Department of Education
Region X - Northern Mindanao
DIVISION OF CAGAYAN DE ORO
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Learning Activity Sheets in Statistics and Probability

SHARED OPTIONS

Senior High Alternative Responsive Education Delivery

Competence.Dedication.Optimism

Preface

It has been elaborated in research and literature that the highest performing education systems are those that combine quality with equity. Quality education in the Department of Education (DepEd) is ensured by the learning standards in content and performance laid in the curriculum guide. Equity in education means that personal or social circumstances such as gender, ethnic origin or family background, are not obstacles to achieving educational potential and that inclusively, all individuals reach at least a basic minimum level of skills.

In these education systems, the vast majority of learners have the opportunity to attain high-level skills, regardless of their own personal and socio-economic circumstances. This corresponds to the aim of DepEd Cagayan de Oro City that no learner is left in the progression of learning. Through DepEd’s flexible learning options (FLO), learners who have sought to continue their learning can still pursue in the Open High School Program (OHSP) or in the Alternative Learning System (ALS).

One of the most efficient educational strategies carried out by DepEd Cagayan de Oro City at the present is the investment in FLO all the way up to senior high school. Hence, Senior High School Alternative Responsive Education Delivery (SHARED) Options is operationalized as a brainchild of the Schools Division Superintendent, Jonathan S. Dela Peña, PhD.

Two secondary schools, Bulua National High School and Lapasan National High School, and two government facilities, Bureau of Jail Management and Penology-Cagayan de Oro City Jail and Department of Health-Treatment and Rehabilitation Center-Cagayan de Oro City, are implementing the SHARED Options.

To keep up with the student-centeredness of the K to 12 Basic Education Curriculum, SHARED Options facilitators are adopting the tenets of Dynamic Learning Program (DLP) that encourages responsible and accountable learning.

This compilation of DLP learning activity sheets is an instrument to achieve quality and equity in educating our learners in the second wind. This is a green light for SHARED Options and the DLP learning activity sheets will continually improve over the years.

Ray Butch D. Mahinay, PhD
Jean S. Macasero, PhD

Acknowledgment

The operation of the Senior High School Alternative Responsive Education Delivery (SHARED) Options took off with confidence that learners with limited opportunities to senior high school education can still pursue and complete it. With a pool of competent, dedicated, and optimistic Dynamic Learning Program (DLP) writers, validators, and consultants, the SHARED Options is in full swing.

Gratitude is due to the following:

- Schools Division Superintendent, Jonathan S. Dela Peña, PhD, Assistant Schools Division Superintendent Alicia E. Anghay, PhD, for authoring and buoying up this initiative to the fullest;
- CID Chief Lorebina C. Carrasco, and SGOD Chief Rosalio R. Vitorillo, for the consistent support to all activities in the SHARED Options;
- School principals and senior high school teachers from Bulua NHS, Lapasan NHS, Puerto NHS and Lumbia NHS, for the legwork that SHARED Options is always in vigor;
- Stakeholders who partnered in the launching and operation of SHARED Options, specifically to the Bureau of Jail Management and Penology-Cagayan de Oro City Jail and the Department of Health-Treatment and Rehabilitation Center-Cagayan de Oro City;

- Writers and validators of the DLP learning activity sheets, to which this compilation is heavily attributable to, for their expertise and time spent in the workshops;
- Alternative Learning System implementers, for the technical assistance given to the sessions; and
- To all who in one way or another have contributed to the undertakings of SHARED Options.

Mabuhay ang mga mag-aaral! Ito ay para sa kanila, para sa bayan!

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ACTIVITY NUMBER	LEARNING ACTIVITY TITLE	DATE	SCORE	ITEM
1	Illustrating a Random Variable			1
2	Distinguishing Between Discrete and a Continuous Variable			5
3	Illustrating a Probability Distribution for a Discrete Random Variable			3
4	Probability Distribution for a Discrete Random Variable (A)			1
5	Probability Distribution for a Discrete Random Variable (B)			1
6	Mean of a Discrete Random Variable			1
7	Variance of a Discrete Random Variable (A)			5
8	Variance of a Discrete Random Variable (B)			3
9	Variance of a Discrete Random Variable (C)			3
10	Illustrating a Normal Random Variable and Its Properties			3
11	Understanding the Normal Curve			3
12	Identifying Regions Under the Normal Curve			1
13	Converting a Normal Random Variable to a Standard Normal Variable			1
14	Computing Probabilities Using the Standard Normal Table			1
15	Computing Percentiles Using the Standard Normal Table			1
16	Illustrating Random Sampling			1
17	Distinguishing between Parameter and Statistic			1
18	Identifying Sampling Distributions Of Statistics			1
19	Finding The Mean And Variance Of The Population			1
20	Finding The Mean And Variance Of The Sampling Distribution Of The Sample Means			1
21	Describing The Sampling Distribution By Computing The Means And Standard Deviation			1
22	Solving Problem Involving Sampling Distribution Of Sample Means			1
23	Solving The Point Estimate			3
24	Confidence Interval Estimator For The Population Mean (A)			3
25	Confidence Interval Estimator For The Population Mean (A)			1
26	Identifying Regions Under the t-distribution			2
27	Computing for the Confidence Interval Estimate			2
28	Computing for the Confidence Interval Estimate			3
29	Identifying Point Estimator for the Population Proportion			1
30	Identifying the Appropriate Form of the Confidence Interval Estimator			3
31	Identifying the Length of a Confidence Interval			2
32	Computing for an Appropriate Sample Size Using the Length of the Interval			1
33	Illustrating Null and Alternative Hypothesis			1
34	Illustrating Type of Errors in Hypothesis Testing			1
35	Calculating the Probabilities of Committing a Type I and Type II Error			1
36	Solving Problems Involving Test of hypothesis on the Population Mean (A)			3
37	Solving Problems Involving Test of hypothesis on the Population Mean (B)			1
38	Solving Problems Involving Test of hypothesis on the Population Mean (C)			4
39	Solving Problems Involving Test of hypothesis on the Population Mean (D)			1
40	Solving Problems Involving Test of hypothesis on the Population Mean (E)			5
41	Computing for the test-statistic of Population Proportion (A)			2
42	Computing for the test-statistic of Population Proportion (B)			1
43	Computing for the test-statistic of Population Proportion (C)			1
44	Illustrating a Bivariate Data			3
45	Constructing a Scatter Plot			1
46	Estimating Strength of Association Between Variables			4
47	Calculating the Pearson's Sample Correlation Coefficient			1
48	Identifying the Independent and Dependent Variable			5
49	Drawing the Best Fit on a Scatter Plot			2
50	Calculating the Slope and Y-intercept of the Regression Line			1
51	Interpreting the Calculated Slope and Y-intercept of the Regression Line			1

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating Random Variable		
Lesson Competency : To illustrate a random variable (M11/12SP-IIIa-1) To find the possible values of a random variable (M11/12SP-IIIa-3)		
References : Statistics and Probability pages 2-5		LAS No.: 3.1

CONCEPT NOTES

A **random variable** is a function that associates a real number to each element in the sample space. It is a variable whose values are determined by chance. Use capital letters to denote or represent a variable.

