veṇvārohāṣt aka, Mānasagaṇitam , Jātakādeś	Authorship of Karaṇap addhati	A member of the Putuvana family of Śivapuram (Thissur). Kar aṇapaddhati has been commented in Sanskrit, Malayalam and Tamil. It manuscripts are available in Tamil and Telugu scripts also.
48	Vāsudevasva mi : Vāsudeva I		Kāaladīpa (treatise on natural astrology)		The manuscrpt of the book was procured from Punnattūrkoṭṭ a Mana, Koṭṭappaṭi, North Malabar

49	Śyāmaḷavaār aṇa Rāja		Commentary on Kāaladīp a of Vāsudevasva mi		Member of Punnattūrko ṭṭa Mana
50	Dāmodara of Bharadvāja Gotra : Dāmodara III		Muhūrtābha raṇa (treatise on auspicious times for functions)		Hails from Tṛpparaṛṛoṭ village in Malabar
51	Kṛṣṇa II		Commentary on Āryabhaṭī ya in Malayalam		

52	Keralīya-dvija		Malayalam commentary on Karaṇottama of Acyuta Piṣāraṭi		
53	Govinda-śiṣya		Commentary called Bālaprabodhini on Jatakapaddhati of Parameśvara of Vaṭaśśeṇi		Hails from Vaikkam
54	Veṇād Brāhmaṇa		Jātakodaya (work on astrology in 103 verses)		
55	Azhvāñceri Taṃprākkaḷ	c. 1725 - 1800	Eleven books in the form of adapted texts and commentaries covering the entire field of astronomy and astrology. Jyotiśśāstra-saṃgraha, Saṃgraha-sādhana-kṛiyā, Jātaka-sāara-saṃgraha, Jātakānīti-mārga, Phalasāra-samuccaya, etc.	Efforts to propagate interest in studies on Jyotiṣa among members of the Nampūtiri communit y	Personal name unknown

56	Vāsudeva of Vaḷḷimana : Vāsudeva II		A metrical commentary on Muhūrta padavi		Hails from Kaṇṇamaṅg alaṃ
57	Tuppan Nampūtiri of Iṭavaṭṭkkāt	c. 1725 - 1800	Muhūrtapad avi VI		Hails from Pāakoḍe in Kunnathuna d
58	Nārāyaṇa of Iṭavaṭṭkkāt : Nārāyaṇa IV	c. 1728 - 1800	Muhūrtapad avi VII		Younger brother of Tuppan Nampūtiri of Iṭavaṭṭkkāt
59	Parameśvara II		Commentari es called Pāram eśvarī on Varāahamihi ra's Horā		

60	Parameśvara, pupil of Śankara : Parameśvara III		Commentaries called Jātakacandrikā on Varāahamihira's Horā		
61	Bhaāradvāja-dvija	c, 1750 - 1800	Gaṇitayuktayah (rati onales of mathematical and astronomical procedures), Karaṇad arpaṇa (advanced manual on astronomical computations)		
62	Nārāyaṇa of Peruvanaṃ (Nārāyaṇa V)		Tantrasāra		
63	Kṛṣṇadāsa (Koccukṛṣṇa n Āāśan)	1756 - 1812	Pañcabodha VIII (Malayalam verse), Bhāṣājātakapa ddhati (Malayalam commentary on Jātakapaddhati of Parameśvara of Vaṭaśśeṇi, Kaṇkkuśās tram (mathematical procedures in Malayalam verse)		Born in the family of Neṭumpayil in Tiruvalla taluk.

64	Śankara of Muktisthala : Śankara IV	17th century	Mantrasāra, Sāmudra sāra, Ārūḍha-praśna, Lāñchana-śāstra (omens, palmistrym, astrological omens, etc.), Āyurpraśna (deal ing with life longevity, Aṣṭamaṅgal a, etc.)			Hails from Mukkola
65	Śaṅkara V		Jātakasāra (III), Praśn asāra (I) (both in Malayalam verse)			
66	Bhutanāthap ura-Somayāji		Praśnasāra (III)			
67	Śrīkumāra, son of Nīlakaṇṭha		Praśnāmṛta			

68	Nārāyaṇan Iḷayatu of Maccāṭ : Nārāyaṇa VI	1765 - 1843	Jyotiṣa-Bhāṣāvalī, Jātakādeśaratna		Patronised by Śaktan Tampurān
69	Parameśvara of Puradahanapura : Parameśvara IV	c. 1775 - 1839	Commentary called Varadīpikā on Muhūrtapadavi II of Māttūr Puruṣottama Naṃpūtiri		Member of Puradahanapura (Purayannur) family in Vaḷḷuvanāṭ
70	Śrīkaṇṭha Vāriyar of Veḷḷrakkaād		Commentary in Malayalam on Jātakapaddhati of Parameśara of Vaṭaśreṇi		The commentary is available in print.
71	Ghaṭigopa	c. 1800 - 1860	Two commentaries on Āryabhaṭīya, one in Malayalam and the other in Sanskrit		A devotee of God Padmanabha, the presiding deity of Thiruvananthapuram

72	Goda Varma, Vidvān Iḷaya Tampurān	1800 - 1851	Commentaries in Sanskrit on Gaṇitādhyāya of Bhāskarīya-gaṇita and the Goladhyāya of Siddhāntaśiromaṇi		Member of the royal family of Kodungallur
73	Śaṅkara Varma of Kaṭattanāṭ	1800 - 1838	Sadratnamālā	The computation of the value of the mathematical constant � correct to 17 decimal places	
74	Subrahmaṇya Śāstri	1829 - 1888	Agaṇita (procedures for computing the positions of planets for a thousand years		Hails from Nalleppalli in Chittur-Cochin
75	Subrahmaṇya of Kunnattu Mana : Subrahmaṇya II	c. 1850 - 1900		Commentary called Bhāvaprakāśika on the Muhūrtadarśana of Vidyāmādhava	Also known as Subrahmaṇyan Tirumunpu of Kunnattu Mana of Payyanur in North Malabar

76	Puruṣottaman Mūssatu : Puruṣottama III	c. 1850 - 1900	Praśnāyana (a comprehensive work in 1018 verses)		
77	Rāma Varma Koyittampurān	1853 - 1910	Jyotiṣapradīpa		
78	Rāma Vāriyar of Kaikkuḷaṅra	1833 - 1897	Sāmudrikaśāstra, Gaụḷiśāstra, commentaries on Horā and Praśnamārga		
79	A. R. Rajaraja Varma Koyittampurān	1853 - 1918	Karaṇapariṣkaraṇa, Pañcāṅgaśuddhipaddhati, Jyotiṣaprakāśna		

80	Vāsuṇṇi Mūsstu of Veḷḷānaśśeri	1855 - 1914	Commentary in Malayalam on Pañchabodha		
81	Punnaśśeri Nampi Nīlakaṇṭha Śarma	1858 - 1935	Jyotiśāstrasubod hini, Pañcabodha kriyā-Bhāṣa		Works published by Bharata Vilasam Press, Thrissur

In conclusion, the exploration of astronomy and mathematics in ancient Kerala unveils a rich scientific heritage that significantly contributed to our understanding of the Cosmos. The astronomers and mathematicians of ancient Kerala, such as Aryabhata II and Madhava of Sangamagrama, made profound contributions that continue to resonate in the annals of scientific history.

The mathematical legacy of Kerala's scholars is characterized by its innovative spirit and advanced methodologies. Notably, the development of infinite series, calculus-like methods, and intricate trigonometry exemplifies the sophistication of mathematical thought in this region. These mathematical concepts were not isolated but intricately woven into the fabric of astronomy, providing the foundation for accurate predictions of celestial events and advancements in observational techniques.

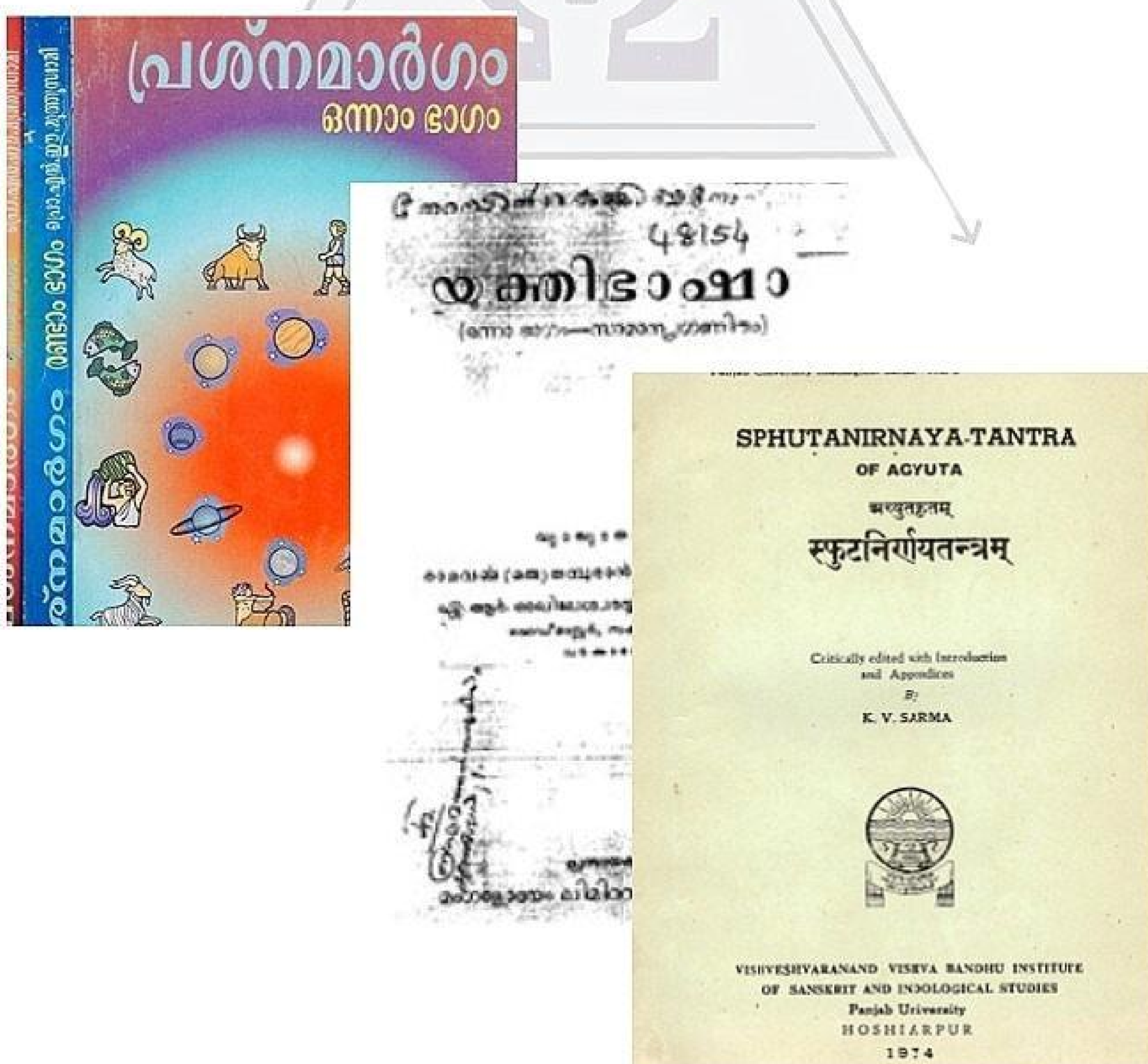
The astronomical achievements of ancient Kerala extend beyond theoretical frameworks. The meticulous observations conducted by Kerala astronomers, coupled with the application of mathematical models, allowed for precise predictions of planetary motion and the creation of sophisticated calendrical systems.

The practical integration of mathematics into maritime activities showcased the holistic understanding of the universe held by Kerala's scholars.

The celestial coordinate systems established in ancient Kerala, based on principles of geometry and trigonometry, stand as enduring contributions to the field of astronomy. These coordinate systems facilitated precise location tracking of celestial objects, marking a significant advancement in observational astronomy.
The transmission of knowledge from ancient Kerala to other regions played a pivotal role in shaping the broader landscape of scientific thought.
Kerala's mathematical and astronomical traditions influenced subsequent developments in mathematics, leaving an indelible mark on the scientific progress of India and beyond.

In essence, the confluence of astronomy and mathematics in ancient Kerala represents a harmonious integration of theoretical understanding and practical application.

The intellectual endeavors of Kerala's scholars exemplify a scientific tradition that transcends geographical boundaries and stands as a testament to the human pursuit of knowledge. As we reflect on the scientific legacy of ancient Kerala, we gain insights not only into the astronomical wonders of the past but also into the enduring and universal nature of mathematical principles that continue to shape our understanding of the cosmos.



ARYABHATIYA

-INT MSC MATHEMATICS 2022

FATHER OF INDIAN MATHEMATICS

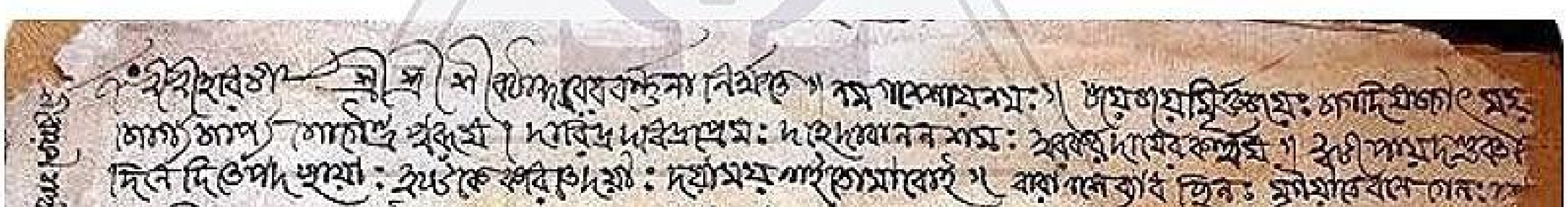
Aryabhatta was the first Indian mathematician, physicist and astronomer who created groundbreaking theories and inventions Aryabhatta was born in a small place called Aryabhata in Bihar during the Gupta dynasty. Aryabhatta worked out the value of pi which is used today by scientists and mathematicians all around the world. It was Aryabhatta who discovered the formula for the area of a triangle and the volume of the sphere which has given birth to various inventions and discoveries in the field of engineering today. Aryabhatta contributed to the development of new deductions and ideas in mathematics. He was the one who computed the estimation of pi, which he discovered to be 3.14, and his contribution to mathematics is unparalleled and cannot be overlooked.

The Aryabhatiya of Aryabhata is an ancient Indian work on mathematics and astronomy, written by the renowned mathematician and astronomer Aryabhata. It is the only known surving work in the 5th century. Philosopher of astronomy Roger Billard estimates that the book was composed around 510 CE. Aryabhatiya is considered one of the most important works on Indian mathematics and astronomy, and it has had a profound impact on the development of these fields in India and beyond. The book covers a wide range of topics, including arithmetic, algebra, trigonometry, geometry, and astronomy. It also includes discussions on the nature of the universe, the movement of the planets, and the calculation of eclipses. It is a brief descriptive work intended to supplement matters and processes which are generally known and agreed upon to give only the most distinctive features of Aryabhata's own system. Many common places and many simple processes are taken for granted. The book vividly addresses topics such as dashagitika, ganitapada, kalakriya and gola in much details. Withstanding many a criticism from people like Brahmagupta on the theories of Aryabhata, this volume through the introductory chapter contends that the Aryabhatiya, on the whole, is quite genuine. It presents Aryabhata as an innovator, thus his difference from Smriti or tradition in his approach to many astronomical matters is fully justified. It also discusses a serious internal discrepancy in the Aryabhatiya about the stationary and revolutionary nature of earth.

