There is no parallel postulate or axioms in sound mathematics!

Mythmaticians of the last 150 years have adopted as dogma the belief that mathematics requires axioms. I suppose when one cannot understand, then all that's left is faith.

Far too long, incorrigible morons known as professors and teachers of mathematics have propagated this lie. Whether they did so sincerely or not, does not change the fact that their claim is a LIE.

Euclid's 5 requirements which are NOT axioms at all, can be derived by ANY sentient being with a reasonable level of intelligence.

We can start with NOTHING. Whoa, you might say ... what about our brains? Well of course we need brains. Note that I said ANY sentient being, you silly human! If you can't think and you don't exist, then obviously you CANNOT realise ANYTHING.

Therefore, with only the ability to reason, we are able to rigorously derive EVERY ONE of the 5 requirements which are incorrectly called 'axioms' by my intellectual inferiors in the mainstream.

Before we begin to reason, we must at least know what it means for a concept to be well defined:

Well-formed concepts.

Knowing what it means to be a *well-formed concept* is not sufficient, but it is necessary!

We begin with the concept of location. In an *empty* universe (excluding our brains and naturally us!), the concept of location can be defined only primitively and developed gradatim.

One cannot even talk of the basic concepts such as direction, path, distance, coordinate at this time. All of these need to be defined gradually starting with the first concept, i.e. location or POINT.

Now the fun begins!

We realise (or "get the idea") of the concept of location when we ask the simple question "Where?" In an empty universe, the first definition of location is primitive and has only two values: "HERE" and "NOT HERE".

HERE is where we are. "NOT HERE" is where we will be if our location changes. Now we are ready to realise the concept of PATH and DIRECTION.

At first, PATH is also primitive, that is, a ***method*** of moving from HERE to NOT HERE. The next step is to introduce another object in the empty universe besides ourselves. That object can be inanimate or just another sentient being, say a friend.

The logical subsequent idea is "How to change location or go from HERE to NOT HERE". So motion is defined as the *change in location*.

The idea of DIRECTION is realised by the question "How to change location or go from HERE to NOT HERE?"

For this, we need a means of sight. We could think of sight in our minds as an ability to identify a particular location, but to make matters simple, I will introduce light in the empty universe, which enables us to see.

Direction is primitively defined as one of two things: TOWARD or AWAY.

We know that being HERE means AWAY from NOT HERE. So if any change in position (or location) results in us being NOT HERE, then we have moved AWAY.

On the other hand, any change in location that results again from NOT HERE back to HERE, means TOWARD HERE.

NOT HERE can mean ANYWHERE else.

We need to identify an object at this stage which is not where we are, that is, HERE.

Now we can define motion a bit more accurately than just a change from HERE to NOT HERE.

Primitively, there are two kinds of motion: one that involves a change in what we see at the destination location from where we are and other that involves no change at all in what we see, not even in the slightest.

At this time we have no concept of plane or space or dimenion yet. Motion as we know it, can be taking place in any random way.

We now reason that between any two locations, it is possible to have more changes or less changes in motion, because if our destination changes just once, then already that particular path will not be equal to the path where what we view remains unchanged. Since we can move at random as we please because we haven't even realised the concept of *boundary* yet, it becomes clear that we cannot determine all the paths possible between any two different locations. I could have written that we cannot determine the "innumerable paths", but I refrained because we are still way off from realising the concept of number! "Numerable" directly implies that the concept of number has been established.

Now we are in a conundrum. There are so many paths and it's daunting to think about the relationship between any two of these paths.

We can call paths the same or equal if the change or movement from source location to destination location involves exactly the same movement (HERE to NOT HERE or vice-versa). From this we realise the concepts of less or more movements (shorter or longer) in any given path. This reasoning leads us to the possibility of the existence of a path requiring the least changes. From this we infer that the path consisting of only those SAME movements from start location to finish location, is the *shortest* path.

The shortest path is the realisation of a **straight line** between two locations. Note that I haven't even talked yet about dimensions such as plane or space!!

In fact, we can't talk about these until we realise the concept of circle which can only be realised after the second requirement.

Till this stage, we have realised the primitive concepts of point (location) and straight line (shortest path).