The set of all possible outcomes of an experiment is called a **sample space.**

EXAMPLE:

Suppose three cell phones are tested at random. We want to find out the number of defective cell phones that occur.

Let D represent the defective cell phone and N represent the non-defective cell phone. If we let X be the random variable representing the number of defective cell phones, can you show the values of the random variable X? Consider the table below.

Possible Outcomes/Sample Space	**Value of the Random Variable X (number of defective cell phones)**
NNN	0
NND	1
NDN	1
DNN	1
NDD	2
DND	2
DDN	2
DDD	3

The values of the random variable are 0, 1, 2, 3. This means that there could be a zero, one, two, or three numbers of defective cell phones in the possible outcomes of an experiment.

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the values of the random variable Y. Make a table to illustrate the situation.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Distinguishing Discrete from Continuous Random Variable		
Lesson Competency : To distinguish between a discrete and a continuous random variable. (M11/12SP-IIIa-2)		
References : Statistics and Probability pages 6-8		LAS No.: 3.2

CONCEPT NOTES

The random variables whose set of possible outcomes is countable are called **discrete random variables.** Mostly, discrete random variables represent count data, such as the number of defective chairs produced in a factory.

A random variable is a **continuous random variable** if it takes on values on a continuous scale. Often, continuous random variables represent measured data, such as heights, weights, and temperatures.

Example:

Classify the following random variables as discrete or continuous.

1. the number of defective computers produced by a manufacturer

Answer: Discrete because number of defective computers can be counted as a whole number

2. the weight of newborns each year in the hospital

Answer: Continuous because the weight of newborns cannot be counted as a whole number but it can be measured.

EXERCISES:

Classify the following random variables as discrete or continuous.

1. the number of siblings in a family of a region
2. the speed of a car
3. the number of female students
4. the number of presidents of the Philippines
5. the amount of sugar in a cup of coffee

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating a Probability Distribution for a Discrete Random Variable		
Lesson Competency : To illustrate a probability distribution for a discrete random variable and its properties. (M11/12SP-IIIa-4)		
References : Statistics and Probability pages 9, 15-16		LAS No.: 3.3

CONCEPT NOTES

A **discrete probability distribution** or a **probability mass function** consists of the values a random variable can assume and the corresponding probabilities of the values.

Properties of a Probability Distribution

1. The probability of each value of the random variable must be between or equal to 0 and 1. In symbol, we write it as $0 \leq P(X) \leq 1$.

2. The sum of the probabilities of all values of the random variable must be equal to 1. In symbol, we write it as $\Sigma P(X) = 1$.

Examples:

A. Find the probability of the following events:

1. Getting an even number in a single roll of a die.

Answer: $\frac{1}{2}$ since there are 3 even numbers in a die.

2. Getting a red ball from a box containing 3 red and 6 black balls.

Answer: $\frac{1}{3}$ since there are 3 out of 9 balls from a box

B. Determine whether the distribution represents a probability distribution.

X	1	5	8
P(X)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Answer: It is a probability distribution because the total or the sum of P(X) is equal to 1.

EXERCISES:

A. Find the probability of the following events:

1. Getting an odd number in a single roll of a die.
2. Getting a tail in tossing a coin.

B. Determine whether the distribution represents a probability distribution.

X	1	3	4	6	8	9
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Probability Distribution for a Discrete Random Variable		
Lesson Competency : To construct the probability distribution of discrete random variable and its corresponding histogram (M11/12SP-IIIa-5) To compute probabilities corresponding to a given random variable (M11/12SP-IIIa-6)		
References : Statistics and Probability pages 9-15		LAS No.: 3.4

CONCEPT NOTES

Steps in constructing the probability distribution of discrete random variable:

1. Determine the sample space.
2. Count the number of the identified outcome in the sample space and assign number to this outcome.
3. Get the possible values of the random variable and compute the probability.
4. Make a table of the probability distribution.

Examples:

Suppose two coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 1 and 2.

Step 1. The sample space for this experiment is: S= {HH, HT, TH, TT}

Step 2. Count the number of tails in the sample space and assign number to this outcome.

Possible Outcomes/Sample Space	Value of the Random Variable Y (number of tails)
HH	0 (no tail)
HT	1 (only one tail)
TH	1 (only one tail)
TT	2 (two tails)

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 1 and 2.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Probability Distribution for a Discrete Random Variable		
Lesson Competency : To construct the probability distribution of discrete random variable and its corresponding histogram (M11/12SP-IIIa-5) To compute probabilities corresponding to a given random variable (M11/12SP-IIIa-6)		
References : Statistics and Probability pages 9-15		LAS No.: 3.5

CONCEPT NOTES

Steps in constructing the probability distribution of discrete random variable:

1. Determine the sample space.
2. Count the number of the identified outcome in the sample space and assign number to this outcome.
3. Get the possible values of the random variable and compute the probability.
4. Make a table of the probability distribution.

Examples:

Suppose two coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 3 and 4.

Step 3. Get the possible values of the random variable and compute the probability.

Number of Tails Y	Probability P(Y)
0	$\frac{1}{4}$ since it occurs only once out of 4 outcomes
1	$\frac{2}{4}$ or $\frac{1}{2}$ since it occurs twice out of 4 outcomes
2	$\frac{1}{4}$ since it occurs once out of 4 outcomes

Step 4. Table1. The Probability Distribution or the Probability Mass Function of Discrete Random Variable Y

Number of Tails	0	1	2
Probability P(Y)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 3 and 4.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Mean of a Discrete Probability Distribution		
Lesson Competency : To illustrate the mean of a discrete random variable (M11/12SP-IIIb-1) To calculate the mean of a discrete random variable (M11/12SP-IIIb-2) To interpret the mean of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving mean of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 21-30		LAS No.: 3.6

CONCEPT NOTES

Steps in computing the mean of a discrete probability distribution:

1. Construct the probability distribution for the random variable.
2. Multiply the value of the random variable by the corresponding the probability.
3. Add the results obtained in Step 2.
4. Interpret the result.

Examples:

Consider rolling a die. What is the average number of spots that would appear?

Solution:

Step 1. Construct the probability distribution for the random variable.

Step 2. Multiply the value of the random variable by the corresponding the probability.

Number of Spots in a Die	Probability P(X)	X • P(X)
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$
Total		$\frac{21}{6}$ or 3.5

.

Step 3. Add the results obtained in Step 2 which is $\frac{21}{6}$ or 3.5

Step 4. Interpret the result.

The value obtained in Step 3 is called the mean of the random variable X or the mean of the probability distribution of X. The mean tells us the average number of spots that would appear in a roll of a die. So, the average number of spots that would appear is 3.5. Although the die will never show a number, which is 3.5, this implies that rolling the die many times, the theoretical mean would be 3.5.