Aryabhatiya was particularly popular in South India, where numerous mathematicians over the ensuing millennium wrote commentaries. The work was written in verse couplets and deals with mathematics and astronomy. Following an introduction that contains astronomical tables and Aryabhata's system of phonemic number notation in which numbers are represented by a consonant-vowel monosyllable, the work is divided into three sections: Ganita ("Mathematics"), Kala-kriya ("Time Calculations"), and Gola ("Sphere").

Aryabhatiya ends with spherical astronomy in Gola, where he applied plane trigonometry to spherical geometry by projecting points and lines on the surface of a sphere onto appropriate planes. Topics include prediction of solar and lunar eclipses and an explicit statement that the apparent westward motion of the stars is due to the spherical Earth's rotation about its axis. Aryabhata also correctly ascribed the luminosity of the Moon and planets to reflected sunlight".

This book is an essential resource for anyone interested in the history of mathematics and astronomy, as well as the development of scientific thought in ancient India. This scarce antiquarian book is a facsimile reprint of the old original and may contain some imperfections such as library marks and notations. Because we believe this work is culturally important, we have made it available as part of our commitment for protecting, preserving, and promoting the world's literature in affordable, high quality, modern editions, that are true to their original work. It presents Aryabhata as an innovator, thus his difference from Smriti or tradition in his approach to many astronomical matters is fully justified. It also discusses a serious internal discrepancy in the Aryabhatiya about the stationary and revolutionary nature of earth. This book helps in introducing Aryabhata and the quintessential of Aryabhatiya to the mathematicians and astronomers of the new generations, for whom the original language Sanskrit and the old processes might be unknown.



LILAVATI

-AISHWARYA JAYAKUMAR

Bhaskara II, renowned ancient Indian mathematician and astronomer, made significant contributions to the fields of mathematics and astronomy during the 12th century. A notable work, titled "Lilavati," served as a comprehensive treatise on arithmetic and geometry, influencing mathematical thought in India for centuries. Lilavati, the daughter of the mathematician Bhaskara II, was born in 1114 CE in Bijapur, Karnataka, India. Raised in an intellectual environment, she developed a deep interest in mathematics from an early age, guided by her father's teachings.One of its significant contributions lies in the realm of number theory and also made notable contributions to geometry, concepts related to shapes, sizes, and measurements. Lilavati's contributions played a crucial role in shaping the future of mathematics in India. From commentaries and cultural references to educational applications and contemporary research. Lilavati's legacy continues to thrive, ensuring that her mathematical brilliance remains an integral part of the ongoing discourse in the world of mathematics.

Lilavati: A Mathematical Masterpiece

"Lilavati," Lilavati's magnum opus, is a comprehensive text that covers various aspects of arithmetic and geometry. The work is structured as a dialogue between Lilavati and her maid, where Lilavati poses mathematical problems and provides solutions. The content includes topics such as number theory, geometry, and algebra. Given that Lilavati's father, Bhaskara II, was an astronomer, the text also includes elements of astronomy. This may involve calculations related to planetary positions, eclipses, and other celestial phenomena.

The mathematical approach is not purely theoretical; it includes practical applications. The problems presented often have real-world significance, emphasizing the utility of mathematics in daily life. The dialogical format of "Lilavati," with Lilavati posing problems to her maid, suggests a pedagogical approach. This style of teaching was not only informative but also engaging, making mathematical concepts more accessible. One of the famous problems in "Lilavati" involves a cup and a flower. Lilavati is said to have accidentally broken a cup, and her father, Bhaskara II, uses the incident to teach her about time and distance, turning it into a mathematical problem and also work delves into chronology, the science of timekeeping. It is also a piece of literary art. The work is composed in poetic verses, adding an aesthetic element to the presentation of mathematical concepts.

Key aspects of Lilavati's mathematical approach:

Lilavati's work delves into number theory, exploring properties and relationships of numbers. This includes discussions on divisibility, factorization and rules for arithmetic operations. "Lilavati" includes a variety of mathematical problems and puzzles. Lilavati posed these problems to her maid, and the solutions provided insights into problem-solving techniques and mathematical reasoning. Lilavati's work involves solving equations, including linear and quadratic equations. Also employs algebraic manipulations to simplify expressions and solve problems. The work involves geometric constructions, showcasing Lilavati's understanding of how to construct various geometric figures using a straightedge and compass.

Commentaries and Expositions:

In the centuries following Lilavati's time, mathematicians and scholars wrote commentaries and expositions on her work. These texts served to elucidate Lilavati's concepts, making them more accessible to a wider audience. These commentaries not only preserved Lilavati's ideas but also helped disseminate them across different regions and cultures.Scholars and mathematicians have played crucial roles in understanding, preserving, and disseminating Lilavati's mathematical legacy. Their works, whether through historical analyses, commentaries, or broader studies of Indian mathematics, contribute to the ongoing appreciation and recognition of Lilavati's contributions to the field. Some notable individuals who have engaged with and done works based on Lilavati:

Professor Subhash Kak, an Indian-American computer scientist, has contributed extensively to the study of ancient Indian mathematics. His works often include discussions on Bhaskara II's contributions, including those found in Lilavati. An American historian of mathematics, Kim Plofker has written extensively on the history of mathematics in India. Late Professor David Pingree, an American classicist and historian of ancient science, made substantial contributions to the study of Indian mathematics. His works include analyses of various Indian mathematical texts, shedding light on the achievements of mathematicians like Bhaskara II. An Indian mathematician, K. S. Shukla, has contributed to the understanding and promotion of ancient Indian mathematics. Various Indian mathematicians and historians have engaged with Lilavati's work in their research, helping to preserve and promote the rich mathematical heritage of India. His works include analyses of various Indian mathematical texts, shedding light on the achievements of mathematicians like Bhaskara II. An Indian mathematician, K. S. Shukla, has contributed to the understanding and promotion of ancient Indian mathematics. Various Indian mathematicians and historians have engaged with Lilavati's work in their research, helping to preserve and promote the rich mathematical heritage of India.The global mathematical community recognizes contributions, and efforts have been made to translate her works into various languages. This has facilitated cross-cultural exchange and enabled mathematicians worldwide to appreciate the depth and significance of Lilavati's mathematical insights. So to conclude from commentaries and cultural references to educational applications and contemporary research, Lilavati's legacy continues to thrive, ensuring that her mathematical brilliance remains an integral part of the ongoing discourse in the world of mathematics and a valuable historical document in the study of Indian mathematics.

YUKTIBHASA

-ANUPRIYA V.V

Ganita-yukti-bhasha (Rationales in Mathematical Astronomy) of Jyesthadeva (c.1530) is a seminal text of the Kerala school of astronomy. It is composed in the Malayalam language and presents detailed yuktis or explanations and demonstrations for the results and processes of mathematical astronomy. The text, comprising fifteen chapters, is naturally divided into two parts, mathematics and astronomy, and purports to give an exposition of the techniques and theories employed in the computation of planetary motions as set forth in the great treatise Tantrasangraha (c.1500) of Nilakantha Somayaji. Even though the importance of Ganita-yukti-bhasa was brought to the attention of modern scholarship by C.M Whish in the 1830s, a critical edition of the entire Malayalam text is published here for the first time along with an English translation and detailed explanatory notes.

The astronomy part is divided into eight chapters. The topics covered are Grahagati (computation of mean and true longitudes of planets), Bhugola and Bhagola (Earth and celestial spheres), Pancadasa-prasna (fifteen problems relating to ascension, declination, longitude, etc.), Chaya-ganita (determination of time, place, direction, etc., from gnomonic shadow), Grahana (eclipses), Vyatipata (when the sun and moon have the same declination), Darsana-samskara (visibility correction for planets) and Candra-srngonnati (phases of the moon).

A distinguishing feature of this work is that it gives a detailed exposition of the revised planetary model proposed by Nilakantha which, for the first time in the history of astronomy, gives the correct formulation of the equation of centre and the latitudinal motion of the interior planets, Mercury and Venus. Another unique feature of Ganita-yukti-bhasa is that it presents systematic derivations of most of the results of spherical astronomy (pertaining to diurnal and shadow problems, parallax, eclipses, and so on) that are discussed in Indian astronomy.

This work is a long-awaited translation of one of the most important and hitherto least accesible works in Indian mathematics, supplemented by detailed explanatory notes and commentary.

INTERVIEW WITH PROF. DR. ROY WAGNER

-KAARTHIKANJANA S KUMAR

On 18th January 2024 I had the amazing opportunity to interview Prof. Dr. Roy Wagner, Full Professor at the Department of Humanities, Social and Political Sciences ETH Zürich on Ancient Kerala Mathematics and his recent works.

Kaarthikanjana[K]: I would like to begin with Vedic Mathematics. Vedic Mathematics is considered to be a system of mathematical concepts that is known for its theory calculations. So, what can you tell about the legitimacy of this Vedic Mathematics?

Roy Wagner[R]: Can you tell me a little bit more about what you mean by Vedic Mathematics because different people use this term in different ways.

K: Vedic Mathematics is usually considered as it is having 16 sutras and 13 sub sutras and we are using it for the speedy calculation. It is, that small formulas that makes it easier for people who have some difficulty in doing calculations. It makes them more comfortable in doing it.

R: So, I am not 100% sure we are talking about the same thing but there is a body of literature that are considered to be Vedic Mathematics and in fact most probably have absolutely nothing to do with Vedic times and Vedic knowledge, namely, the knowledge really from those centuries before Christ where the Vedic literature was developed.There is a lot of critical literature explaining that is rather a very late creation. The people who claimed to find it in fact would not supply the manuscript sources , and this is an attempt to take some nice calculation shortcuts and impose on them the aura of old sacred knowledge. Now, there is real Vedic Mathematics, the Sulvasutras are very famous which includes a lot of interesting geometric information and reasoning. However, this has nothing to do with those rules of calculation. Again, I qualify, if we are talking about the same thing. I am not 100% sure. There are papers for example by Shukla, who analyses this carefully and explains why he thinks that this has nothing to do with Vedic ethics. And this is an interesting cultural phenomenon, taking knowledge and trying bestow an aura on it. And then we have to ask, why, what's the point? Why not just teach these nice rules as they are, or acknowledge that they are coming from many different places and many different times? Why try to embed them in the Vedas?

K: Your work's based onKriyakramakari, Vernacular mathematics in medieval South India and Citrabhanu's 21 algebraic problems. So what inspired you in this area of research.

R: So, my interest in Indian mathematics in general was due to the Kerala school. I thought, when I learned about it for the first time, I realized not enough people know about it. It is a very interesting and very exciting kind of mathematics. And it should definitely be explored more and shared more within the research community and outside the research community. In terms of my approach, when I was studying the history of European mathematics, in particular, early modern algebra, I realized that in order to understand the innovation of early modern European mathematics, or the kind of thing that happened in the 16th, 17th century in Europe, it is hard to understand it if you do not understand a sort of lower or more popular level of mathematics from which it emerged. This is called the Abaco school. These were schools of mathematics in northern Italy in the 14th, 15th, 16th century. And the kind of knowledge they produced, although often considered not very original, not very innovative, set the ground for the emergence of new mathematics. And I thought that what is missing, perhaps, in the research on Indian mathematics, in particular Kerala mathematics, is understanding the relation between this higher, innovative, exciting mathematics and what people tend to ignore, which is the more basic everyday kind of school mathematics. This was not researched at all in the context of India or Malayalam.So what I chose to do is focus on a family of educational texts that were prevalent in the Tamil and Malayali speaking area, which are rather basic in terms of the mathematics, but probably the infrastructure or the base knowledge against or with respect to which, some of the higher mathematics that created in Kerala came up. The research is still rather preliminary, and I don't have any broad or general thesis about how it is related to higher mathematics, or maybe it might even be not related to the higher mathematics. There is no necessary connection between two kinds of mathematics, which happen to be in the same place and time. I try to understand if and what the relations might be. This is the kind of approach I have been taking. Instead of focusing, like most researchers, on the most high-end texts to try to go back to another more popular layer of mathematics.

K: And you mentioned in your paper, Kriyakramakari, about the practitioners of other sciences in the same social system or elsewhere. and it requires a deeper study about this.

R: Yeah, what we know about Indian mathematics and in particular Kerala mathematics is usually at the level of mathematical reconstruction, namely, how do we translate their knowledge to the modern mathematical language? And many very clever and very thorough and very interesting scholars have worked on that and we have a pretty good view of what kind of mathematics they were able to do when phrased in modern mathematical terms.

What we're missing is historical perspective. Historical view means to understand why they did the kind of mathematics that they were doing, what helped them generate the ideas or questions that they were pursuing and to understand their cultural context. So, we need to understand not just their mathematics but other things that they were interested in, like many of the same people wrote devotional texts, astrological texts, medical texts, philosophical texts. So, to understand a Mathematician we really need to understand his or her broad interests and situate the mathematical questions or methods or ideas in this context. We need to understand who they are writing for. We need to understand how they made their living, who paid them for their Mathematics, If anyone paid them for their mathematics - maybe they made their living in other ways. All of this social situating of mathematics is hardly done today in terms of researching to the history of Indian Mathematics. This is partly because it is very difficult. I mean, just taking one mathematical text and reading it carefully and understanding it is much less demanding than trying to read many different texts, understanding the cultural setting, understanding the historical setting, which requires much more knowledge . And then relating everything to the mathematics is a much bigger and more difficult project. It makes sense that it comes later, but I think this is where research on Indian mathematics should go today.

K : Sir, we know that the chain of Kerala school, that is starting from Madhavacharya and then Parameshwara, Damodara, but we don't know about the people who may have contributed to this before them. So, in your research have you ever included about them and their contributions? Would you share anything about them?

R : So, we do know a lot about what kind of Indian mathematics there was before the Kerala mathematics and there is a whole list of canonical literature starting from Aryabhata onwards. But it is not clear to what extent this list of major authors that you can see in many histories of Indian mathematics is really representative of what was going on. It is presented like a sort of bubble. There were these mathematicians, sometimes they are 100 or 200 years apart and it is like there is nothing to connect them. They are just flashes of mathematical light that appear and disappear and then taken over 100 or 200 years later. This is clearly not a historical picture. This is not how knowledge is transmitted and processed in a real life environment. So, what we need is to understand the context. Who transmitted this knowledge? Who used this knowledge? Maybe this knowledge was never actually used. We often assume that mathematics has a practical application. It is not necessarily so. Sometimes people do mathematics precisely because it is not practical, because they are interested in the spiritual or the logical or reason or the mental and they want to stay away from applications. We need to understand what is the educational system? Who is teaching this? Who is learning this? Whether the culture is more written or more oral? Are these really schools or just a single mentor-student kind of educational situation?

K : Sir, in your recent paper,Kanakathikaram, which is purely based on the Malayalam manuscripts. So, what was your working experience?

R: So, my working experience is that this is a very strange kind of text. So, on the one hand, it looks like a school text. Which is supposed to teach rather elementary mathematics to rather beginner students. It seems to assume only the knowledge of numbers and fractions and to be able to do basic calculations. But the language is so difficult to decipher. And at the beginning, I thought it was just my problem because, you know, I am not a native speaker. My knowledge of Malayalam is very limited. But then, as I try to work with other scholars with much better linguistic skills than mine, I see that this is a real problem,that this language is difficult even for the most professional researchers. It is a strange mix of Malayalam and Tamil with a lot of archaisms, which is very difficult to decipher. And it is not just the modern scholars who have difficulties. Might think the language changed, so even the best contemporary scholars have difficulties with it. But it is very clear that the commentators who wrote prose commentary between the verses of the Kanakathikaram also have great difficulty understanding what is going on.And some of the information there looks like the kind of very basic knowledge you would need in a practical setting, like knowing units of measurements and solving some basic problems. But some of the verses are really strange. They are systems of measurement that clearly do not make practical sense, or systems of fractions that clearly do not make practical sense. They are not useful. They are not designed to be useful. The gradation of fractions is so weird using such weird, denominators or going into such small numbers that have no use in practice. It is not clear why they are there. Maybe there is some religious or mystical or philosophical meaning that I just don't understand.