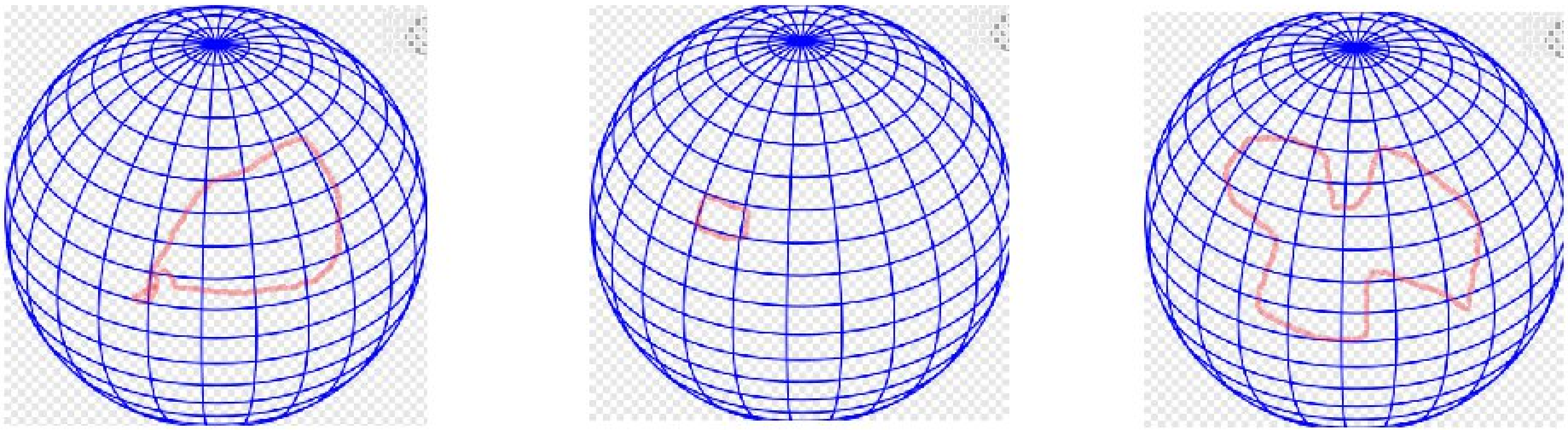
I have been moving very slowly because it's very difficult to derive the first 3 requirements, ie. straight line, extended line and circle.

To show that any path between two points can be extended is elementary because at any time during the motion from one point to another, we can make our HERE the destination point which would mean the path is shorter. This applies to any path, not just a straight line. But since a straight line is just a special path, we are done. So a path can be extended or diminished.

Now we have enough prior knowledge to realise the concept of circle.

This might at first appear to be easy, that is, a path from which any path to a given "centre" is the same straight line. However, paths produced in this way may look nothing like a circle. For example, any complete path on a sphere will meet these requirements.

Any complete path on sphere is a circle by this definition.



This is where requirement 2 (extending a straight line) comes in to play.

We can realise a circle path as a path from which the shortest path to a given point (called a centre) is the same straight line AND that if any such straight line is extended by the same straight line, it will meet the circle path again.

At this stage, we can give the special straight line that is used to define circles the name **radius**, and if the straight line is extended from one point on a circle to another, we call the extended straight line a **diameter**.

And now we can realise the concept of **plane** as follows:

A plane consists of paths which are part of the same circle, if the circle is extended indefinitely in any direction.

We have now established the first 3 requirements of Euclid's Elements systematically. Each requirement uses the previous.

The fourth requirement requires the previous three, but quite a few definitions are needed before we can establish the 4th requirement which states that all right angles are equal. For starters, we need to define angle which is easy, but not so straight forward as you might imagine.

Symmetry is the attribute of being made up of exactly the same paths (comprised of motion from one location to another).

An **arc** is any part of a circle's path.

How do we know that a circle's paths from one end of a diameter to the other end are the same?

We know that the arcs (semi-circle circumference paths) formed on either side of a circle's diameter are equal because either can be generated by the same radius.

Define an angle primitively as the ***ratio*** of any given arc of a circle to its radius, ie. angle = **arc : radius**. Later, we can define angle as a number, ie. angle = $\frac{\textbf{arc length}}{\textbf{radius length}}$

At this stage we don't care about measure of arc or radius paths because we do not have numbers yet. All that **ratio** means is that the **aliquot path parts being compared**, that is, **arc** and **radius** are in a certain relation. The result of the comparison is irrelevant for now. All that we need to know is that we can compare any paths qualitatively (without numbers), disregarding accuracy.