EXERCISES:

The probabilities that a customer will buy 1, 2, 3, 4 or 5 items in a grocery store are $\frac{3}{10}, \frac{1}{10}, \frac{1}{10}, \frac{2}{10}, and\ \frac{3}{10}$, respectively. What is the average number of items that a customer will buy? Follow the steps in computing the mean of the discrete random variable.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.7

CONCEPT NOTES

The variance and standard deviation describe the amount of spread, dispersion, or variability of the items in a distribution.

Steps in Finding the Variance and Standard Deviation:

1. Find the mean of the probability distribution.
2. Subtract the mean from each value of the random variable X.
3. Square the results obtained in Step 2.
4. Multiply the results obtained in Step 3 by the corresponding probability.
5. Get the sum of the results obtained in Step 4 to obtain variance.
6. Get the square root of the variance to get the standard deviation.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 1 and 2.

Solution:

Step 1. Find the mean of the probability distribution in column 3.

Step 2. Subtract the mean from each value of the random variable X in column 4.

Number of Cars Sold (X)	Probability P(X)	X• P(X)	X -μ
0	$\frac{2}{10}$	0	0- 2.2 = -2.2
1	$\frac{2}{10}$	$\frac{2}{10}$	1-2.2 = -1.2
2	$\frac{3}{10}$	$\frac{6}{10}$	2-2.2 =-0.2
3	$\frac{2}{10}$	$\frac{6}{10}$	3-2.2 =0.8
4	$\frac{2}{10}$	$\frac{8}{10}$	4-2.2 =1.8
$\mu = \Sigma X \bullet P(X) = \frac{22}{10} = 2.2$			

EXERCISES:

When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 1 and 2.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.8

CONCEPT NOTES

The variance and standard deviation describe the amount of spread, dispersion, or variability of the items in a distribution.

Steps in Finding the Variance and Standard Deviation: (Continuation of LAS 3.7)

3. Square the results obtained in Step 2.
4. Multiply the results obtained in Step 3 by the corresponding probability.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 3 and 4.

Solution:

Step 3. Square the results obtained in Step 2 and write in column 5.

Step 4. Multiply the results obtained in Step 3 by the corresponding probability and write in column 6.

Number of Cars Sold (X)	Probability P(X)	X• P(X)	X -μ	$(X-\mu)^2$	$(X-\mu)^2 \cdot P(X)$
0	$\frac{2}{10}$	0	0- 2.2 = -2.2	4.84	0.484
1	$\frac{2}{10}$	$\frac{2}{10}$	1-2.2 = -1.2	1.44	0.288
2	$\frac{3}{10}$	$\frac{6}{10}$	2-2.2 =-0.2	0.04	0.012
3	$\frac{2}{10}$	$\frac{6}{10}$	3-2.2 =0.8	0.64	0.128
4	$\frac{2}{10}$	$\frac{8}{10}$	4-2.2 =1.8	3.24	0.648

EXERCISES:

When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 3 and 4.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.9

CONCEPT NOTES

Steps in Finding the Variance and Standard Deviation: (Continuation of LAS 3.8)
Step 5. Get the sum of the results obtained in Step 4 to obtain variance.
Step 6. Get the square root of the variance to get the standard deviation.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 5 and 6.
Solution:

Step 5. Get the sum of the results obtained in Step 4 to obtain variance.
Answer: The variance is 1.56. In symbol, $\sigma^2 = \Sigma(X - \mu)^2 \bullet P(X) = 1.56$

Step 6. Get the square root of the variance to get the standard deviation.
The standard deviation is $\sigma = \sqrt[2]{1.56}$ = 1.25.

EXERCISES:

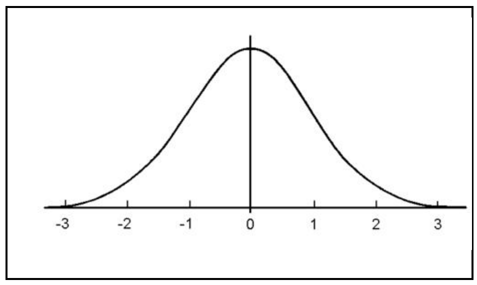
When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 5 and 6.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating a Normal Random Variable and Its Characteristics		
Lesson Competency : To illustrate a normal random variable and its characteristics (M11/12SP-IIIc-1)		
References : Statistics and Probability pages 50-53		LAS No.: 3.10

CONCEPT NOTES

A **normal curve** is defined by an equation that uses the population mean (μ) and the standard deviation (σ). It has a very important role in inferential statistics. It provides a graphical representation of statistical values that are needed in describing the characteristics of populations as well as in making decisions.

Properties of the Normal Probability Distribution:



1. The distribution curve is bell-shaped.
2. The curve is symmetrical about the center.
3. The mean, median, and the mode coincide at the Center.
4. The width of the curve is determined by the standard deviation (σ) of the distribution.
5. The tails of the curve flatten out indefinitely along the horizontal axis, always approaching the axis but never touching it. That is, the curve is asymptotic to the base line.
6. The area under the curve is 1. Thus, it represents the probability or proportion, or the percentage associated with specific sets of measurement values.

EXERCISES:

Fill in the blanks with appropriate word or phrase to make meaningful statements.

1. The area under a normal curve is ____________ .
2. The important values that best describe a normal curve are _________ and __________.
3. The area under normal curve may also be expressed in terms of ____________ or ____________ or ______________.
4. The mean, the median, and the mode of a normal curve are ____________.
5. The curve is ______________ about its center.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Understanding the Standard Normal Curve		
Lesson Competency : To find the area that corresponds to the z-score. To construct a normal curve (M11/12SP-IIIc-2)		
References : Statistics and Probability pages 54-60		LAS No.: 3.11

CONCEPT NOTES

A standard normal curve is a normal probability distribution that has mean μ =0 and a standard deviation $\sigma = 1$.

Four-Step Process in Finding the Areas Under the Normal Curve Given a z-Value:

Step 1. Express the given z-value into a three-digit form.
Step 2. Using the z table (found in the next page), find the first two digits on the left column regardless of the sign of the z-score.
Step 3. Match the third digit with the appropriate column on the right.
Step 4. Read the area (or probability) at the intersection of the row and the column. This is the required area.

EXAMPLE:

1. Find the area that corresponds to z=1.

Solutions:

Finding the area that corresponds to is the same as finding the area between z =0 and z=1.

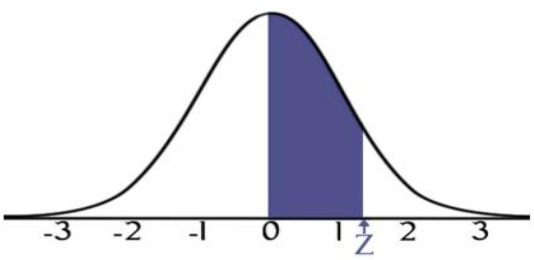
Steps	Solution
1. Express the given into a three-digit form.	z = 1.00
2. In the table, find the Row z =1.00	
3. In the table, find the column with the heading .00	
4. Read the area (or probability) at the intersection of Row 1.0 and the Column .00	The area is 0.3413. This is the required area.