Maybe it is an exercise in virtuosity. I mean, in many cultures, including mathematical cultures, we see that people gain prestige by being able to describe strange useless knowledge and their skill is shown by memorizing the difficult verses and the difficult numbers. So maybe that's part of it. Maybe it is some sort of way for the teacher to impress the students or the client. It is as if they are saying" Look at these strange, mysterious things that I know and I could teach you". There are a lot of sociological reasons to include this kind of knowledge, but again, we are lacking enough context and enough comparative studies to figure out why this kind of knowledge was there. There are also many word problems in this literature. Word problems of the kind that you sometimes see in school textbooks today. Two people going from two cities in different speeds, when will they meet? This is a tradition where some problems go back thousands of years, at least to old Babylonian times, so 2000 years B.C, and we see this kind of problems transmitted to many different cultures. We see them popping up in Mesopotamia, China, India, , Egypt Greece, Europe, all over the place. So, it's clear that there is a kind of a culture of transmission of problems. Now, it's very interesting to see how Malayalam authors engage with those problems and again, we see that sometimes, in the more difficult problems, the people who copy the problems and try to solve them do not really understand them. So again, there's an interesting social setting of transmitting mathematics that is not always well understood by the people who transmit it. And this is the elementary mathematics, not the high-end difficult mathematics. So, there are a lot of open questions about what kind of social role this mathematics plays and how does it relate to other areas of knowledge and of course the whole difficult question of the relation between Sanskrit and vernacular. How do the Sanskrit texts relate to the vernacular texts in languages like Malayalam, Tamil and other languages, because each area has its vernacular literature(not always well preserved, but almost every area has vernacular literature). So, and researchers usually focus on the Sanskrit mathematics and most of the editions we find and most of the analysis we find is about the Sanskrit mathematics, but don't think you can understand the Sanskrit mathematics in isolation without putting it in the context of the vernacular.

K: Sir, what is your point of view about the concept of language barriers when it comes to Sanskrit? Even Sanskrit is not a globally accepted language.

R: So again, we're lacking a lot of knowledge. There's this one approach that says Sanskrit was only known by Brahmins, and only studied by Brahmins, and was a separate culture. But if you look more carefully, you find a lot of evidence of people who are not Brahmins, and sometimes not even Hindus, who understand Sanskrit.

Try to understand who had access to Sanskrit literature, and what kind of relations you have between the Sanskrit literature and the vernacular literature. Including Indian language and also non-Indian language, but Arabic or all sorts of combinations, like Arabic and Malayalam, these are completely open questions. We really don't know enough about the relations between Sanskrit and the vernacular. We're very ignorant in the context of mathematics, but in fact I'm not sure these questions are well-researched even in the context of other kinds of knowledge like medicine or philosophy. So there's really a lot of open questions which require going back to the manuscripts, reading them carefully and patiently, reading a lot of them, which requires large communities of research, this be done by one person. So there's a lot of work to be done.

K: Sir, are there any specific challenges you face while doing this research?

R: Yes, of course the main challenge is linguistics, because my level of Malayalam is quite basic, I can't even read a newspaper. My proficiency is restricted to this mathematical texts, I know the vocabulary and the structures of this mathematical text, and I can read them. This is of course very, very limiting, I'll give you an example. There is one verse in the Kanakatikaram, which is about units of measurement of volume. And the larger units of volume are units that we know were actually used and the ratios between those units of measurement are more or less what we expect to find based on other evidence.

But then there are extremely small sub-units with very strange names, like smoke, and smell, and milk, and drop. And then when I presented it in some workshop, Professor Gurukkal was there and he said, these are precisely the stages of producing ghee. You first have the smell, then you have the smoke, then you see the milk bubbling or evaporating or whatever, these are all elements from that context's not a practical system of units, nobody can measure anything with that. But it has a clear cultural context. Someone looked at the process of making ghee and decided to take inspiration from there to create a system of fantastically small units of measurement of volume. This is precisely the kind of knowledge that as an outsider I don't have. I never saw ghee being produced. I have no knowledge about the production of ghee or the cultural uses of ghee. I mean, I know the taste, I like the food, but I don't know anything about the cultural context. And this means that I miss a lot of these clues, which means that for the research to develop well, it really has to involve local people who are much better embedded into the culture, both as scholars but also as people who are part of this culture, have an intuitive knowledge of this culture.

There are all sorts of strange numbers in those verses and those problems, and maybe some of these numbers would immediately trigger a certain association for someone who understands the culture better, because they clearly belong to some religious story or mantra or ritual or whatever. I don't know. So, you know, these are an difficulties. I'm confined to slow reading of texts. I'm confined to secondary literature in English.

K: How are you overcoming these challenges?

R: I'm looking for collaborators who can work with me and had a lot of help from many experts and colleagues. But it's very difficult to find. Because people are not necessarily interested in that or don't necessarily have the skills. So in order to do that, you need to have some access to mathematics. You don't have to be a mathematician, but you have to like and be interested in mathematics, which is already a bit rare. And you also have to be interested in linguistics. You have to understand how the language works and the history of the language and be able to analyse a language. What I see very often, people read the text and give some sort of intuitive translation, but they cannot explain the translation. They cannot break it down. And then if I point out some syntactic or grammatic problem with the translation, they just don't seem to have the tools to engage with it. So you need someone who is linguistically trained, interested in Middle Ages period, likes or is interested in mathematics. And, of course, can spend time doing his work because, I mean, people have to make a living also and very few people are given the opportunity to make their living by following this kind of research. And it seems that today, from what I hear from colleagues in universities in Kerala, students who are interested in language are usually interested or almost exclusively interested in modern language, modern Malayalam. I understand that modern Malayalam is very exciting:

Malayalam literature, Malayalam cinema, Malayalam culture is very vibrant and amazing. They are interested in modern social issues. At best, they are interested in colonial time, but going to pre-colonial time and medieval history of Malayalam, it doesn't seem to be very attractive for people in Kerala today, for students in Kerala today. There are very few older experts. They are dying out or retiring and there doesn't seem to be a new generation that picks up where they leave. And those people who do, who are interested in older history, usually are interested in the Sanskrit rather than in Malayalam. And then you lose a huge amount of knowledge and culture and literature. Looking at the catalogue of the Kerala University Oriental Research Institute manuscript library, you have thousands of manuscripts that nobody has looked at. And we don't have an infinite time. I mean, these are perishable manuscripts. Their lifespan is maybe 200 years. Whenever I open a manuscript, some little pieces break. I mean, it's not going to stay there. So, there is some urgency in starting to engage with those manuscripts. Otherwise, a whole chunk of knowledge about how Kerala became what it is, is going to be lost.

K: What do you think are the challenges and opportunities in choosing this field?

R : So I think the challenges by most of it are more of less covered. You need a broad knowledge of language and affinity to mathematics and interest in medieval cultures or early modern culture, pre colonial.

And a lot of time to read very different kinds of text, mathematical and non mathematical, and it's very challenging. And you need people can devote their time to it, and for that you need PhD positions and post doc positions and professor positions.

And you know better than I do the situation of higher education- Now these are not the kind of areas that get the most funding.

K: So are there any specific theories or areas that you found fascinating.

R: Yeah. So I think what should definitely be looked at is the relation between astronomy and astrology, one of the major motors for the development of astronomy was astrology. Today there's, not a lot of respect for astrology it's considered superstition and boring. And nobody wants to study that. But I think that in order to understand mathematics and specifically Indian mathematics or Kerala mathematics well, one really needs to look at the link between astronomy and astrology.

K: So how far is the contributions of Kerala School have been utilized.

R: "There is the question of whether this knowledge was transferred to Europe, because there are some interesting similarities between Kerala mathematics and later European mathematics". However, we don't find any hard evidence. We don't find anything that looks like a proper translation. We don't find any written evidence of Europeans who say" I saw this kind of knowledge in India". So the question is open.

K: What can you tell about the accuracy of these contributions of Kerala School?

R: The results are extremely high quality. There's no doubt about that. So in that sense they are really top notch. In another paper I wrote with a colleague we suggested another area where something may have been transmitted.

But again we don't have any hard evidence, and we might never have, because people sometimes transmit knowledge orally. When we think about knowledge transmission, we think, there is a text in one language, and then it is translated, to another. But that is how Knowledge is transmitted. In fact, very often this is exactly not always it is transmitted is transmitted orally in parts and pieces.

K: Sir, it's been a couple of months our students are working for this. So what's a message for us?

R : The main message is that there's a treasure of untapped knowledge and resources to be looked at. And then the problem is how to find the means to really get into it. Because, it doesn't just require good will, but also resources, to go to the Trivandrum Library and start reading.

K: So thank you so much, sir. I'm deeply honored for this.

R: I'm very happy to to see people interested in that. Because, as I said, this is not something that can be done by simply by outside research and we need a local research community to develop. There are still professors and experts who know a lot. But I'm not sure that they have the infrastructure to teach and create a new generation of people who really have good understanding of the linguistic and historical content.

INTERVIEW WITH DR. EASWARAN NAMBUDIRI T C

-KAARTHIKANJANA AND DHRUV

On 22nd January 2024 we had the amazing opportunity to interview Dr. Easwaran Nambudiri T C, retired Associate Professor and HoD from Government Brennan College, Thalassery, on Ancient Kerala Mathematics.

Kaarthikanjana[K]: The main aim of our magazine is to study about the mathematical contributions of ancient Kerala. So we began our study from Madhavacharya and our studies are still going on. In your opinion what is the relevance of exploring our ancient mathematics in this current scenario?

Easwaran sir [E]: There are 4 well known series expansions attributed to Madhavacharya. First of all, the pi/4 series, which was the infinite series expansion of circumference of a circle in terms of its diameter. This can then be modified into the popular Gregory series. But the interesting thing is, the proof of this claim is highly geometric. The way in which he arrived at this result and its proof for calculating 1/8th of the circumference of the circle is a mystery. The verse for the series expansion of pi /4 is

व्यासे वारिधिनिहते रूपहृते व्याससागराभिहते।
त्रिशरादिविषमसङ्ख्याभक्तमृणं स्वं पृथक् क्रमात् कुर्यात्॥

This verse is ending with 'prithak kramath kuryat' and because of this only, we are calling it an infinite series expansion. Actually it aims to get better and better approximations for pi/4. The procedure involves the bhuta sanghya system.

Terms like 'varidhi' and 'sagara' mean the number 4, tri for 3, sara for 5 etc in this system. So to read the palm leaf texts, one must be familiar with the number systems like the bhuta sankhya system and the katapayadhi system. In kerala, the katapayadhi system was more popular. In the later versions of Madhava's contributions such as the Karanapaddhati of Puthumana Somayaji and the Sadratnamala of Sankaravarman, the usage of katapayadhi number system can be found. Madhava also looked into the expansions of the arc tangent, sine and cosine functions. They were making all these inventions for astronomical purposes, mainly to correct the errors occurring in the earlier calendar system they were following. The main object, they thought probably, needed for understanding the planetary motion was the ratio between the circumference and the diameter of a circle, what we call pi now a days. They realized that in the path followed by the planets there is a minor circle and a major circle. So Madhava tried to generalize the idea of calculation of the circumference into the calculation of the partial arc length of a circle. Once this was done, a finer calculation in the planetary path was needed, and for that he developed infinite series expansions of sine and cosine functions. This infinite series expansions also corresponded to numbers which are not rational. Hence they were well aware of the irrationality of numbers, but proceeded by using natural numbers in the process. In my opinion, the methodology they followed in such investigations is highly relevant in the current scenario also.

Dhruv[D]: According to you sir what are the relevant information we should take care to add in our magazine so as to get maximum impact?

E: As a first step, you can introduce and explain the main contributions by Madhavacharya, namely the four infinite series. Among these, the first one is the Gregory - Madhava series involved in the verse 'vyase varidhi nihate…'. Explain how the expansion of pi /4 is arising from this verse. This may not be known for a general audience. Similarly, if you look into the arc tangent series expansion, there appears a warning regarding the choice of x and y coordinates. Specifically the verse insists that y must be less than x, and this actually controls the convergence of the series. So they were aware of the convergence as well as divergence of series and were discussing only about convergent infinite series. In the next step, verses of sine and cosine series can be explained. The mathematical aspects of these infinite series may be highlighted, since whenever I read such an article, it is usually about the historical aspects. Mathematical explanations are found very less in popular articles.

K: Sir,what inspired you to do you in medieval Kerala mathematics?

E: Actually my topic of interest was not any aspect of history mathematics. I was doing research in harmonic analysis, specifically in wavelet frame analysis. Surprisingly, all the main concerns in my study were about infinite series expansions of various objects appearing as mathematical functions. This motivated me to look into the historical origins of infinite series and functions. On the way, I landed upon the contributions of the genius, Madhavacharya. It was somewhat accidental.

D: Sir, you had mentioned earlier that ancient Kerala mathematics was influenced by astronomy. Could you elaborate it?

E: As far as I know, the first major contribution in Indian astronomy was by Aryabhatta. He and his followers were trying to explain the climatic changes, based on the observations on movements of planets. For this, they prepared calendars which are not equivalent to the modern Gregorian calendar followed nowadays. The Gregorian calendar contains 365 days in a year and these 365 days are divided into 12 months named as January, February etc. However, there is no special significance in beginning a year from January 1. But Aryabhatta and his followers were preparing the calendar by observing the planetary movements. They understood that the movements of the Moon and the Sun are mainly influencing the major climatic changes and with respect to this observation, they tried to prepare the calendar. For this, they were in need of the speed of movement of these planets and Aryabhatta succeeded a lot in developing this idea. He and his followers suggested that as is moving towards the future, we could move towards the past also. So they predicted that the occurrence of a moment when all the planets will come along a single straight line. They identified this moment as the beginning of a 'Yugam'. As per this, currently we are in the Kaliyugam. Also, they developed the Kalidina sanghya system which was nowhere available in Europe at that time. It was a surprising discovery by Aryabhatta. The calendar and all the related climatic predictions made by Aryabhatta are somewhat to-date. In this direction, another significant contribution was by Madhava. I believe that he was trying the level best to minimize the errors occurring in the preparation of the calendar and was aware of the influence of the irrationality of the number pi causing this. This irrationality of pi is not only mathematical but also highly philosophical, because we are talking about a number which is not accessible by human beings.

K: What do you think about the influence that the contributions of ancient Kerala have in modern mathematics?

E: Directly, there may be no influence at all, because what we study in now a days mathematics is completely based on European methodology. Even as a teacher, I was not aware of this Madhava series and I was teaching it as Gregory series. I had no opportunity to mention that this was a contribution by our great Madhava. This is how our education system is going on in our place. A novel "Francis Itty Cora" was written in Malayalam, in which, the main background of the novel was the ancient Kerala mathematics. I feel this novel is narrating negatively, the mathematical contributions from ancient Kerala.

D: Sir, if a student is interested in learning about ancient Kerala mathematics, according to you how should they approach the subject? What are the resources that you would recommend?