Now we have already established that a circle's **diameter** partitions the circle path (known as **circumference**) into symmetrical paths (known as **semi-circle arcs**).

We know that the **centre** partitions the diameter into equal radii since any radius which is extended, has the same centre point in common.

The radius from the centre point of a diameter which partitions a semi-circle arc into symmetrical paths, is called a **perpendicular** (*) to that diameter. If such a perpendicular is extended, it will also partition the other semi-circle into symmetrical paths, so that all the arcs created by a perpendicular to a diameter are all equal.

(*) We know this is possible because of circle symmetry. It's easy to see when we form the semi-circle arcs since both rest on the diameter.

What is true of one arc formed in such a way is also true of the semi-circle arcs.

Proof: If you want to argue otherwise, let it be so that the semicircular arcs are not divided into equal parts, but this very thing is impossible because it would contradict the symmetry of the semicircular arcs. It's that simple.

This proof was added because of a challenge by Comuniune cu Osho – Campul Budic.

The 4th requirement: Angles and Right Angles. - YouTube

Now we are ready to define any **right angle** as the ratio of an arc formed by the partitioning of a circle's circumference into equal parts using only a diameter and its perpendicular.

Observe that no concept of measure or number has been used at all. Once we establish number, we can count the right angles in a circle.

In 17 pages, we have established the first FOUR requirements.

Next we need to define parallel lines before we can derive the 5th or last requirement of Euclid's Elements. But to do this, we need to realise a few more concepts. For the realisation of parallel lines, we realise the concept known as a chord or secant.

A chord is any straight line that has endpoints (locations) which are part of the circle circumference.

Definition of parallel line:

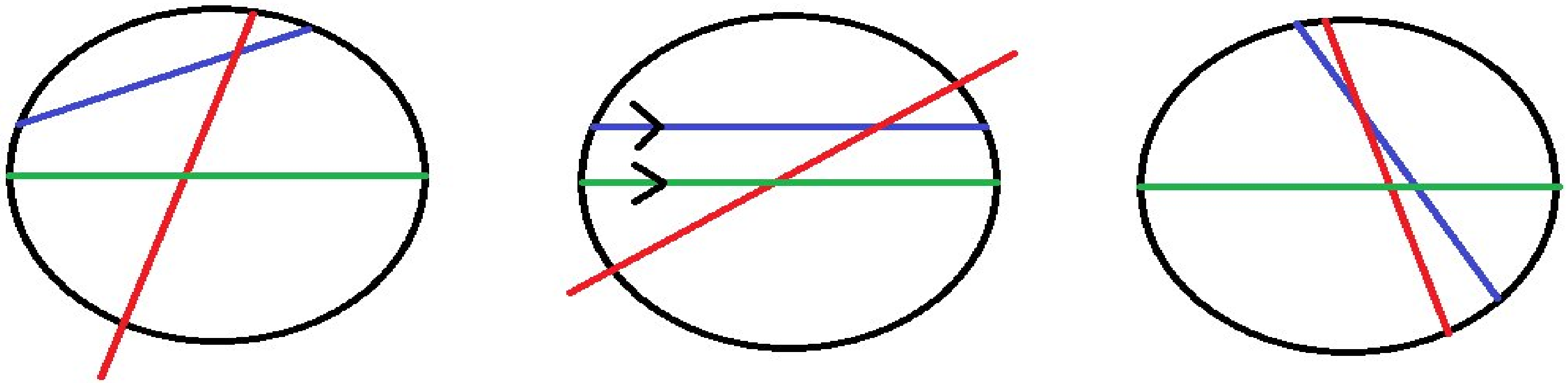
If the arcs formed by a chord (on either side of a perpendicular) between the chord and a diameter are equal (symmetrical), then the chord is **PARALLEL** to the diameter.

Thus, to establish in a rudimentary fashion if two STRAIGHT lines are parallel, either straight line must coincide (or overlap) at least partly over the diameter and the other over a circle chord or secant.

Now we need some more definitions and a theorem before we can derive the 5th and final requirement.