EXERCISES:

1. Find the area that corresponds to z = 1.36.
2. Find the area that corresponds to z = -2.58.
3. Find the area that corresponds to z= 2.18.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Understanding the Standard Normal Curve		
Lesson Competency : To find the area that corresponds to the z-score. To construct a normal curve (M11/12SP-IIIc-2)		
References : Statistics and Probability pages 54-60		LAS No.: 3.11

(Distribute this copy of Standard Normal Table (z) to the students)



STANDARD NORMAL TABLE (Z)

Entries in the table give the area under the curve between the mean and z standard deviations above the mean. For example, for $z = 1.25$ the area under the curve between the mean (0) and z is 0.3944.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0190	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2969	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3513	0.3554	0.3577	0.3529	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Identifying Regions of Areas Under The Normal Curve		
Lesson Competency : To identify regions under the normal curve corresponding to different standard normal values. (M11/12SP-III-c-3) To construct a normal curve (M11/12SP-IIIc-2)		
References : Statistics and Probability pages 67-73		LAS No.: 3.12

CONCEPT NOTES

We see that using the z-table, we can determine specific regions under the normal curve. Since the z-table provides the proportion of the area (or probability or percentage) between any two specific values under the curve, regions under the curve can be described in terms of area.

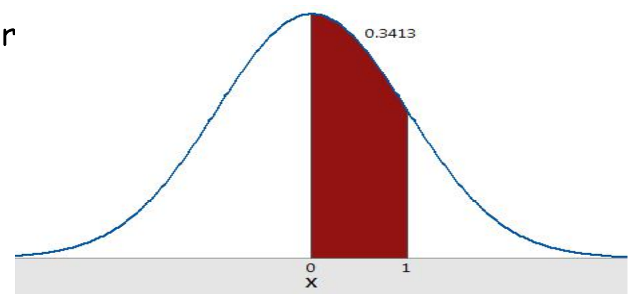
EXAMPLES:

1. Find the area of the region between z =0 and z =1.
2. Find the area of the region between z=1 and z= 2.

Solution:

1. Use the z-table and construct a normal cur

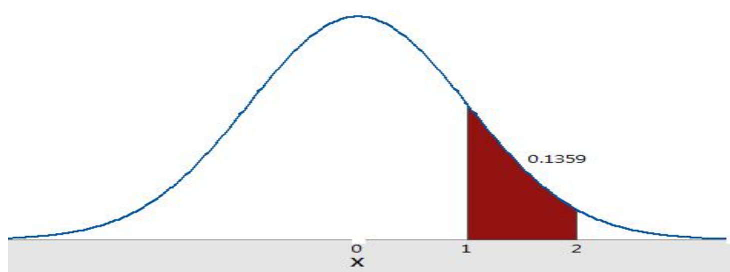
In the z table, the area of z= 1 is 0.3413.



2. Use the z-table. Locate the area of z =1, and z= 2. Then, subtract the area of z=1 from the area of z = 2. Construct a normal curve.

The area of z= 1 is 0. 3413. The area of z = 2 is 0.4772.

0.4772 – 0.3413 = 0.1359. So, the area of the region between z = 1 and z=2 is 0.1359.



EXERCISES:

1. Find the area of the region between z =0 and z = 3.
2. Find the area of the region between z = 2 and z = 3.
3. Find the area of the region between z = -1 and z = 1.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Identifying Regions of Areas Under The Normal Curve		
Lesson Competency : To convert a normal random variable to a standard normal variable. (M11/12SP-III-c-4)		
References : Statistics and Probability pages 61-66		LAS No.: 3.13

CONCEPT NOTES

The areas under normal curve are given in terms of z-values or scores. Either the z-score Locates X within a sample or within a population.

The formula for calculating z is:

$z = \frac{X-\mu}{\sigma}$ (z-score for population data)

$z = \frac{X-\bar{X}}{s}$ (z-score for sample data)

where: X = given measurement
μ = population mean
$\sigma =$ population standard deviation
$\bar{X}$ = sample mean
s = sample standard deviation

EXAMPLES:

1. Given the mean, μ = 50 and the standard deviation, $\sigma = 4$ of a population of Reading scores. Find the z-value that corresponds to a score X = 58.

Steps	Solution
1. Use the computing formula for finding z-scores of a population data.	$z = \frac{X-\mu}{\sigma}$
2. Check the given values. Since these are population values, the z-score locates X within a population.	μ=50 , σ =4 , X = 58.
3. Substituting the given values in the computing formula.	$z = \frac{58-50}{4}$
4. Compute the z- value	$z = \frac{8}{4}$=2 **Thus, the z-value that corresponds to the raw score 58 is 2 in the population distribution.**

EXERCISES:

Given μ=62 and $\sigma = 8$. Find the z-score value that corresponds to each of the following scores up to two decimal places:

1. X= 70
2. X= 78
3. X= 42

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Computing Probabilities Using the Standard Normal Table		
Lesson Competency : To compute probabilities using the standard normal table. (M11/12SP-III-c-5)		
References : Statistics and Probability pages 74-82		LAS No.: 3.14

CONCEPT NOTES

The following notations for a random variable are used in various solutions concerning the normal curve.

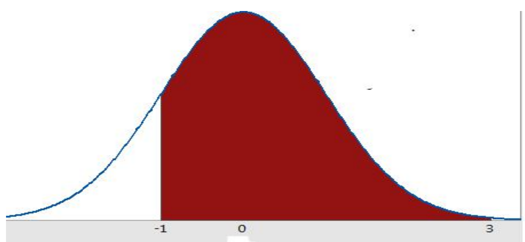
$P\ (a\ < z\ < b)$ denotes the probability that the z-score is between a and b.

$P\ (z\ > a)$ denotes the probability that the z-score is greater than a.

$P\ (\ z < a)$ denotes the probability that the z-score is less than a.

EXAMPLES:

Find the proportion above z = -1.

Steps	Solution
1. Draw a normal curve. 2. Locate the z-value 3. Draw a line through the z-value 4. Shade the required region.	
5. Consult the z-Table and find the area that corresponds to z=-1.	z=-1 corresponds to an area of 0.3413
6. Examine the graph and use probability notation to form an equation showing the appropriate operation to get the required area.	The graph suggests addition. The required area is equal to 0.3413 + 0.5 = 0.8413 That is, $P\ (z >\ -1) = 0.8413$
7. Make a statement indicating the required area.	The proportion or probability of the area above is 0. 8413.

EXERCISES:

1. Find the proportion of the area greater than z =1.
2. Find the proportion of the area below z = 1.5.
3. Find the area between z = -2 and z = -1.5.