E: Till the last century, the main mathematical text in all over India was Leelavati written by Bhaskaracharya II. Many commentaries are available in Malayalam for this text. Vataseri Parameswara, a student of Madhava, wrote an independent commentary of Leelavati in Sanskrit which also contains new contributions from Kerala. Many such commentaries are also available as palm leaf manuscripts and some of them are available in Malayalam also. So as a first step, it is better to look into such texts to get some familiarity in reading and decoding. In addition, the popular 16th century Malayalam text "Yukthibhasha" by Jyeshtadevan is also available in the printed form. Though it was written in Malayalam, it contains many verses from Leelavati and Tantrasangraham of Nilakanta Somayaji. There you can notice how sincere were these people to their predecessors. Jyestadevan mentions the original author of the verse wherever it is cited. I strongly recommend Yukthibhasha for any beginner who wants to learn ancient Kerala mathematics.

K: What are the future opportunities in this area of research?

E: In Kerala, opportunities are less. But there are opportunities outside Kerala as well as in abroad. But you must be very sincere while doing research. One can do research work by praising the contents of our ancient mathematical palm leaf scripts. But according to me, blind praising and blind rejection are equally harmful in the research field. You have to justify impartially the content of the manuscript under study.

D: What are the challenges they must be prepared to deal with?

E: First of all it is very difficult to read through an old palm leaf text. It is not as easy as reading our regular text books. Knowledge in Sanskrit as well as in old Malayalam are very much essential for analyzing Kerala mathematical contributions. Getting relevant documents is another challenge in this field. Many palm leaf texts are available in various repositories which are not even looked into.

K: Are there any specific challenges you have faced and how did you overcome them?

E: Getting relevant documents other than the popular ones was the main challenge. For instance, Prof. K V Sharma mentioned that the popular verses of Madhava are available in a commentary of Tantrasangraha by Sankara Variar. Nilakantha Somayaji wrote Tantrasangraha and Sankara Variar gave a commentary on it namely, Tantrasangraha Vyaghya. The original palm leaf text of this commentary is available some where in Tripunithura but I am unable to get it. So getting the original text is a challenge. We can research the popular documents. The original text written down by scholars like Nilakantha Somayaji may be copied down many times by others and such copies may contain errors. On the other hand, such copies may also contain some additional contributions by the scribe. So a copy of Tantrasangraha that we are looking into may not be an exact copy of Tantrasangraha.

D: What would be your message for all the students who have been working for this magazine for the past couple of months?

E: First of all I express my deep appreciation. When I was a student I was not aware of this great heritage. But you have got a golden opportunity for learning something from our own heritage. It is really a great thing. This will motivate you for learning it rigorously. If you simply study infinite series as the part of the course work, you may not be getting the real taste of it. But if you study the same series as given by Madhavacharya, you will be excited and there by get new ideas regarding it. In addition, you can transfer the knowledge of our great heritage. I congratulate you all.

K: Sir do you think including this history in our curriculum is important?

E: Yes. Not only our history, but the history of mathematics is also very important.

We would like to conclude our interview report by expressing our sincere gratitude to Dr. Easwaran sir for sparing his valuable time and to the department of mathematics for providing us with this opportunity.

Interview with Dr. Easwaran Nambudiri T C

AN INTERVIEW WITH DR. VENKATESWARA R PAI

-SREELEKSHMI AND RIJA

We got a chance to have an amazing fruitful conversation with Dr. Venkateshwara R Pai, Associate Professor, Humanities and Social Sciences, IISER Pune on 24th of January 2024

Q(Sreelakshmi): Sir, when we searched about you in google for we noticed that you were a part of small and unique group of researchers that had knowledge in the field of Mathematics, Astronomy and Sanskrit. Why is this group still small sir ?

Answer : This field requires deep knowledge in all these three subjects to do something big in it. Just knowing Maths and Astronmy concepts won't be enough. Knowledge in Sanskrit language is very necessary for research in this are. And the sole knowledge of Language is also not what we fully require, it's a mix of all these three areas what is needed. Knowledge in Sanskrit language along with malayalam will be an add on to it as there are a lot of manuscripts were language is malayalam and script is sanskrit.

Q(Rija): Sir, What made you choose this particular area when you had a whole lot of options in front of you ? What was that one fascinating factor that made you interested in this topic / area ?

Answer : I have done my Masters in Physics and I had pretty good knowledge in the sanskrit language right from my childhood. Instead of just looking for a mere job after completing my masters, I decided to do something out of the box to use my full passion I had for the subject by doing something more in the field. My keen interest and passion was what led me to such a destination.

Q(Sreelakshmi): Sir, When some discoveries are made by Indians and some other from the west and the credit is given to the person from the west. The tendency of today's youth in India is to believe that it was the person from the west is the one who did it and not the other way. Why is it so sir ?

Answer : Though you may not be the best person to speak about the same, the need to educate Indians in this field is so important. The curriculum should include that as well. So atleast through that people might be able to recognize and understand these things in detail, about the rich history of our nation in these fields.

Q(Sreelakshmi): Sir who are those organisations that credit people for their works in this field of mathematics ?

Answer : So, basically there are lot of agencies like that. And all these conversations are through journals , that is when knowledge about a particular topic lately found is spread across the world which later leads to recognition from different parts of the world. It's not solely an organisation crediting anything.

Q(Sreelakshmi): Sir who is that one mathematician from the history of mathematics if given a chance you would meet and what will be the topic of discussion between both of you ?

Answer : I'll be discussing about the subjects, what are the difficulties they faced while discovering and what are the exact methodologies that they have used. You find so many computations how exactly they did the computation. Did they have an instrument?. We have now pen and paper. Were they using sand and stick or what exactly they were using?. What was their exact mental processes which were happening? This may not be a rational question, but were there any extra the kind of, you know, abilities that they had which is which could be explained through science or any other way. Did they have, you know, the kind of special capabilities?

Q(Sreelakshmi): Sir why do you feel today's youth is not attracted towards topics like history of mathematics or astronomy?

Answer : I took up this purely due to passion but I feel people will surely think of job opportunities after doing this which is scarce actually. That maybe a reason why people are not taking it up.

Q(Sreelakshmi): Sir do you think if British have not come and taken over India, We would have done further researches and studies on the existing knowledge we had from 14th and 15th century?

Answer : As far as Indian Knowledge System is considered, they were not materialistic. They were least worried about making their life comfortable. Even if they could have chances for inventing further things, they were least bothered about that. They were happy with what they had. So as far as the case of current indian mathematicians using the knowledge, it depends on a lot of factors. It had deep connections with astronomical science and not with maths and physics.

Q(Rija): A piece of advice for people who feel like this particular branch of mathematics related to history is having low scope ?

Answer: There are a few research areas. First one being primary texts, which is pretty interesting.

Q(Sreelakshmi): Do you think Britishers purposely destroyed our nation to show us as uncivilized and uneducated to the world so that they can rule our us easily ?

Answer : Defintiely, the invasions made by foreign countries have gravely effected our nation. It changed the mentality of people in such a way that today we feel not knowing English is a shame. That is the level of manipulation britishers had on us. They made it about knowing a mere language over immense knowledge in other fields like Physics and Maths.

Q(Sreelakshmi): Sir while we were doing research for this magazine and interview we went through few articles and videos were it said that few theories were invented by the kerala mathematicians in the 14th or 15th century itself but never got the credit for it as someone who found the similar theory after decades got through with the credit. Why is it so sir even after having clear proof that the kerala mathematicians were the first to discover it ?

Answer : But now few of them are getting credited like the Madhava series. Madhava was clearly the first to discover it precisely 2 centuries before James Gregory and Gottfried Wilhelm Leibniz. There are different theories which originated independently over time, Madhava being the first.

Q(Sreelakshmi): Sir is there any one invention or discovery we found from the manuscripts which we acquired while we already knew it as a part of modern mathematics ?

Answer : Fibonacci Series is something which falls into the category along with continued fractions and Chakravahas.

Q(Sreelakshmi): Is there any big discovery made from the information we got from the ancient scripts? If not, is there any further chances ?

Answer : No and I feel their are limitations for the information we had from the ancient manuscripts. And today's science is far more developed than that. Their are whole lot of new concepts now which limits the use of information from ancient times.

References

Since this magazine is a comprehensive collection of information on Ancient Kerala Mathematics here we have included all the references used in each of the articles

- A history of the Kerala school of Hindu Astronomy by K.V. Sarma,Vishvesharanand Institute, Hoshipur (1972)
- Kerala school of Astronomy and Mathematics by M.D.Srinivas, Mathematics Newsletter(March 2012 & June 2012)
- https://www.slideshare.net/Rhea66/madhava-of-sangamagrama
- https://mathshistory.st-andrews.ac.uk/Biographies/Madhava/
- https://www.wikiwand.com/en/Parameshvara_Nambudiri
- https://mathshistory.st-andrews.ac.uk/Biographies/Jyesthadeva/
- https://academic-accelerator.com/encyclopedia/melpathur-narayana-bhattathiri
- https://en.wikipedia.org/wiki/Achyutha_Pisharadi
- https://en.wikipedia.org/wiki/Nilakantha_Somayaji
- Lilavati and Vedic Mathematics,Jayanta Acharya
- P Devaraj
- Lectures 1,2 and 3 by P P Divakaran at ICTS
- NPTEL: Mathematics in India from Vedic Period to Modern Times-IIT Bombay, Lecture 1
- Kerala mathematics and its possible transmission to Europe by Dennis Francis Almeida and George Gheverghese Joseph
- Namboothiri Websites Trust.
- https://youtu.be/pYiRq0rHvKE?si=NsygQRWjb2gfY3Xk
- Picture credits to pexels

ARCHIVE

INAUGRAL FUNCTION OF MATHSPACIA

-SREELAKSHMI MURALI

Post the hardships created by the covid pandemic, our department decided to revive the activities and events which were once actively their in the campus. As part of the same the then-HOD Dr. Archana Nair along with the class representatives decided to call for a meeting which eventually led to a fruitful result. That was the formation of our Maths Club named Mathspacia. The Club was born with 4 subclubs which were Arts, Games, Seminar and Quiz club.

As a part of inauguration of the club which was done by Dr.U Krishnakumar (Dean, School of Arts, Humanities, Spiritual and Cultural Studies) , it was decided that the games and arts club will together conduct an event in the last week of November. The Arts club had an amazing mehendi corner and hand art corner along with few exhibits. The Games club chimed in with some games which had basic maths principles and logics as it's base idea. To make things more interesting a small fee was decided for every activity by setting up a ticket corner.

There were 7 to 8 simple games based on basic mathematical principles and ideas which led to a lot of people revisiting their childhood playing games and immersing themselves in a fun mood like never before in the campus post Covid. Students from all other departments decided to give their best and enjoy themselves grabbing the rare opportunity with open arms.

The inaugural function was a grand success providing a sense of satisfaction to all the people who were a part of organising and participating in it. The grand success of the event inspired a lot of other fests that came up in our college in the next few months really proving that we have to take risk and prove something so that others can be eventually gain the confidence to take risk and do something big on their own.

After all if this is not the time we are learning to take risk and try new things which inspire us, then when are we going to do these things, when are we going to work hard for something we like something apart from studies.

SASTRAMRITAM

- ANUGRAHA MANOJ

"Get ready to embark on an intellectual journey as the Annual Science Day Celebration, organized by the departments of Mathematics and Sciences, under the School of Physical Sciences takes center stage. With a lineup of esteemed speakers, captivating science exhibitions, and thought-provoking debates, the event promises to ignite a passion for discovery and innovation. Join us as we delve into the realms of scientific exploration and embrace the spirit of curiosity and wonderment."

It's an electrifying event designed to inspire a deep appreciation and passion for discovery and innovation. Through engaging talks by distinguished speakers, captivating science exhibitions, and thought-provoking debates, the event seeks to ignite the spirit of curiosity and wonderment in attendees. By showcasing the power and beauty of Mathematics and Sciences, the celebration aims to foster a deep and abiding enthusiasm for exploration, creativity, and the pursuit of knowledge.

The SASTRAMRITAM 2023 was a resounding success, featuring a plethora of events that showcased the incredible talents and skills of the students and faculty. Coordinated by Dr. K. Sreekanth and Dr. Sreedevi R. Mohan, the fest was a celebration of academics, culture, and creativity. The fest commenced on March 10th, graced by the presence of esteemed visiting professor Dr. Vellat Krishnakumar from the Kerala School of Mathematics, Kozhikode.

The inauguration set the tone for an intellectually stimulating series of events, starting with the enlightening Vidyamritam session led by Dr. V. M. Nandakumaran, a visiting professor from the International School of Photonics at CUSAT. Another highlight was the inter-departmental paper presentation competition on March 24th, themed "Global Science for Global Wellbeing." It allowed students to exchange cutting-edge ideas on addressing global challenges through scientific advancements. The fest continued to dazzle on March 30, offering a diverse range of activities including science exhibitions, a photography exhibition, engaging cultural events, a delectable food court, and thrilling games. The event provided a platform for students to express their artistic talents and foster a sense of community spirit. As the fest approached its conclusion, March 31 saw the culmination of the intercollegiate quiz competition, testing participants' knowledge and quick thinking. Furthermore, a captivating Vidyamritam session was conducted by Dr. Shyama Narendranath, a noted scientist from the Space Astronomy Group at the U R Rao Satellite Centre, ISRO, Bengaluru.

The event, organized under the able guidance of overall coordinators Mr.Parameswaran R and Dr.Archana Nair, shown a spotlight on the incredible contributions of Indian mathematicians, fostering an environment of learning and appreciation.

Club 1, under the leadership of quiz head Shubhada and coordinators Lakshmi Girish madam and Manjusha madam, hosted an exhilarating quiz competition that tested the participant's knowledge and quick thinking. The event saw active participation from students eager to showcase their intellect and grasp of diverse topics. Club 2, led by games head Sreelakshmi and co-head Rijakrishna, along with coordinators Lakshmipriya madam, offered a myriad of engaging games that added a dash of joy and excitement to the fest. The spirit of friendly competition and sportsmanship was on full display as students immersed themselves in various entertaining activities. Under the stewardship of seminar head Dhruv and coordinators Sreekanth sir, Senthil sir, and Remya madam, Club 3 organized enlightening and thought-provoking seminars that delved into the world of Indian mathematicians.

The seminars served as a platform for students to gain insight into the rich legacy and groundbreaking contributions of these mathematical pioneers. Club 4, with models head Panchami and coordinators Supriya madam and Remya madam, captivated the audience with awe-inspiring models and presentations highlighting the profound impact of Indian mathematicians on various aspects of society and science. The fest brought forth a vibrant tapestry of events that not only celebrated the brilliance of Indian mathematicians but also showcased the diverse talents and interests of the students.

It was a testament to the collective efforts of the club heads, coordinators, and participants who made the fest a resounding success. It left a lasting impression, igniting a passion for learning and exploration while honouring the invaluable contributions of Indian mathematicians to the world.The fest drew to a fitting close with a valedictory function, acknowledging and celebrating the achievements of all participants and organizers.

SEMINAR COMPETITION ON GLOBAL SCIENCE FOR GLOBAL WELL BEING

- SREELAKSHMI MURALI

Conducted for the very first time as a part of Sastramritam 2023, the seminar competition was really a breath of fresh air for all the students who where interested in seminars. The topic being 'GLOBAL SCIENCE FOR GLOBAL WELLBEING' made it easier for students from other departments also to participate and showcase their talents. We had a total of 4 participants including 1 from BBA. The competition was judged by Dr.Senthilkumar, Dr.Sreekanth (HOD of Mathematics Department) and Dr.Sindhu(HOD of Physics Department). Diversity was maintained while choosing interest areas from given topic.

The first participant, Anhas Hasan (BBA B Batch 2021) choose to speak about the wellbeing surveys that are conducted by various organisations across the world. The seminar highlighted how wellbeing is not always happiness. Coming from non-maths background, he impressed the judges with his understanding about the topic.