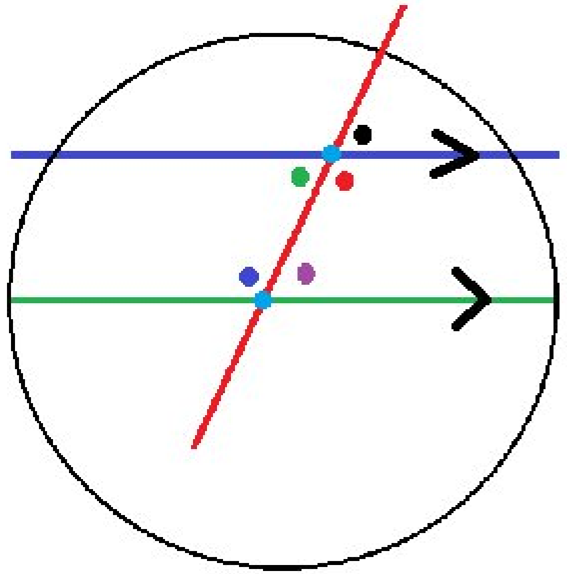
We define a **transversal** as a straight line which meets any secant line and diameter in different points.

Red lines are transversal lines. Blue lines are chords or secant lines. The green lines are diameters.



We can call the point where two lines of an angle meet, a **vertex** since we also want to talk about angles not at circle centres.

Given any two parallel lines and a transversal crossing them, we need to give names to the angles formed:



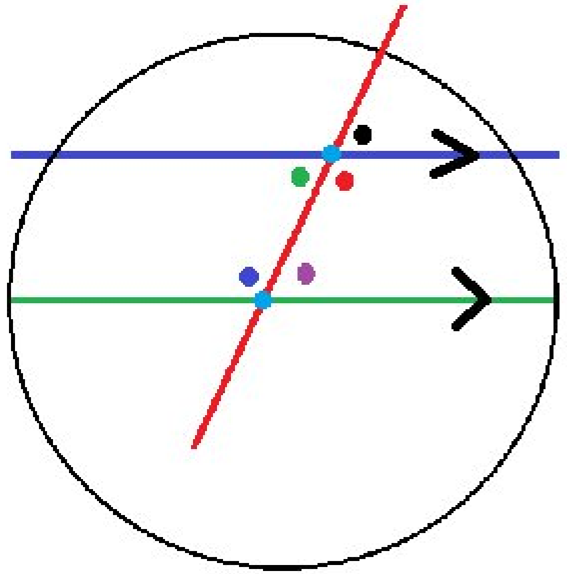
We call the red and purple angles cointerior angles.
The dark blue and green angles are also cointerior angles.

The black and green angles are called vertically opposite angles.

Finally, the black and purple angles are called corresponding angles.
Next, we have to prove that the given pairs of angles are equal.

We can conclude that the black and green angles are equal, because a circle with centre at the light blue point (on the blue parallel line) has the same the same transversal at its centre, the same parallel line and the same arc on which the black and green angles are subtended. Similarly we can reason that the black and purple angles are equal because of two equal circles with centres at the light blue points.

The alternate green and purple angles are equal because they are both equal to the black angle.



We know that the sum of supplementary (*) angles is the same as two right angles because we proved this earlier. If the light blue points are centres, the result is immediately evident.

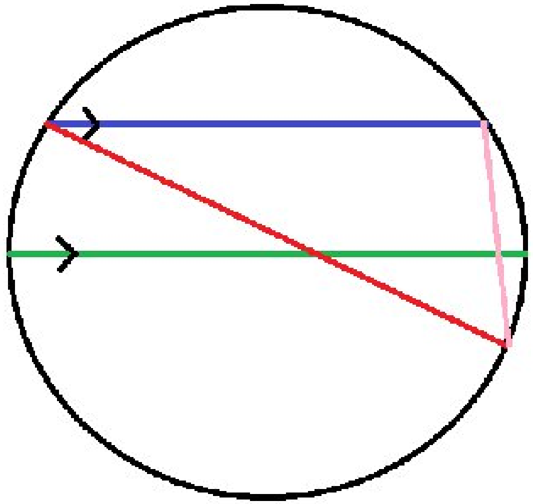
Moreover, we can prove that the sum of the cointerior angles (say red and purple) is equal to two right angles because the purple angle is equal to the green angle (alternate angles) and the green and red angles are supplementary.

(*) Angles with the same vertex that lie on a straight line.

We are still not ready to derive the 5th and last requirement.

A triangle is defined as that path which is formed by joining the endpoints of a secant line that is parallel to a diameter(*), to another point on the circle's circumference such that the joined paths are straight lines. Again, note that I said nothing about the number of points or vertices because I have not yet derived number.