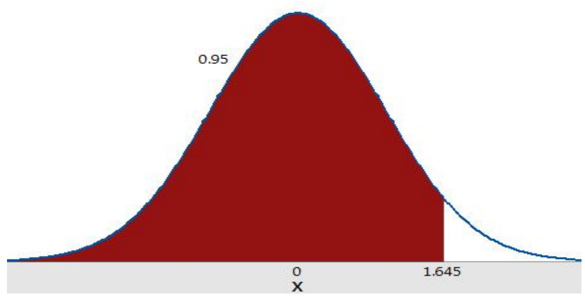
Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Computing Percentiles Using the Standard Normal Table		
Lesson Competency : To compute percentiles using the standard normal table. (M11/12SP-III-c-5)		
References : Statistics and Probability pages 83-88		LAS No.: 3.15

CONCEPT NOTES

For any set of measurements (arranged in ascending or descending order), a **percentile** is a point in the distribution such that a given number of cases is below it. For example in the test in Algebra, you got a score of 82 and you want to know how you fared in comparison with your classmates. If your teacher tells you that you scored at the 90^{th} percentile, it means that 90% of the grades were lower than yours and 10% were higher.

EXAMPLES:

Find the 95^{th} percentile of a normal curve.

Steps	Solution
1. Draw the appropriate normal curve.	
2. Express the given percentage as probability.	95% is the same as 0.9500
3. Split 0.9500 into 0.5000 and 0.4500	0.9500 = 0.5000 + 0.4500
4. Shade 0.5000 of the sketch of the normal curve in Step 1.	
5. Refer to the z-Table. Locate the area 0.4500 in the body of the table.	This area is not found in the table. It is between the values of 0.4495 and 0.4505
6. Find the z-score that corresponds to 0.4500 on the leftmost column,	Find z by interpolation, as follows.
7. Find the z-value that corresponds to 0.4505.	0.4505 is z =1.65
8. Find the z-value that corresponds to 0.4495.	0.4495 is z = 1.64

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Computing Percentiles Using the Standard Normal Table		
Lesson Competency : To compute percentiles using the standard normal table. (M11/12SP-III-c-5)		
References : Statistics and Probability pages 83-88		LAS No.: 3.15

9. Find the average of the two z-values.	$z = \frac{1.65 + 1.64}{2} = 1.645$
10. Locate z= 1.645 under the curve in Step 1 and make a statement.	The 95^{th} percentile is z = 1.645.
11. Draw a line through under the curve in Step 1.	Do this under the sketch of the curve in Step 1.
12. Shade the region to the left of z = 1.645	Do this under the sketch of the curve in Step 1.
13. Describe the shaded region.	The shaded region is 95% of the distribution.

EXERCISES:

1. Find the 99^{th} percentile of a normal curve.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Illustrating Random Sampling and Computing the Mean		
Lesson Competency : Illustrates random sampling and computes their corresponding mean		
References : Statistics and Probability, pg.		LAS No.: 3.16

CONCEPT NOTES

A random sampling distribution is a probability distribution of a statistic obtained through a large number of samples drawn from a specific population. The sampling distribution of a given population is the distribution of frequencies of a range of different outcomes that could possibly occur for a statistic of a population.

Example:

A population consists of the numbers 2,4,9. List all possible samples of size 2 from this population and compute the mean.

sample	mean
2,4	3
2,9	5.5
4,9	6.5

The number of samples of size n that can be drawn from a population of size N is given by ${}_{N}C_{n}$.

Exercise:

A population consists of the five numbers 2,3,6,8, and 11. Consider the samples of size 2 that can be drawn from this population.

a. List all possible samples and the corresponding mean.

Sample	Mean

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Distinguishing Between Parameter and Statistic		
Lesson Competency : Distinguishes between parameter and statistic.		
References : Statistics and Probability, pg.		LAS No.: 3.17

CONCEPT NOTES

Parameters are numbers that summarize data for an entire population.

Statistic are numbers that summarize data from a sample, i.e. some subset of the entire population.

Example:

Identify both the parameter and the statistic in the study.

1) A researcher wants to estimate the average height of women aged 20 years or older. From a simple random sample of 45 women, the researcher obtains a sample mean height of 63.9 inches.
   Parameter : average height of all women aged 20 years or older.
   Statistic: average height of 63.9 inches from the sample of 45 women.

2) A nutritionist wants to estimate the mean amount of sodium consumed by children under the age of 10. From a random sample of 75 children under the age of 10, the nutritionist obtains a sample mean of 2993 milligrams of sodium consumed.
   Parameter : mean amount of sodium consumed by children under the age of ten.
   Statistic: mean of 2993 milligrams of sodium obtained from the sample of 75 children.

Exercise:

Identify both the parameter and the statistic in the study.

1) An education official wants to estimate the proportion of adults aged 18 or older who had read at least one book during the previous year. A random sample of 1006 adults aged 18 or older is obtained, and 835 of those adults had read at least one book during the previous year.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Identifying Sampling Distributions of Statistics (Sample Mean)		
Lesson Competency : Identifies the sample distributions of statistics (sample mean).		
References : Statistics and Probability, pg.		LAS No.: 3.18

CONCEPT NOTES

A **sampling distribution of sample means** is a frequency distribution using the means computed from all possible random samples of specific size taken from a population.

Probability distribution of the sample means is also called the sampling distribution of the samples.

Example:

Based on the given example in LAS 1 Illustrating the random sampling and computing their corresponding mean, identify the sampling distribution of sample mean.

Sample mean	Frequency	Probability
3	1	$\frac{1}{3} = .33$
5.5	1	$\frac{1}{3} = .33$
6.5	1	$\frac{1}{3} = .33$
Total	n= 3	1.00

Exercise:

Based on the given exercise in LAS 1, identify the sample distributions of statistics.

Sample Mean	**Frequency**	**Probability**
Total		

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Finding the mean and variance of population		
Lesson Competency : Finds the mean and variance of population.		
References : Statistics and Probability, pg.		LAS No.:3.19

CONCEPT NOTES

Statisticians describe the variation of the individual data values about the mean of the population.

Example:

Consider a population consisting of 1,2,3,4 and 5. Suppose samples of size 2 are drawn from this population. Find the mean and variance of population.

1. Compute the mean of the population (μ).

$$\mu = \frac{\sum x}{N}$$

$$= \frac{1+2+3+4+5}{5}$$

$$= 3.00$$

So, the mean of the population is 3.00.

2. Compute the variance of the population (σ).

X	X - μ	$(X - \mu)^2$
1	-2	4
2	-1	1
3	0	0
4	1	1
5	2	4
		$\sum (X - \mu)2 = 10$

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$

$$= \frac{10}{5}$$

$$= 2$$

So, the variance of the population is 2.