The Second Participant, Sreelakshmi Murali (Int MSc Maths 2020) took up a complex topic of graph brain theory network which included a rough overview of developments in the field of graph theory which had various connections with neuropsychiatric disorders and it's adverse impacts on people across the world.

The Third Participant, Dhruv S N (Int MSc Maths 2021) decided to discuss about a very general idea regarding the use of concepts in linear algebra in the field of medicine. It included the applications in the field of Radiology, Genetics and much more. An excellent performance earned him the second prize in the competition.

The fourth and final participant, Meghana Chandra (Int MSc Maths 2022) delivered and excellent seminar on the purpose of raising public appreciation of scientific issues in global context which has a bearing upon global well being. She did a fabulous job while presenting which earned her the first prize.

The seminar was an overall success with students finding a platform to showcase their presentation skills. Opportunities are limited in such a competitive world and brave are those who capture each one of them with whole heart and open arms.

UNVEILING THE ENIGMA: EXPLORING THE INTRICACIES OF MATHEMATICAL CRYPTOGRAPHY

-SARADA

In an era dominated by digital communication and information exchange, the need for secure communication channels has never been more pressing. Mathematical cryptography, a fascinating and dynamic field at the intersection of mathematics and computer science, plays a pivotal role in safeguarding sensitive information from prying eyes. This article delves into the fundamental principles and key concepts that underpin mathematical cryptography, shedding light on the ingenious methods used to protect data in the digital age.

Symmetric Cryptography:

Symmetric cryptography, the bedrock of many cryptographic protocols, relies on a shared secret key between communicating parties. The process involves using the same key for both encryption and decryption. The Advanced Encryption Standard (AES), a widely adopted symmetric encryption algorithm, exemplifies the elegance of mathematical constructs in providing robust security.

Public Key Cryptography:

While symmetric cryptography is efficient, the challenge lies in securely exchanging secret keys. Public key cryptography, a revolutionary concept introduced by Whitfield Diffie and Martin Hellman in 1976, addresses this challenge by employing a pair of keys: a public key for encryption and a private key for decryption. The security of public key systems is built upon the difficulty of certain mathematical problems, such as factoring large numbers or computing discrete logarithms.

RSA Algorithm:

One of the most celebrated public key algorithms is the RSA algorithm, named after its inventors Ron Rivest, Adi Shamir, and Leonard Adleman. RSA relies on the difficulty of factoring the product of two large prime numbers. The security of RSA hinges on the impracticality of factoring the product back into its constituent primes, even with powerful computing resources.

Elliptic Curve Cryptography (ECC):

Emerging as a cornerstone of modern cryptographic systems, Elliptic Curve Cryptography leverages the mathematics of elliptic curves over finite fields. ECC offers equivalent security to traditional public key systems but with shorter key lengths, making it more efficient in terms of computation and bandwidth. Its implementation is particularly well-suited for resource-constrained environments.

Hash Functions:

Hash functions play a crucial role in ensuring data integrity and authenticity. These mathematical algorithms transform variable-length input into a fixed-length output, often referred to as a hash value or checksum. Cryptographic hash functions, like SHA-256 (Secure Hash Algorithm 256-bit), are designed to be one-way functions, making it computationally infeasible to reverse the process and deduce the original input.

Challenges and Future Trends:

While mathematical cryptography has made tremendous strides in securing digital communications, new challenges and threats continuously emerge. Quantum computers, with their potential to break widely used encryption schemes, pose a looming threat to the existing cryptographic landscape. Researchers are actively exploring post-quantum cryptographic algorithms to withstand the computational power of quantum machines.

In conclusion, Mathematical cryptography stands as a testament to the power of mathematical concepts in shaping the landscape of secure communication. From symmetric ciphers to public key algorithms and hash functions, the field continues to evolve in response to emerging challenges. As we navigate the complex web of digital interactions, the fusion of mathematics and cryptography remains indispensable in fortifying the foundations of privacy and security.

MATHEMAPHOBIA

-RIJAKRISHNA AND GAYATHRI MANOJ

Students often face anxiety and fear of failure in math.To address this, creating a positive learning environment, breaking down complex problems into smaller steps, and emphasising the process over the result can help build confidence and reduce anxiety.

One common problem students face in math is a lack of foundational understanding. If fundamental
concepts are not grasped, it can lead to difficulties in more advanced topics.

Measure can be taken:

1. Build Strong Foundations: Ensure a solid understanding of fundamental concepts before moving on to more advanced topics.
2. Real-World Applications: Connect math to real-life situations to demonstrate its practical relevance and make it more engaging.
3. Interactive Learning: Incorporate hands-on activities, group discussions, and interactive tools to make lessons more dynamic and engaging.
4. Visual Aids: Use visual aids like diagrams, charts, and graphs to illustrate concepts and enhance comprehension.
5. Step-by-Step Approach: Break down complex problems into smaller, manageable steps, guiding students through the problem-solving process.
6. Encourage Questions: Foster a classroom environment where students feel comfortable asking questions to clarify doubts and deepen their understanding.
7. Varied Teaching Methods: Employ a mix of teaching methods to cater to different learning styles, such as auditory, visual, approaches.
8. Positive Reinforcement: Recognise and celebrate students' efforts and achievements to boost their
confidence and motivation.
9. Personalised Support: Provide individualised assistance when needed, addressing specific challenges students may face.
10. Practice and Repetition: Regular practice reinforces learning and helps students build confidence in their math skills.

To alleviate the fear of math in students:

1. Positive Mindset: Encourage a positive attitude toward math, emphasising that mistakes are a natural part of the learning process.
2. Real-World Connections: Show how math is used in everyday life, making it more relevant and less abstract.
3. Interactive Learning: Use interactive methods, games, and practical applications to make math enjoyable and engaging.
4. Step-by-Step Approach: Break down problems into smaller, manageable steps, ensuring students understand each component before moving on.
5. Encourage Questions: Create an environment where students feel comfortable asking questions and seeking clarification without fear of judgment.

6.Relatable Examples: Use examples that relate to students' interests or experiences, making math more relatable.
7. Build Confidence: Celebrate small victories and progress, reinforcing the idea that improvement is achievable.
8. Peer Support: Foster collaborative learning environments where students can help each other, reducing feelings of isolation.
9. Effective Communication: Clearly explain concepts and provide multiple explanations if needed, ensuring students grasp the material.
10. Individualised Support: Offer additional help or resources for students struggling with specific concepts, addressing their unique needs.

SUMMER SCHOOL IN MATHEMATICS: NURTURE 2023 AT CUTN

- ANNIE JOHN

Central University of Tamilnadu [CUTN] is a central university established by the government of India in 2009. CUTN is the organising body of NURTURE. Each year the 21 days workshop is conducted for second and third year UG students and in 2023 it was conducted in memory of the late mathematician Srinivasa Ramanujan. The main aim of the workshop is to develop independant learning among the budding mathematicians.

A total of two students were selected for this camp from Amrita Vishwa Vidyapeetham, Kochi Campus. Now let's dive into the exciting highlights of Nurture Camp and how it can help you achieve personal development like never before. The main aim of the camp was to provide participants with a nurturing environment where they can explore their strengths, overcome challenges, and develop valuable skills. The camp focused on holistic development by combining theoretical knowledge with practical experiences.

One of the primary objectives of Nurture Camp was to foster personal growth. Through various activities and workshops, participants were encouraged to step out of their comfort zones and push their boundaries.

This helped them build resilience, self-confidence, and adaptability - qualities that are essential for success in today's fast-paced world. Another key objective was to facilitate expert sessions led by professionals. By working together towards common goals, participants learn how to effectively communicate ideas, resolve conflicts, and leverage each other's strengths. The objectives of the camp revolved around empowering individuals with essential life skills while creating an atmosphere conducive for personal growth and professional excellence.

The Nurture Camp organized by the Central University of Tamil Nadu was an enriching experience for all participants.

One of the key highlights of the camp was the opportunity for participants to gain valuable insights from experts like Dr V P Rameshan, Prof .R.Thangadurai ,Prof .B Ramakrishnan. These experts shared their knowledge and experiences, providing invaluable guidance on various subjects like algebra, linear algebra, real analysis.

Participants were encouraged to actively participate in discussions and ask questions. This created a lively learning environment where ideas could be exchanged freely. The class ran from 9.00 in the morning to 5.30 in evening. Throughout the day there would be interaction sessions, live classes, discussion hours and team building activities. In addition to expert sessions, personal development activities played a significant role in making this camp memorable. Participants engaged in team-building exercises that helped foster camaraderie and collaboration among them. Furthermore, networking opportunities were abundant during this camp. The key highlights of the Nurture Camp included engaging expert sessions, hands-on personal development activities, and valuable networking opportunities. It truly provided a platform for growth and learning that will benefit participants long after they left the camp!

Now that we have explored the objectives and key highlights of the Nurture Camp organized by Central University of Tamil Nadu, let's see about my experience. Participating in the Nurture Camp was a truly transformative experience for me.

The expert sessions were incredibly insightful and provided me with valuable knowledge and skills that I can apply to my personal development journey. Interacting with like-minded individuals from diverse backgrounds also broadened my perspectives and helped me develop essential networking skills. I am grateful for the opportunity to be a part of the camp organized by CUTN .

The camp exceeded my expectations in terms of content quality and organization. The expert sessions were delivered by industry professionals who shared practical insights and techniques that I can implement in my professional life. It was an enriching experience that has empowered me to take charge of my personal growth.

The Nurture Camp conducted by Central University of Tamil Nadu proved to be an exceptional platform for individuals seeking personal development
opportunities through expert sessions led by industry professionals. Participants had the chance to enhance their knowledge and develop new skills.

MATHEMATICS TRAINING AND TALENT SEARCH [MTTS]: ONLINE FOUNDATION COURSE IN MATHEMATICS

MTTS TRUST

- DHRUV S N

Mathematics Training and Talent Search Programme (MTTS) is the most popular undergraduate/graduate training programme in Mathematics running in India. It conducts workshops of duration from 1 week to 4 weeks throughout the year in different locations in India. The students are selected from the region of the programme or from the national level depending on its aims. The MTTS Trust is formed for conducting these programmes. It also conducts teacher training programmes titled Pedagogical Training for Mathematics Teachers (PTMT) and other academic discussions benefitting a large class of mathematicians in India.

One of the most loved and significant features of MTTS camps is its two-week Foundation Course for Mathematics at Level O. It is conducted for second year UG students and I was fortunate enough to get an oppurtunity to participate in this program.

About 500 students from all across the country participated who were divided into numerous batches and classes were conducted online. My class teachers were Dr. Vikram T Aithel, one of the founding directors of MTTS along with Dr. Arusha C, post doctoral fellow at IIT Bombay. The students were further divided into groups with a mentor for group discussion session after class which helped negate any deficiency in understanding and encouraged students to interact with others. We even had a live session with Kumaresan Sir who addressed the various questions raised by students regarding mathematics in general.

One of the key features of this program was the gamified quizzes conducted on a weekly basis which were thouroughly enjoyed by the students. Assignments were also given to test the understanding of students and to help them practice writing proofs in mathematics. These were unlike the normal assesments conducted in collage. There were no marks, no topper, no rank holder its sole purpose was to help identify our weaknesses and help others with our strengths.

Overall the classes were very fruitfull and I thoroughly enjoyed it. It was an amazing oppurtunity to develop my skills not only in mathematics but also my social skills. I highly recommend all my dear friends to attend further programs by MTTS as it would be extremely benificial for you.

MATHEMATICS WORKSHOP FOR UG STUDENTS: NIT PUDUCHERRY

-ANJANA ANIL

National Institute of Technology Puducherry (NIT Puducherry or NITPY) is an autonomous public technical and research university located in the city of Karaikal in Union Territory of Puducherry. The Department of Mathematics of NIT Puducherry has been conducting the 'Mathematics Workshop for UG students' to introduce students to diverse mathematical disciplines.

In 2023 the workshop was conducted from 27th September to 1st October for 2nd and 3rd year undergraduate students of mathematics which covered broad spectra of areas including Calculus, Algebra, Analysis and Differential Equations. The workshop also assisted the participants in their preparation for IIT-JAM exam as well. It consisted of lectures section as well as tutorial sections where the lectures were taken up by great teachers from various central universities as well NIT's and IIT's .The total number of students who attended the workshop was 121 from various states of the country.

The students were separated into different groups for their tutorial section where every group has a mentor who is studying in nit for helping them in solving the practice questions assigned during the tutorial sections. Incuding myself a total of 4 students from Amrita Vishwa Vidyapeetham, Kochi Campus participated in the event. The lecture series taken by Dr Barani Balan ,Dr N Annamalai and by Dr G S Mahapatra were the most interesting sessions for me. Attenteding the workshop was very informative for me and also helped in improving my knowledge in various topics of mathematics.This worship also helped me in networking with various people and their mode of thinking and solving problems and helped creating an exposture for a better understanding in mathematics.

MATLAB SIMULINK WORKSHOP

-LAKSHMI UNNIKRISHNAN

In the ever-evolving landscape of mathematics and technology, MATLAB stands out as a powerful tool that has transformed the way mathematical problems are approached and solved. Recently, I had the opportunity to attend a mathematics workshop focused on MATLAB, an event that proved to be both enlightening and enriching. This article aims to share the highlights and insights gained from this workshop.

The workshop was structured to cater to a diverse audience, ranging from engineering students to mathematics enthusiasts seeking to enhance their analytical skills. The 3 day workshop was conducted by Amrita Vishwa Vidyapeetam, Amritapuri campus on behalf of their yearly tech fest 'Vidhyut' .The event promised a deep dive into the functionalities of MATLAB and its applications in various mathematical domains.

Day 1 focused on the introduction to Matlab. The session covered basic commands, variables, and data types, laying a solid foundation for the more advanced topics to follow. It became apparent how the software could streamline complex mathematical computations.

The second day of the workshop focused on advanced commands and the participants were guided and demonstrated with different matlab commands. One of the key takeaways was the integration of MATLAB with Simulink, a companion product that facilitates the modeling and simulation of dynamic systems.
The final day of the workshop centered on vast array of plotting functions and Simulink commands.