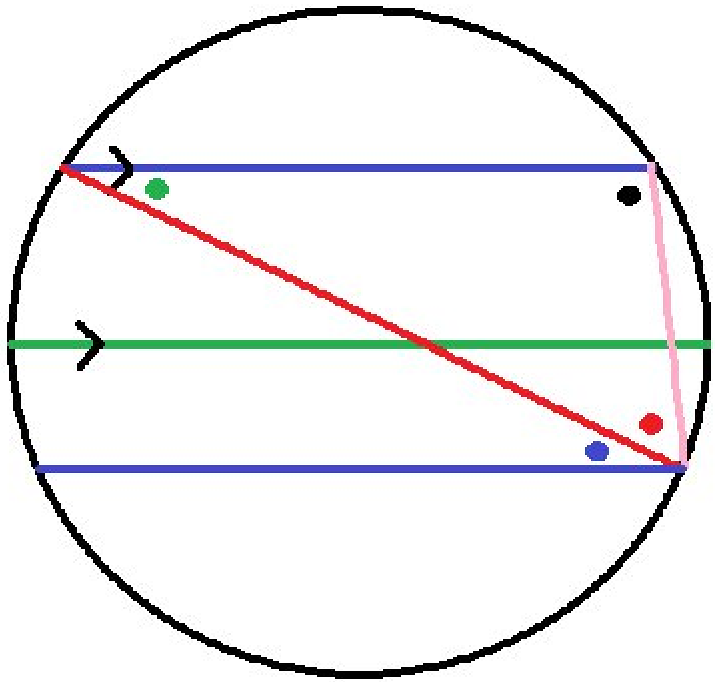
In the diagram, the triangle path consists of the blue, red and pink lines.



(*) It is easy to show that for any given secant line, there is a diameter line that is parallel.

We know that a parallel blue secant line exists that is parallel to the diameter at the point where the pink and blue lines meet below the diameter.

Since the black, red and blue angles have a sum that is two right angles, it follows that the sum of the red, green and black angles must also be two right angles because the green angle is equal to the blue angle (alternate angles on parallel lines are equal).



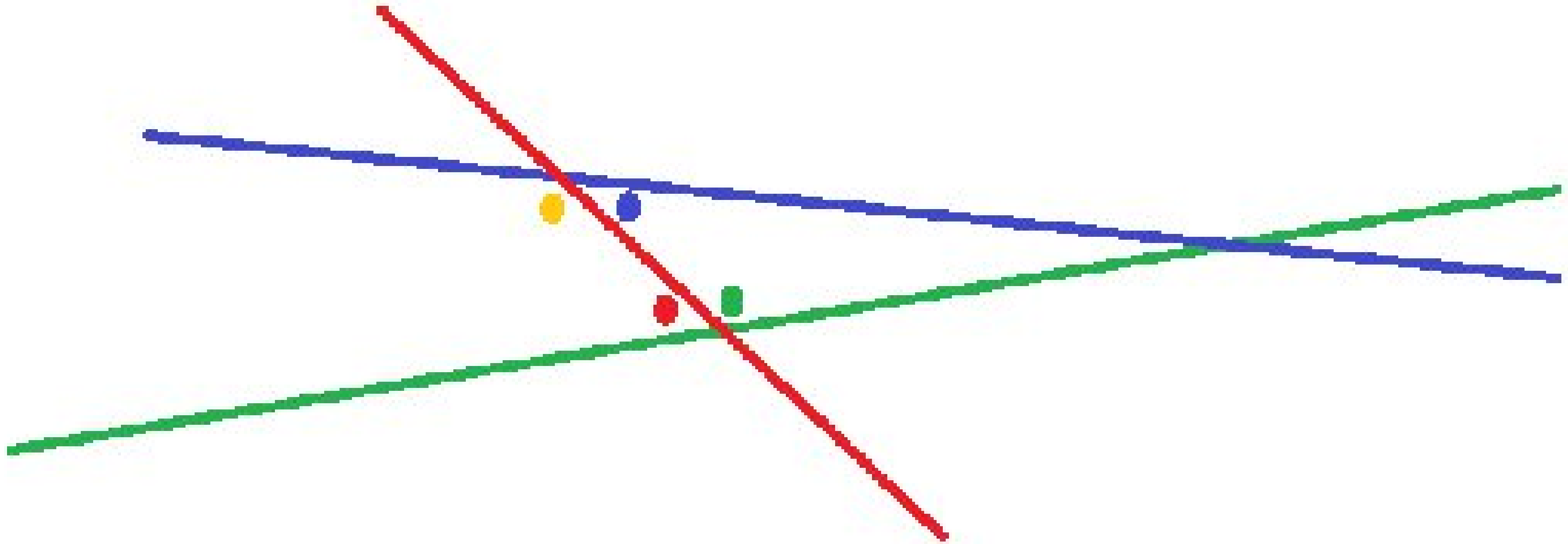
So the theorem proved states that the sum of any triangle is equal to two right angles.