Exercise:

Consider a population consisting of 1,2,3 and 4. Suppose samples of size 2 are drawn from this population. Find the mean and variance of population.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Finding the Mean and Variance of the Sampling Distribution of the Sample Mean.		
Lesson Competency :Finds the Mean and Variance of the Sampling Distribution of the Sample Mean.		
References : Statistics and Probability, pg.		LAS No.:3.20

CONCEPT NOTES

Steps in finding the mean and variance of the sampling distribution of the sample mean:

1. Determine the number of possible samples size.
2. List possible samples and their corresponding means.
3. Construct the sampling distribution of the sample means.
4. Compute the mean of the sampling distribution of the sample means ($\mu_{\overline{X}}$).
   a. Multiply the sample mean by the corresponding probability.
   b. Add the results.
5. Compute the variance ($\delta^2{}_{\overline{X}}$) of the sampling distribution of the sample means.
   a.Subtract the population mean (μ) from each sample ($\overline{X}$). Label this as $\overline{X}$ $-\mu$.
   b. Square the difference. Label this as $(\overline{X} -\mu)^2$.
   c. Multiply the results by the corresponding probability. Label this as $P(\overline{X}) \bullet (\overline{X} -\mu)^2$
   d. Add the results.

Example:

Consider a population consisting of 1,2 and 3. Suppose samples of size 2 are drawn from this population. Describe the sampling distribution of the sample means.

- What is the mean and variance of the sampling distribution of the sample means?

Sample	Sample Mean	Frequency	Probability $P(\overline{X})$	$(\overline{X}) \bullet P(\overline{X})$	$\overline{X} - \mu$	$(\overline{X} - \mu)^2$	$P(\overline{X}) \bullet (\overline{X} - \mu)^2$
1,2	1.5	1	$\frac{1}{3} = .33$	0.495	-0.5	0.25	0.0825
1,3	2	1	$\frac{1}{3} = .33$	0.66	0	0	0
2,3	3	1	$\frac{1}{3} = .33$	0.99	1	1	0.33
Total				2.145			0.4125

So, the mean of the sampling distribution of the sample means ($\mu_{\overline{X}}$) is 2.145 and the variance of the sampling distribution ($\delta^2{}_{\overline{X}}$) is 0.4125.

Exercise:

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Finding the Mean and Variance of the Sampling Distribution of the Sample Mean.		
Lesson Competency :Finds the Mean and Variance of the Sampling Distribution of the Sample Mean.		
References : Statistics and Probability, pg.		LAS No.:3.20

Consider a population consisting of 2,3 and 4. Suppose samples of size 2 are drawn from this population. Describe the sampling distribution of the sample means.

- What is the mean and variance of the sampling distribution of the sample means?

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Sample Means and Standard Deviation		
Lesson Competency :Describes the sampling distribution of the sample means by computing its mean and standard deviation.		
References : Statistics and Probability, pg.		LAS No.:3.21

CONCEPT NOTES

The standard deviation ($\delta_{\overline{X}}$) of the sampling distribution of the sample means is also known as the standard error of the mean. It measures the degree of accuracy of the sample mean ($\delta_{\overline{X}}$) as an estimate of the population mean (μ).

Example:

A population has a mean of 60 and a standard deviation of 5. A random sample of 16 measurements is drawn from this population. Describe the sampling distribution of the sample means by computing its mean and standard deviation.

We shall assume that the population is infinite.

Steps	Solution
1. Identify the given information.	Here $\mu = 60, \delta = 5, and\ n = 16.$
2. Find the mean of the sampling distribution. Use the property that $\mu_{\overline{X}} = \mu$	$\mu_{\overline{X}} = \mu$ = 60
Find the standard deviation of the sampling distribution. Use the property that $\delta_{\overline{X}} = \frac{\delta}{\sqrt{n}}$	$\delta_{\overline{X}} = \frac{\delta}{\sqrt{n}}$ $= \frac{5}{\sqrt{16}}$ $= \frac{5}{4}$ = 1.25

Exercise:

Answer the following problem.

The heights of male college students are normally distributed with mean of 68 inches and standard deviation of 3 inches. If 80 samples consisting of 25 students each are drawn from the population, what would be the expected mean and standard deviation of the resulting sampling distribution of the means? Assume that the population is infinite.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Central Limit Theorem and Problem Solving Involving Sampling Distribution of the Sample Means		
Lesson Competency : Illutrates the central limit theorem. Solves problems involving sampling distribution of the sample means.		
References : Statistics and Probability, pg.		LAS No.:3.22

CONCEPT NOTES

The Central Limit Theorem

If random samples of size n are drawn from a population, then as n becomes larger, the sampling distribution of the means approaches the normal distribution, regardless of the shape of the population distribution.

Example:

The average time it takes a group of college students to complete a certain examination is 46.2 minutes. The standard deviation is 8 minutes. Assume that the variable is normally distributed.

a. What is the probability that a randomly selected college student will complete the examination in less than 43 minutes?

Steps	Solution
1. Identify the given information.	Here $\mu = 46.2, \delta = 8, and\ X = 43.$
2. Identify what is asked for.	$P(X < 43)$
3. Identify the formula to be used.	Here we are dealing with an individual data obtained from the population. So, we will use the formula $Z = \frac{X-\mu}{\delta}$ to standardize 43.
4. Solve the problem.	$Z = \frac{X-\mu}{\delta}$ $= \frac{43-46.2}{8}$ We shall find P(X<-0.40) by getting the area under the normal curve. P(x<43)= P(z< -0.40) = 0.5000 -0.1554 = 0.3446
5. State the final answer.	So, the probability that a randomly selected college student will complete the examination in less than 43 minutes is 0.3446 or 34.46%.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Central Limit Theorem and Problem Solving Involving Sampling Distribution of the Sample Means		
Lesson Competency : Illutrates the central limit theorem. Solves problems involving sampling distribution of the sample means.		
References : Statistics and Probability, pg.		LAS No.:3.22

Exercise:

Solve the following problem.

The average number of milligrams (mg) of cholesterol in a cup of a certain brand of ice cream is 660 mg, and the standard deviation is 35 mg. If a cup of ice cream is selected, what is the probability that the cholesterol content will be more than 670 mg? Assume the variable is normally distributed.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Point Estimation		
Lesson Competency :Illustrates point estimations.		
References : Statistics and Probability, pg.		LAS No.:3.23

CONCEPT NOTES

A point estimate is a specific numerical value of a population parameter. This sample mean $\bar{x}$ is the best point estimate of the population mean. To find the point estimate, compute the mean of all means or the overall mean.

Example:

Mr. Santiagocompany sells bottled of coconut juice. He claims that a bottle contains 500 ml of such juice. A consumer group wanted to know if his claim is true. They took six random samples of 10 such bottles and obtained the capacity, in ml, of each bottle. The result is shown as follows:

Sample 1	500	498	497	503	499	497	497	497	497	495
Sample 2	500	500	495	494	498	500	500	500	500	497
Sample 3	497	497	502	496	497	497	497	497	497	495
Sample 4	501	495	500	497	497	500	500	495	497	497
Sample 5	502	497	497	499	496	497	497	499	500	500
Sample 6	496	497	496	495	497	497	500	500	496	497

Assuming that the measurements were carefully obtained and that the only kind of error present is the sampling error, what is the point estimate of the population mean?