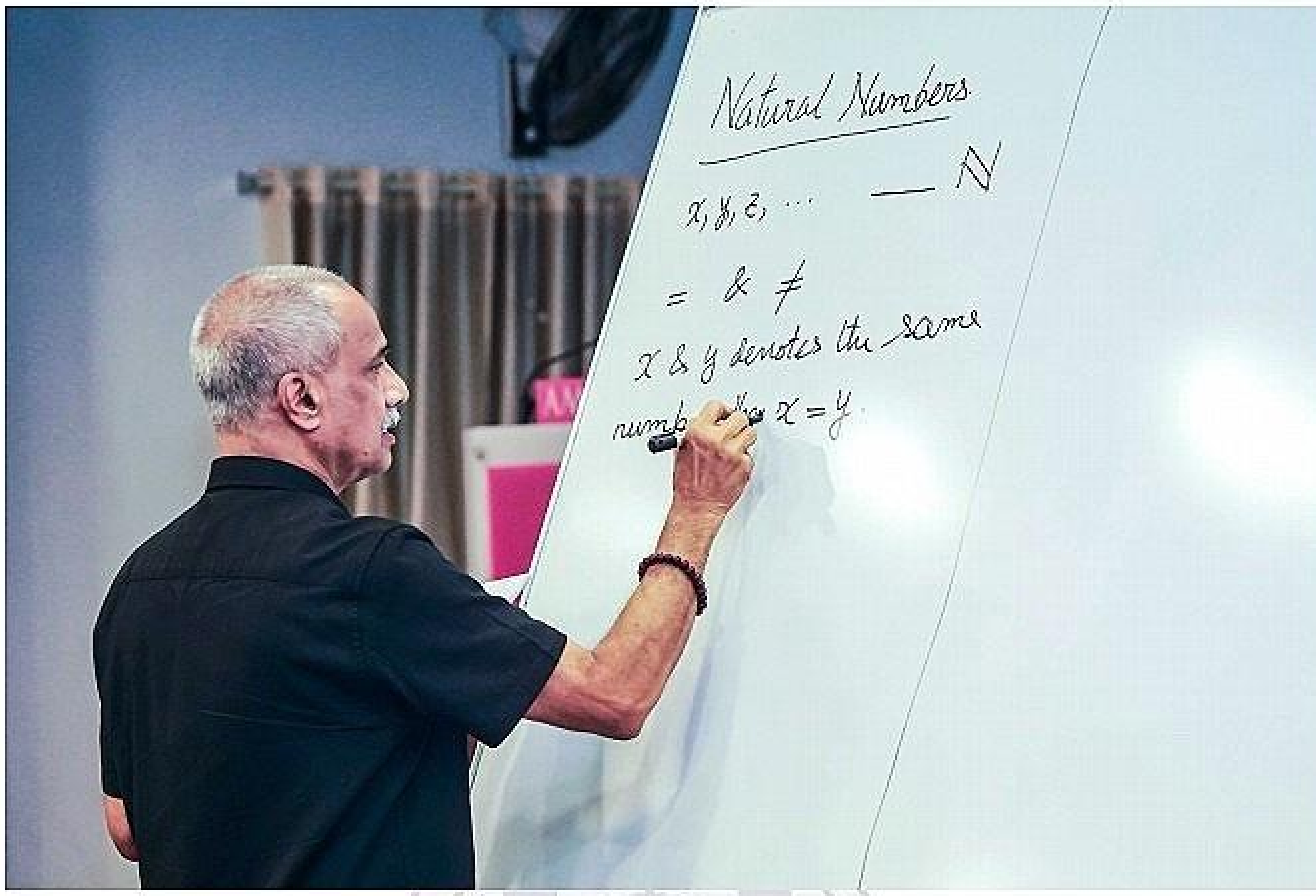
As participants, we not only gained proficiency in MATLAB but also developed a newfound appreciation for its potential in various fields. The workshop served as a catalyst for embracing technology in mathematical exploration, empowering us to apply these skills in our academic pursuits.

ALBUM

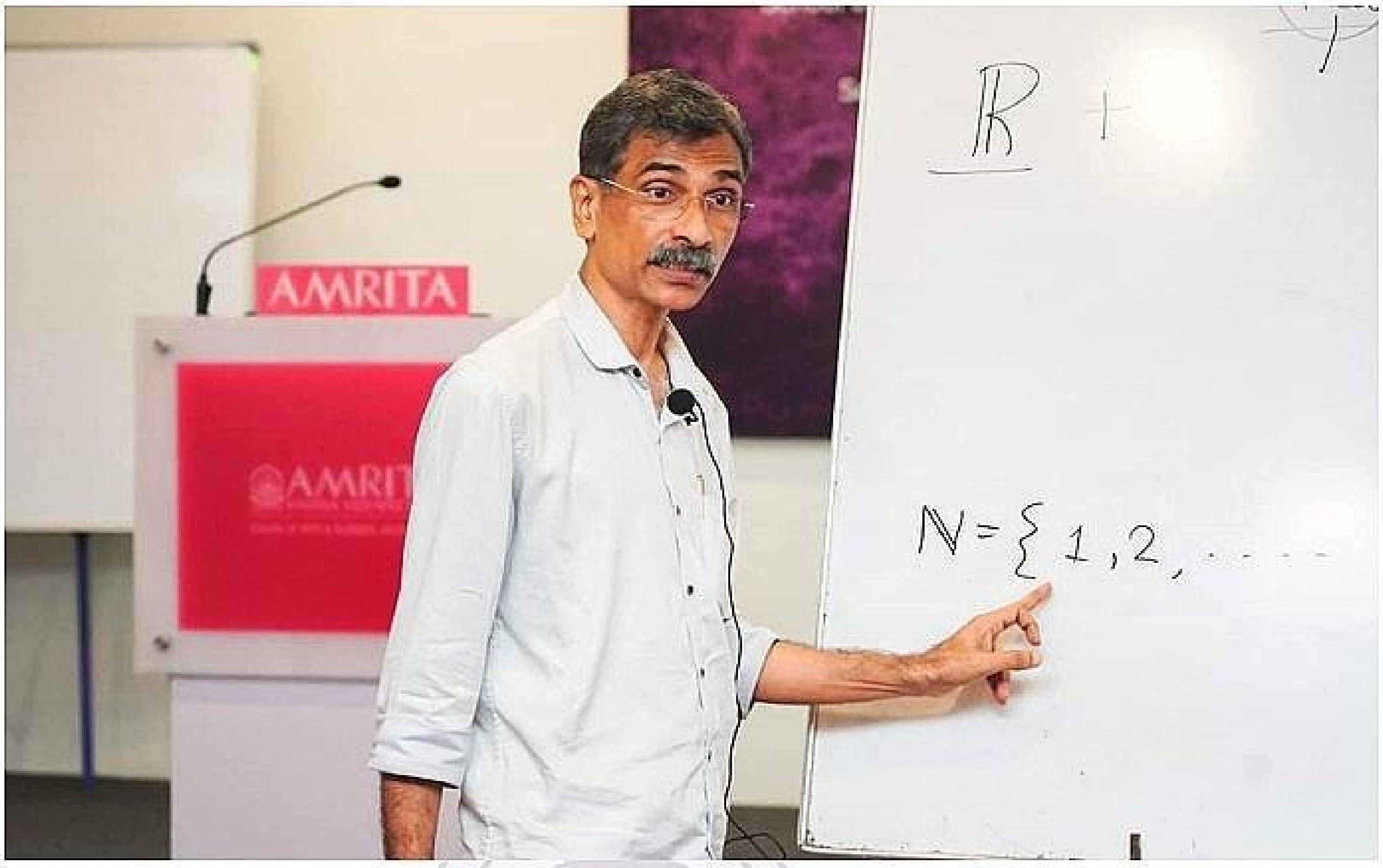
VIDYAMRITAM SESSIONS 2023

Dr. Vellat Krishna Kumar

On "An Excursion Through The Number System"



Dr. Sajith G
On Topology, Geometry and Analysis



Dr. Vellat Krishna Kumar
On Diagonalization

SASTRAMRITAM 2023

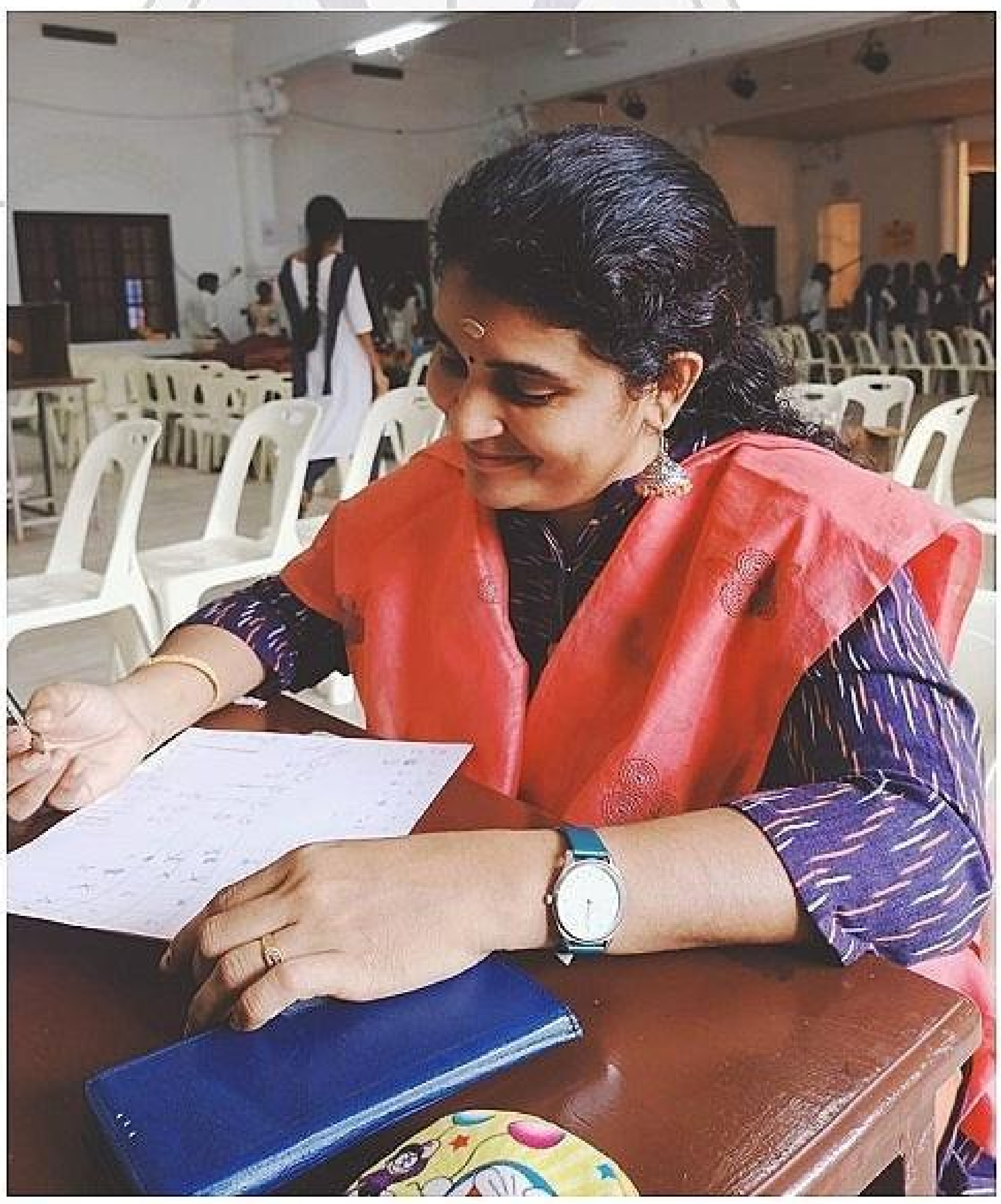
Games club inauguration by Dr.Narayanankutty Karuppath