Now we are ready to derive the 5th requirement.

The 5th requirement states that the sum of cointerior angles on either side of a transversal crossing ANY two lines, is constant.

Do you see anything about "parallelism" in the statement? In actual fact the 5th requirement in the original Greek is stated without using the word parallel at all. So how do we derive the 5th requirement from the previous four?

Before I continue, the SUM of angles does not require any arithmetic, only the joining of arcs on a circle circumference.

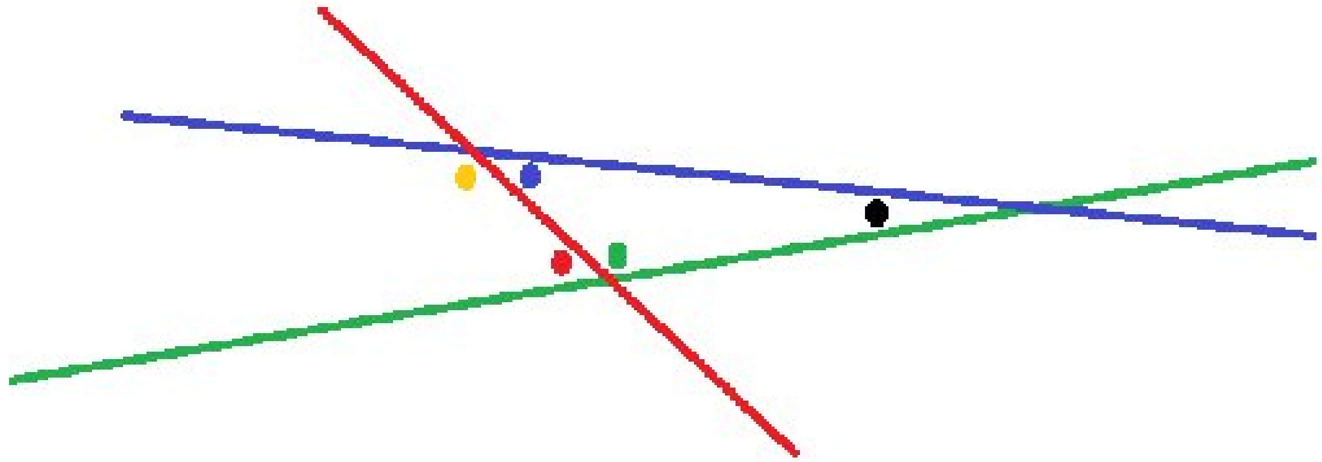


The 5th requirement states that as long as the red line (transversal) crosses both the blue and green lines, the sum of yellow and red angles remains constant. Also the sum of the blue and green angles remains constant.

To prove that the 5th requirement is true, is fairly straight forward.

Since the green, blue and black angles are part of a triangle, their sum is two right angles. Therefore, blue and green angles will always be supplementary to black angle and hence their sum cannot change.

But the yellow and red angles are supplementary to the blue and green angles respectively, therefore their sum also cannot change.



As you can see, we have already 26 pages to demonstrate the systematic derivation of the 5 requirements.

Using only straight lines, we can complete the systematic derivation of the number concept. See chapter on How we got numbers in my free eBook:

How we got numbers - the true story

My eBook contains a summary of these 26 pages that does not explain in as much detail as I have done so here.

I am certain the Ancient Greeks were able to systematically derive all the requirements as I have done. This means they are NOT axioms.

Far too long, the false belief of axioms being necessary for mathematics prevailed.

If you are of average intelligence and you still don't see the truth of my claims, I can only conclude that you are an incorrigible crank.

Be certain to download my free eBook explaining the first rigorous formulation of calculus in human history.