Sample Row	sum of Scores	Mean
1	4980	498
2	4984	498.4
3	4972	497.2
4	4979	497.9
5	4984	498.4
6	4971	497.1
	Overall mean/ point estimate	497.83

Exercise:

Find the point estimate of the population parameter of the scores in a long test in Science.

78	75	86	82	70	85	83	86
80	92	82	85	80	88	84	86
90	88	90	78	83	90	86	84
75	85	77	88	85	90	85	83

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : T- Distribution		
Lesson Competency : identifies the appropriate form of the confidence interval estimator for the population mean when: (a) the population variance is known, (b) the population variance is unknown, and (c) the Central Limit Theorem is to be used.		
References : Statistics and Probability, pg.		LAS No.:3.24

CONCEPT NOTES

The general expression for the confidence interval when σ is unknown is given by:
$\overline{X} \pm t\left(\frac{s}{\sqrt{n}}\right)$ and the distribution of values is called t- distribution.

Example:

A researcher wants to estimate the number of hours that 5-year old children spend watching television. A sample of 50 five-year old children was observed to have a mean viewing time of 3 hours. The population is normally distributed with a population standard deviation $\alpha = 0.5\ hours$, find:

a. The best point estimate of the population mean

Solution:

a. Point Estimate

Steps	Solution
1. Describe the population parameter of interest.	The parameter of interest is the mean μ of the TV viewing time of all 5- year old children.
2. Specify the confidence interval criteria. a. Check the assumptions. b. Determine the test statistic to be used to calculate the interval c. State the level of confidence	The sample size of 50 children is large enough for the Central limit Theorem to hold. So, the sampling distribution of means is normal. The test statistic is the z, using $\delta = 0.5$. The questions asks for a 95% confidence,or $\alpha = 0.05$. This means that if more random samples were taken from the target population, and an interval estimate is made for each sample, then 95% of the intervals will contain the true parameter value.
3. Collect an present sample evidence. a. Collect the sample information.	The sample information consists of $\overline{X = 3,}\ n = 50\ and\ \delta = 0.5$
b. Find the point estimate.	The point estimate for the population mean is 3 (the sample mean).

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : T- Distribution		
Lesson Competency : identifies the appropriate form of the confidence interval estimator for the population mean when: (a) the population variance is known, (b) the population variance is unknown, and (c) the Central Limit Theorem is to be used.		
References : Statistics and Probability, pg.		LAS No.:3.25

CONCEPT NOTES

Based on the discussion from LAS 3.24, consider the example below.

Example:

A researcher wants to estimate the number of hours that 5-year old children spend watching television. A sample of 50 five-year old children was observed to have a mean viewing time of 3 hours. The population is normally distributed with a population standard deviation $\alpha = 0.5\ hours$, find:

a. The 95% confidence interval of the population mean

a. 95% Confidence Interval

1. Determine the confidence interval a. Determine the confidence coefficient. b. Find the maximum error E. c. Find the lower and upper confidence limits.	The confidence coefficient is 1.96 $E = z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$ = 1.96 $\left(\frac{0.5}{\sqrt{50}}\right)$ = 1.96 (0.07) = 0.14 $\overline{X}$ - $z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < X + \overline{X}$ + $z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$ $\overline{X} - 1.96\left(\frac{0.5}{\sqrt{50}}\right) < \mu < \overline{X}$ +1.96 $\left(\frac{0.5}{\sqrt{50}}\right)$ 3 - 0.14 to 3 + 0.14 2.86 to 3.14
d. Describe the results.	Thus, we can say with 95% confidence that the interval between 2.86 hours and 3.14 hours contain the population mean μ based on 80 five year old children's TV viewing time.

Exercises:

Do the following:

Given the information: the sampled population is normally distributed, $\overline{X} = 36.5, \delta = 3, and\ n = 20$.What is the 95% confidence interval estimate for μ? Are the assumptions satisfied?

SHARED OPTIONS
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GRADE 11 DLP LEARNING ACTIVITY SHEET

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Identifying Regions Under the t-distribution		
Lesson Competency :. identifies regions under the t-distribution corresponding to different t-values. **M11/12SP-IIIg-4**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 26

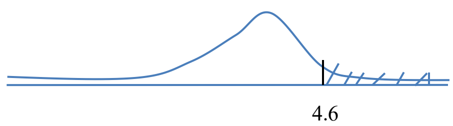
CONCEPT NOTES

The **t distribution** is a kind of symmetric, bell-shaped distribution that has a lower height but a wider spread than the standard normal distribution. The units of a *t* distribution are denoted with a lower case 't'.

Example on how to find area under t-distribution:

1.Suppose T has a t distribution with 4 degrees of freedom, what is P(T> 4.6)?

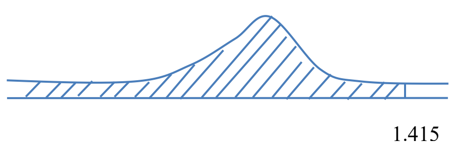
Solution:



From t-distribution table, the area to the right of 4.6 is 0.005.

2. Suppose T has t distribution with7 degrees of freedom, what is P(T< 1.415)?

Solution:



From t-distribution table, the area to the left is 1.00-0.1=0.9.

Exercises:

1. Given a t-distribution with 15 degrees of freedom, what is P(T> 0.866)?

2. Given a t-distribution with 5 degrees of freedom, what is P(T>4.032)?

3. Given a t- distribution with 25 degrees of freedom, what is P(T< 2.060)?

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for the Confidence Interval Estimate		
Lesson Competency :computes for the confidence interval estimate based on the appropriate form of the estimator for the population mean.**M11/12SP-IIIh-1**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 27

CONCEPT NOTES

When you want to find percentiles for a t-distribution, you can use the t-table. A percentile is a number on a statistical distribution whose less-than probability is the given percentage for example, the 95th percentile of the t-distribution with n – 1 degrees of freedom is that value of whose left-tail (less-than) probability is 0.95 (and whose right-tail probability is 0.05).

Example:

Suppose you have a sample of size 10 and you want to find the 95th percentile of its corresponding t-distribution. You have n – 1= 9 degrees of freedom, so, using the t-table, you look at the row for df = 9. The 95th percentile is the number where 95% of the values lie below it and 5% lie above it, so you want the right-tail area to be 0.05. Move across the row, find the column for 0.05, and you get 1.833.

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073

Exercises: Use the t-table above to answer the following:

1. Find the corresponding t-value for 90^{th} percentile of df=15?
2. Find the corresponding t-value for 50^{th} percentile of df=10?
3. Find the corresponding t-value for 99^{th} percentile of df=3?

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for the Confidence Interval Estimate		
Lesson Competency :computes for the confidence interval estimate based on the appropriate form of the estimator for the population mean,solves problems involving confidence interval estimation of the population mean and draws conclusion about the population mean based on its confidence interval estimate. **M11/12SP-IIIh-1, M11/12SP-IIIh-2, M11/12SP-IIIh-3**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 28

CONCEPT NOTES

A confidence interval, (CI) is an indicator of your measurement's precision. It is a range of values that is used to estimate a parameter.There are three commonly used confidence intervals: the 90%, 95% and the 99% confidence intervals.Follow steps below to calculate, CI.