Exhibition


M A L H A

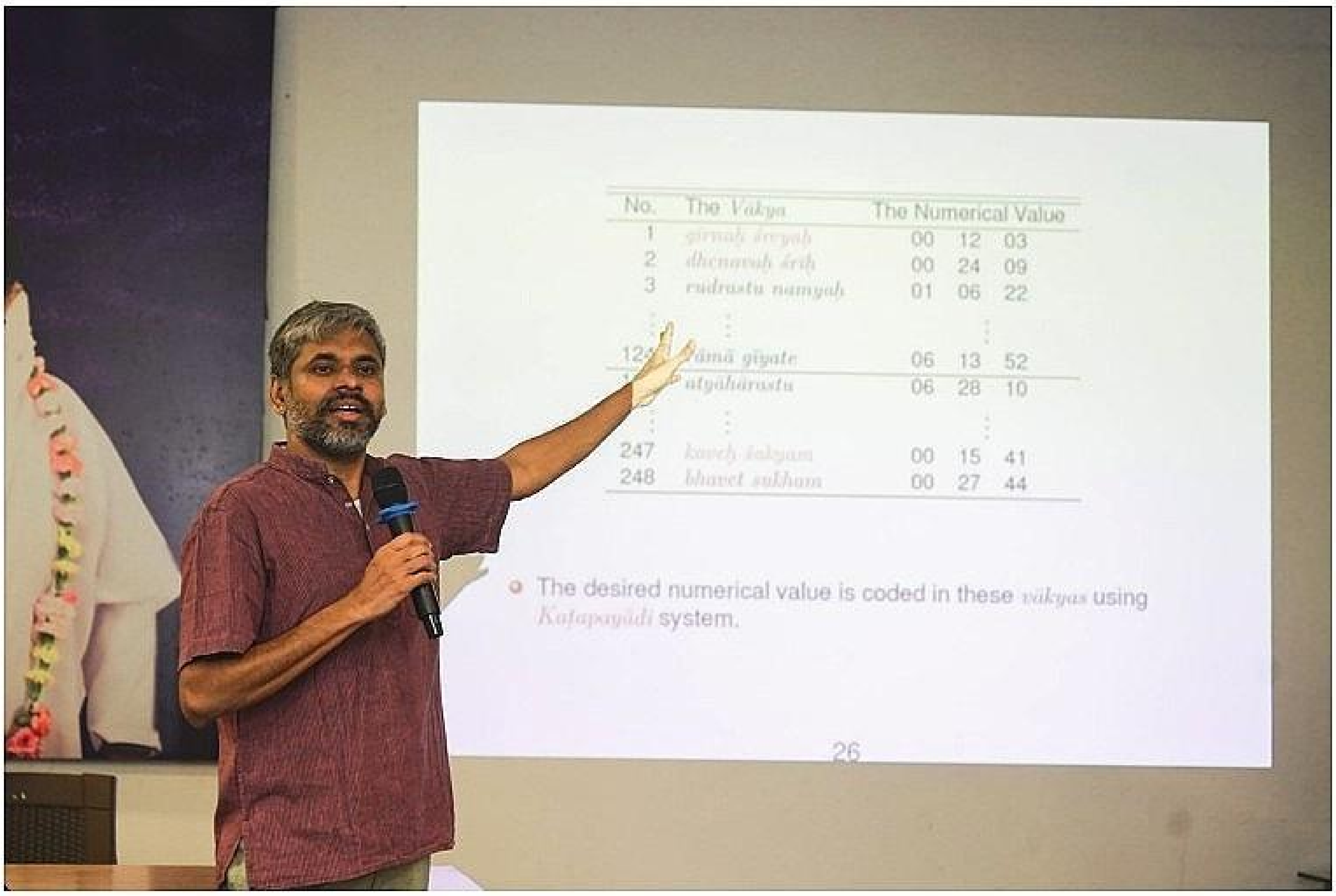

YAMAHA

GAMES CLUB ACTIVITIES

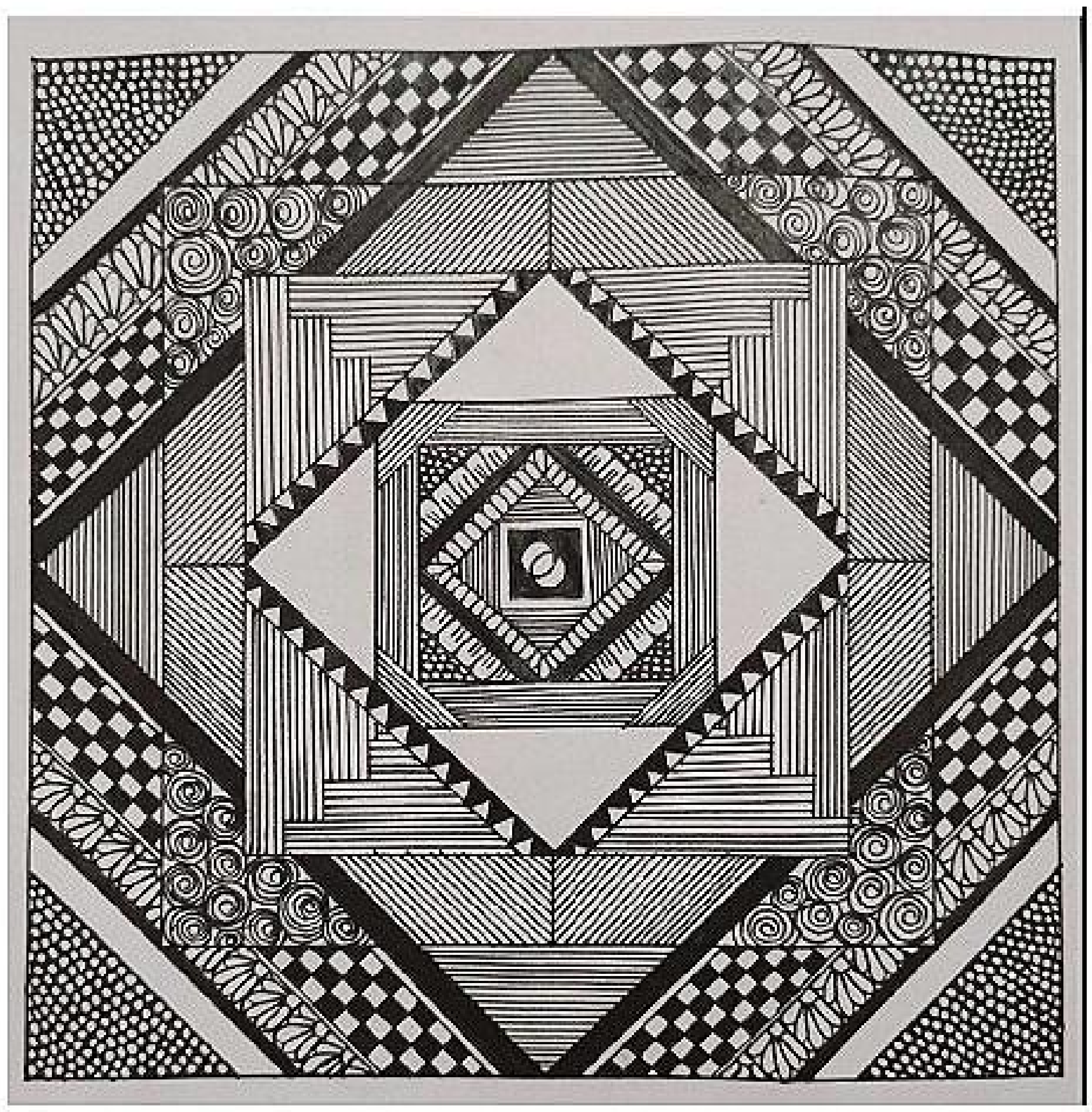


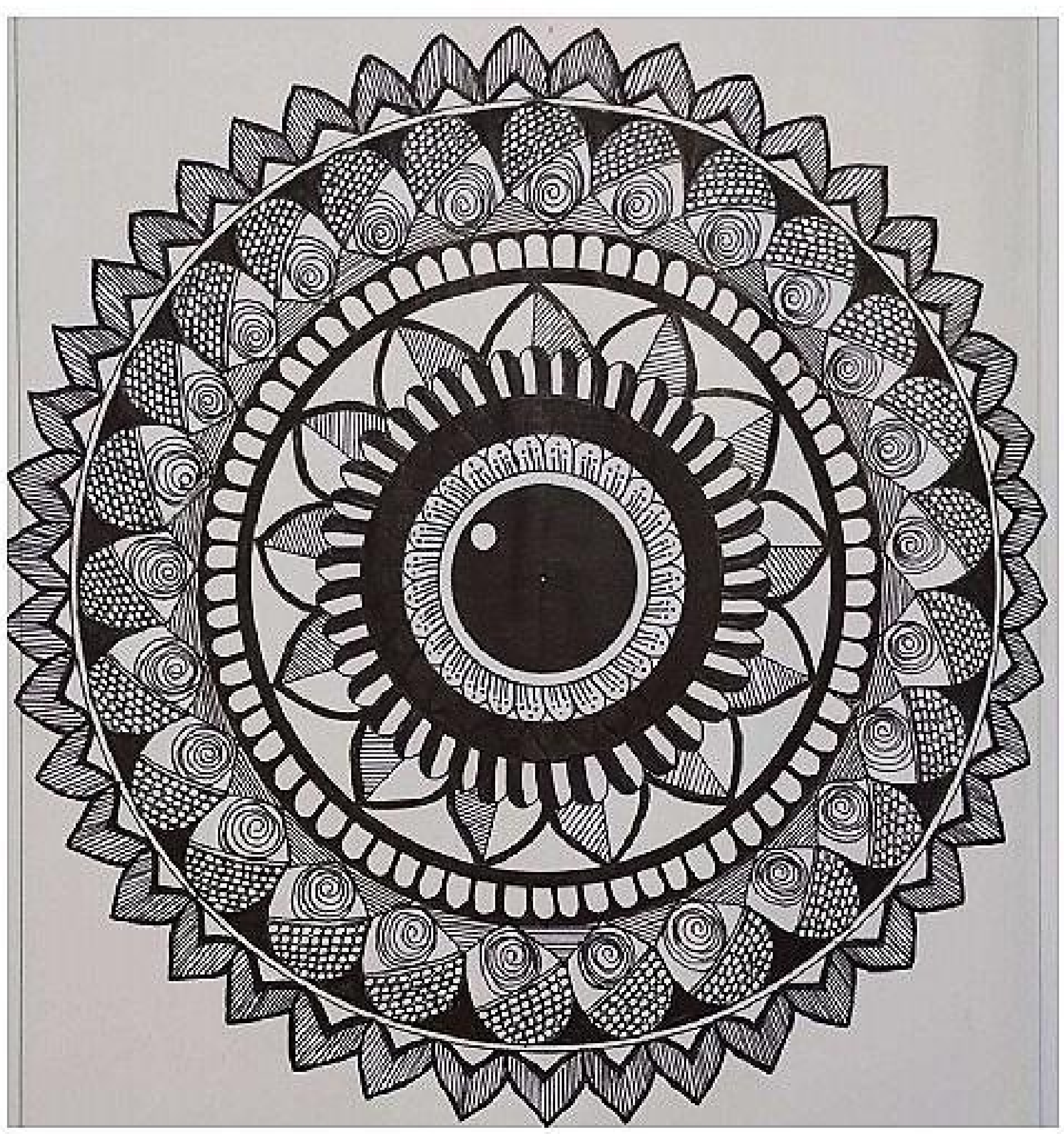
Dr. Venkateshwar R Pai

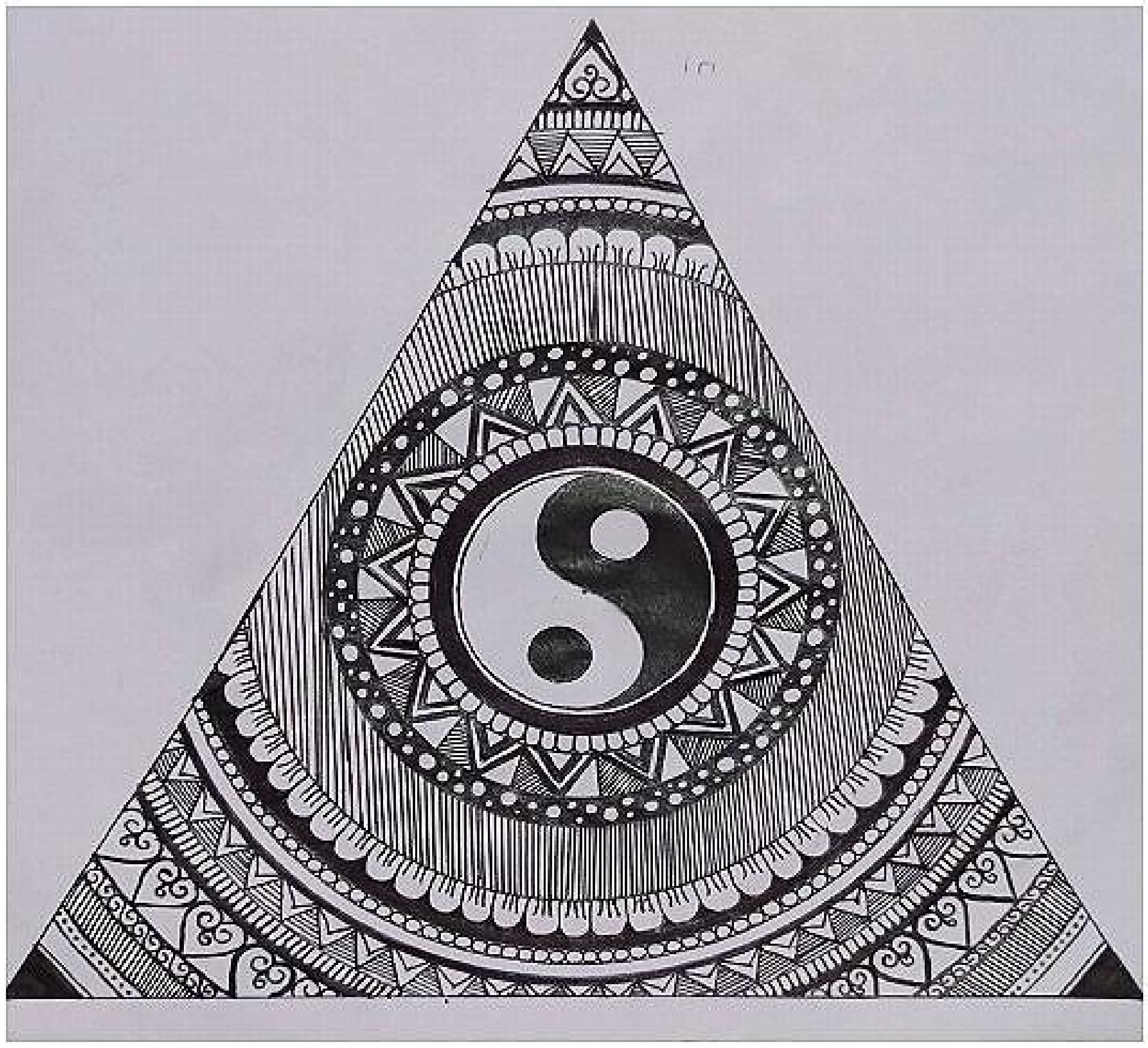
On Candravakyas: A Mathematical Model Expressed Through 248 phrases

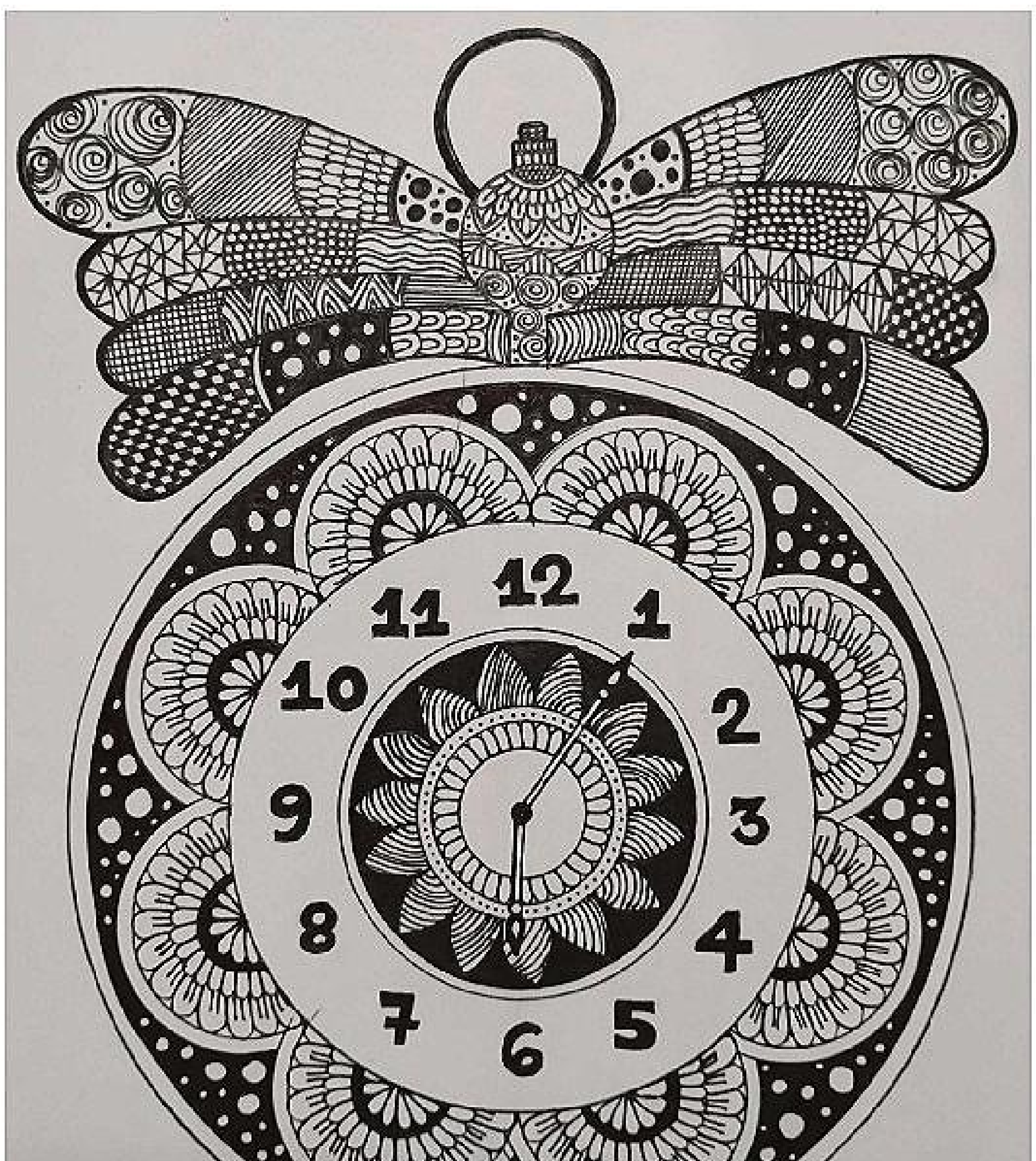


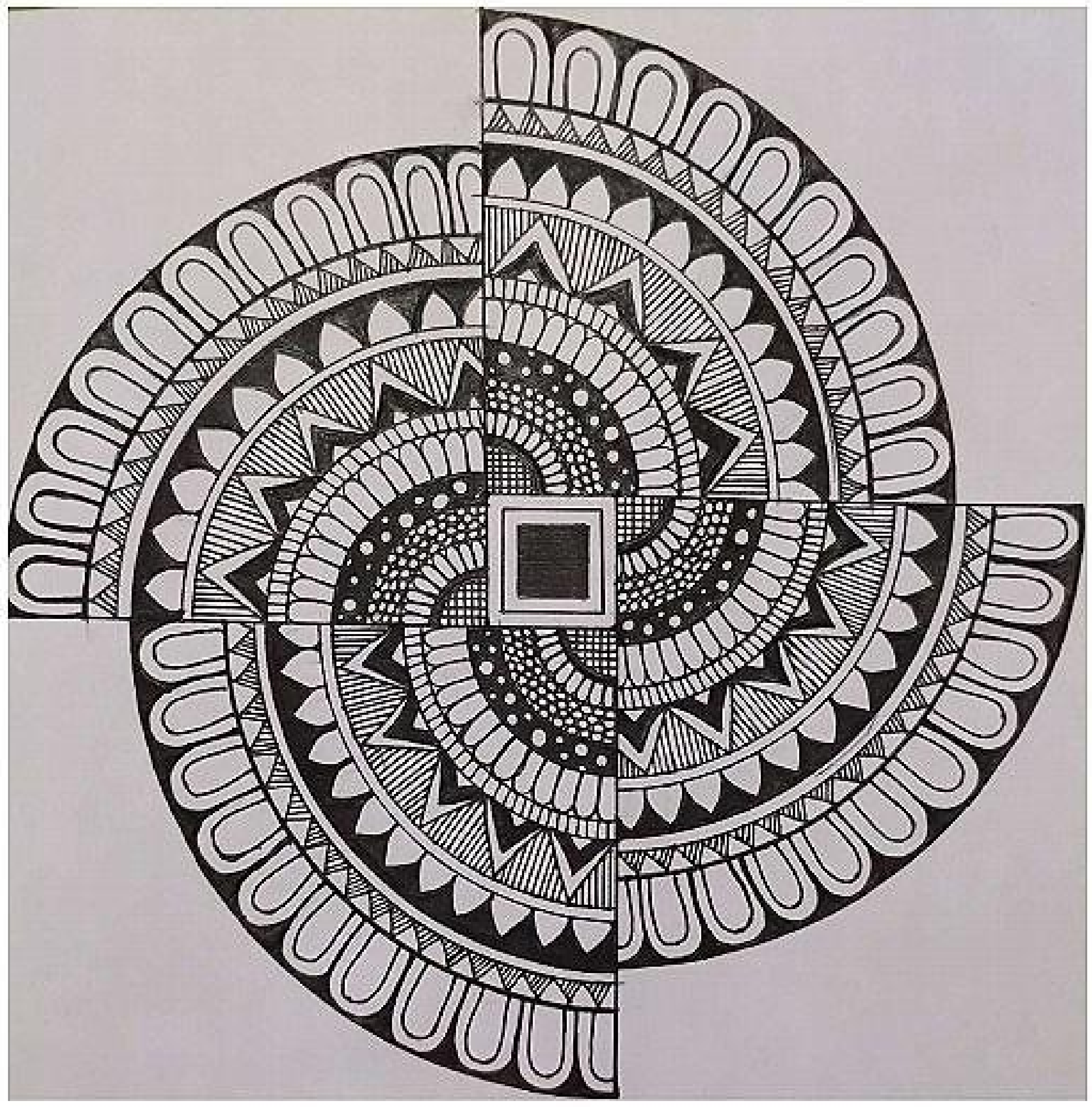
Geoart








12
1
2
3
4
5
6
7
8
9
10
11



MATHSPACIA MEMBERS 2022- 2023

PRESIDENT : R .SARADA (INT MSC MATHS 2019)

VICE PRESIDENT : ADVAITT ASHOK(INT MSC MATHS 2020)

SECRETARY : ANJU S (INT MSC MATHS 2018)

QUIZ CLUB:

HEAD: SHUBHADA (MSC ASDA 2022)

CO HEAD: ADVAITT ASHOK (INT MSC MATHS 2020)

GAMES CLUB:

HEAD : SREELAKSHMI MURALI (INT MSC MATHS 2020)

CO HEAD : RIJA KRISHNA (INTMSC MATHS 2020)

ARTS CLUB:

HEAD : PANCHAMI S.A (INT MSC MATHS 2020)

CO HEAD : GAYATHRI MANOJ (INT MSC MATHS 2020)

SEMINAR CLUB :

HEAD: DHRUV (INT MSC MATHS 2021)

CO HEAD: KAARTHIKANJANA S KUMAR (INT MSC MATHS 2019)

MATHSPACIA

MATHSPACIA MEMBERS 2023-2024

PRESIDENT : KAARTHIKANJANA (INT MSC MATHS 2019)

VICE PRESIDENT: SREELEKSHMI MURALI (INT MSC MATHS 2020)

SECRETARY: SARANYA MURALI(INT MSC MATHS 2019)

GAME CLUB:-

HEAD : NOEL THOMAS VALLAVANTHARA (MSC MATHS 2022)

CO HEAD : AJEESH M UNNI (INT MSC MATHS 2020)

ARTS CLUB:-

HEAD : RIJA KRISHNA(INT MATHS 2020)

CO HEAD : ANEETA M J(ASDA 2023)

SEMINAR CLUB:-

HEAD : ANNIE(INT MATHS 2021)

CO HEAD : MEGHNA (INT MATHS 2022)

QUIZ CLUB:-

HEAD : SHILPA KRISHNAN(ASDA 2023)

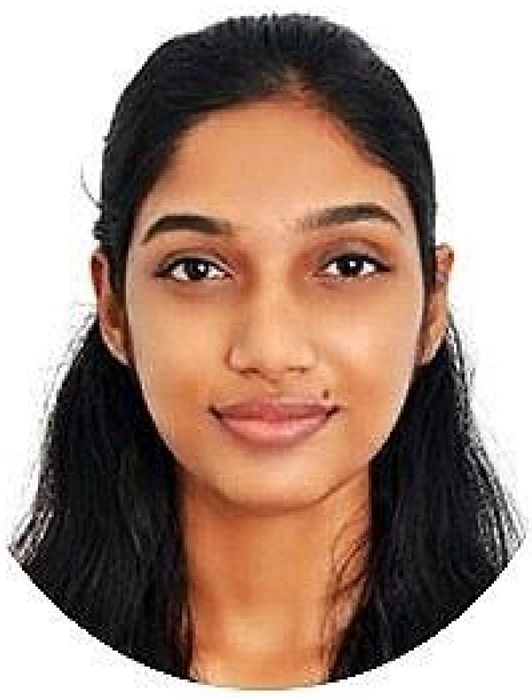
CO HEAD : AKHILA ABHILASH (INT MSC MATHS 2022)

MAGAZINE HEAD : DHRUV(INT MATHS 2021)

EDITORAL BOARD

KAARTHIKANJANA S KUMAR
EDITOR IN CHIEF

CO-EDITORS



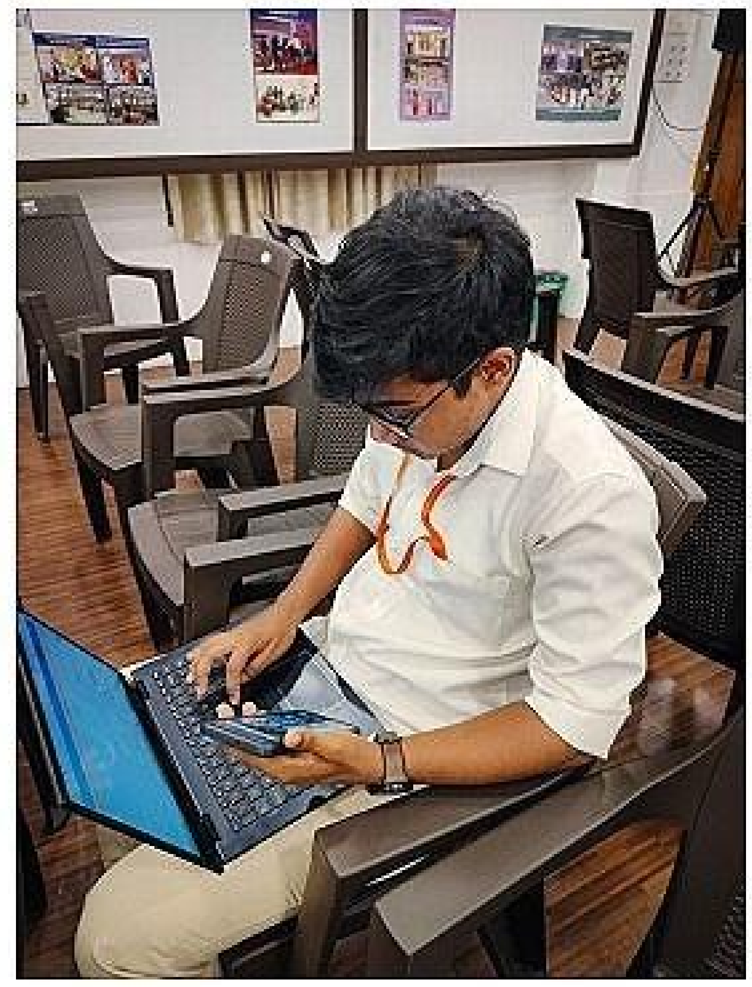
SARANYA MURALI

DHRUV

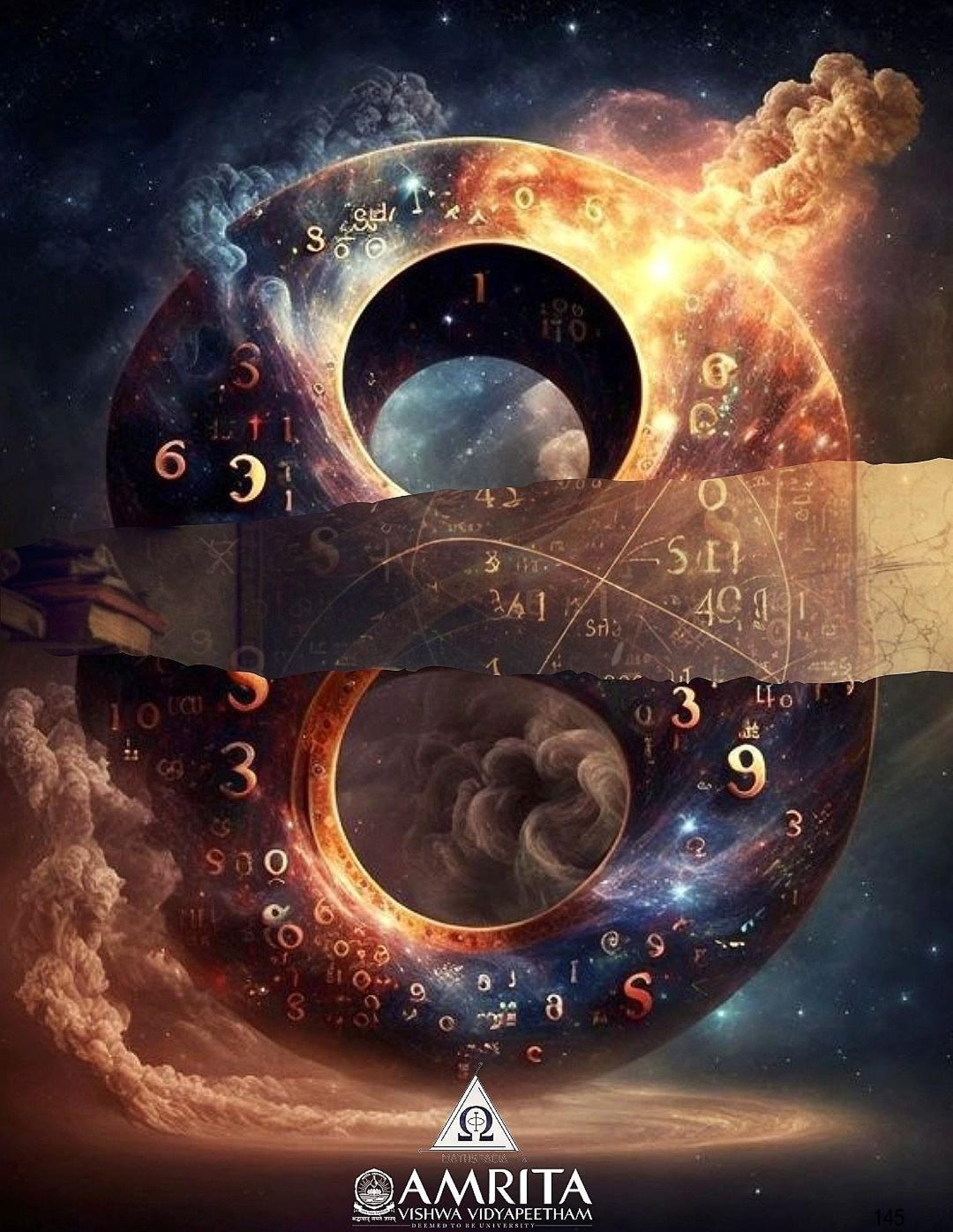
SREELAKSHMI MURALI

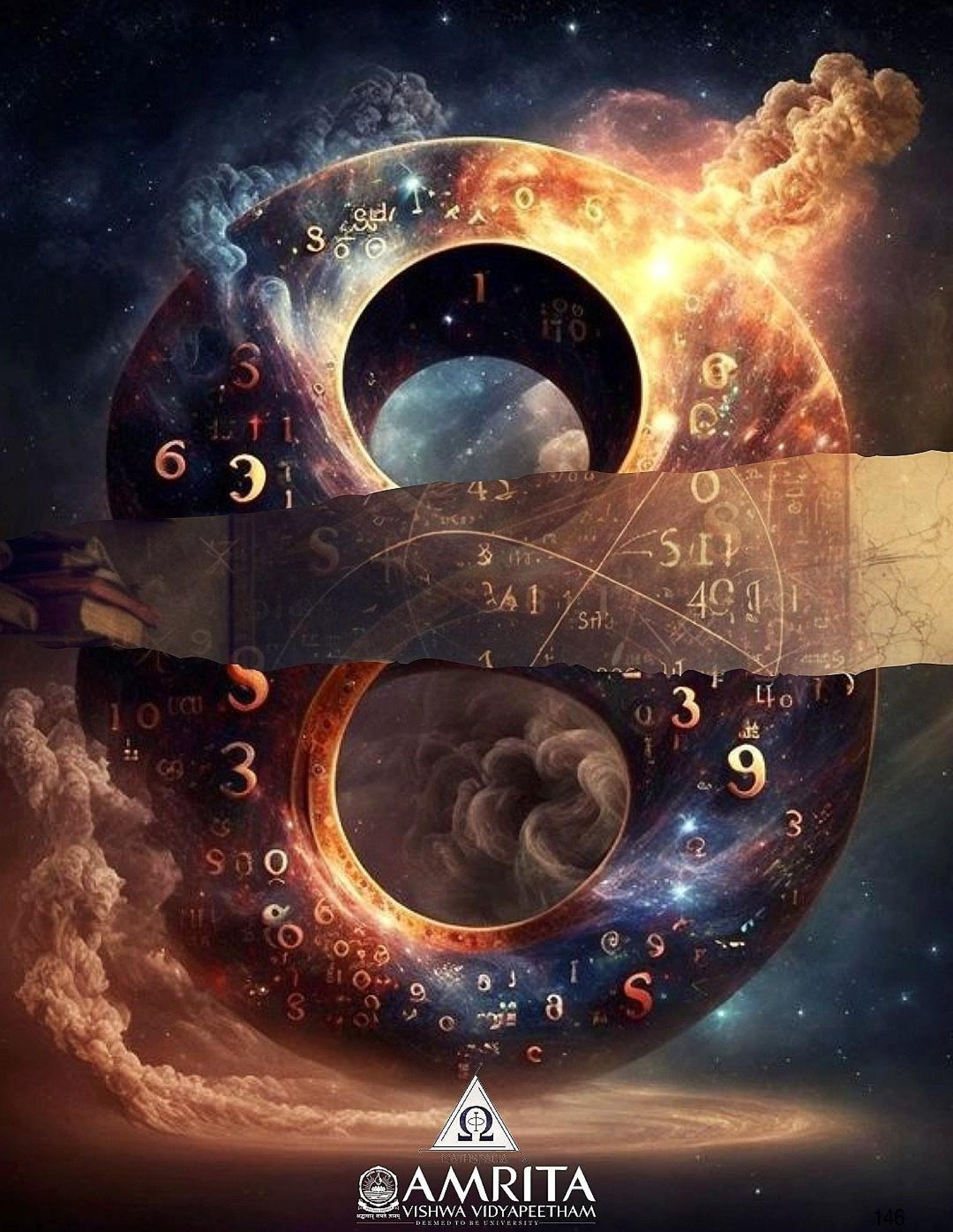
VISHNU BABU


isual Media & Communication
English Language & Literatu




English Language & Literature


AMRITA
VISHWA VIDYAPEETHAM
DEEMED TO BE UNIVERSITY



AMRITA
VISHWA VIDYAPEETHAM
DEEMED TO BE UNIVERSITY