Example:

A researcher wants to estimate the number of hours that 5-year old children spend watching television. A sample of 50 five year old children was observed to have a mean viewing time of 3 hours. The population is normally distributed with a population standard deviation σ=0.5 hours, find the a. point estimate of the population mean, b. the 95% confidence interval of the population mean and c. draws conclusion about the population mean based on its confidence interval estimate.

Steps	Solution
1. Describe the population parameter of interest.	The parameter of interest is the mean μ of the TV viewing time of all 5-year old children.
2.Specify the confidence interval criteria. Check the assumption, determine the test statistic to be used to calculate the interval and state the level of confidence	The sample is normal as guaranteed by the CLT. The test statistic is the z with $\sigma = 0.5$ The question asks for a 95% confidence, or a=0.05
3. Collect and present sample evidence. Collect the sample information. Find the point estimate.	The sample information consist of $\bar{x} = 3, n = 50$ and $\sigma = 0.5$. The point estimate for the population mean 3(the sample mean)
4. Determine the confidence interval. The confidence coefficient. Find the max error, the upper and the lower confidence limits.	The confidence coefficient is 1.96. $E = z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 1.96\left(\frac{0.5}{\sqrt{50}}\right) = 1.96(0.07) = 0.14$
Draw Conclusion: Thus, we can say with 95% confidence that the interval between 2.86 hours and 3.14 hours contain the population mean, μ based on 50 five year old children's TV viewing time.	$\bar{x} - z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{x} + z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)$ 3-1.96 $\left(\frac{0.5}{\sqrt{50}}\right)$<$\mu < 3 + 1.96\left(\frac{0.5}{\sqrt{50}}\right)$ 3-0.14 to 3+0.14 = 2.86 to 3.14

Exercises:A researcher conducts a random selection of 40 entering Mathematics majors who has the following GPA's below. Assume that $\sigma = 0.46$, estimate the true mean GPA with 99% confidence.

4.0	3.5	3.0	3.3	3.8	3.1	3.6	4.0	3.9	3.5	3.2	2.9	3.0	2.8
3.2	3.0	3.5	3.2	3.0	3.2	4.0	3.0	3.4	3.0	4.0	3.7	3.0	3.3
3.0	2.8	5.6	3.0	3.2	3.5	3.2	2.8	3.3	3.1	3.2	2.8		

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Identifying Point Estimator for the Population Proportion		
1. Lesson Competency :identifies point estimator for the population proportion and computes for the point estimate of the population proportion. **M11/12SP-IIIi-1, M11/12SP-IIIi-2**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 29

CONCEPT NOTES

In a binomial experiment, a point estimator for population proportion, p is given by a statistic, $\frac{x}{n}$ where x represents the number of successes in n trials. Thus, the sample proportion, $\hat{p} = \frac{x}{n}$ will be the estimator for the parameter, p.

Example

There are 320 students taking up mathematics and 125 of these students take up Statistics. From this class, a random sample of 100 was selected and 67 of the students take up statistics.

1. What is the proportion, p of students who took up mathematics?

2. Compute for the sample proportion, $\hat{p}$?

Solution:

1. Proportion of students, p = $\frac{125}{320}$
2. Sample proportion, $\hat{p} = \frac{67}{100}$.

EXERCISES

There are 500 students taking up mathematics and 300 of these students take up Statistics. From this class, a random sample of 250 was selected and 175 of the students take up statistics.

1. What is the proportion, p of students who took up mathematics?

2. Compute for the sample proportion, $\hat{p}$?

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Identifying the Appropriate Form of the Confidence Interval Estimator		
Lesson Competency :Identifies the appropriate form of the confidence interval estimator for the population proportion based on the CLT, computes for the confidence interval estimate of the population proportion andsolves problems involving confidence interval estimation of the population proportion.draws conclusion about the population proportion based on its confidence interval estimate.**M11/12SP-IIIi-3, M11/12SP-IIIi-4, M11/12SP-IIIi-5, M11/12SP-IIIi-6**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 30

CONCEPT NOTES

Recall that the margin of error, E for the confidence interval of a population proportion is $E = z_{a/2}\sqrt{\frac{pq}{n}} \approx z_{a/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$ so, the formula for computing a large sample confidence interval for a population, p is $\hat{p} \pm z_{a/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$. If $\hat{p}$ is used for an estimate for p, then we can be (1-a) 100% confident that the error will not exceed a specified amount of error, E.

Example

In a random sample of 500 people eating lunch every Friday at a school canteen, it was found out that x = 160 people preferred to eat seafood. Find the 95% confidence interval for the actual proportion of people who eat seafood every Friday. Describe the result.

Solution:

Given that x = 160 and n = 500, so $\hat{p} = \frac{160}{500} = \frac{8}{25}$.

$\hat{q} = 1 - \hat{p} = 1 - \frac{8}{25} = \frac{25-8}{25} = \frac{17}{25}$,since (1-a)100% is 95%, $z_{a/2} = 1.96$.

$$\hat{p} - z_{a/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{a/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}.$$ Therefore we have

$$\frac{8}{25} - 1.96\sqrt{\frac{\frac{8}{25}\left(\frac{17}{25}\right)}{500}} < p < \frac{8}{25} + 1.96\sqrt{\frac{\frac{8}{25}\left(\frac{17}{25}\right)}{500}}$$

$$0.28 < p < 0.36$$

Conclusion: Thus, with the 95% confidence, we can state that the interval from 28% to 36% contains the true percentage of all people preferred to eat seafood.

Exercises

1. A new launching system is being considered for a development of small short range launches. The existing system has p = 0.8 as the probability of a successful launch. A sample of 40 experimental launches is made with the new system, and 34 of these experimental launches are successful. Construct a 95% confidence interval for p.
2. In a random sample of 1000 houses in a city, 628 of the houses have air conditioning units.
   a. Find the 98% confidence interval to estimate the proportion of houses with conditioning units.
   b. How large should a sample be if you wish to be 98% confident that the sample proportion is within 0.05 of the true proportion of houses that have air conditioning units?

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Identifying the Length of a Confidence Interval		
Lesson Competency : identifies the length of a confidence interval andcomputes for the length of the confidence interval **.M11/12SP-IIIj-1 , M11/12SP-IIIj-2**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 31

CONCEPT NOTES

Definition. If a confidence interval for a parameter, p is: *Lower< p <Upper*, then the **length of the interval** is simply the difference in the two endpoints. That is:*Length = Upper – Lower*

$$\text{Length}, L = \left[\bar{x} + z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right] - \left[\bar{x} - z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right] = 2z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

(1) As the population standard deviation σ decreases, the length of the interval decreases.
(2) As the sample size *n* increases, the length of the interval decreases.
(3) As the confidence level decreases, the length of the interval decreases. (Consider, for example, that for a 95% interval, *z* = 1.96, whereas for a 90% interval, *z* = 1.645.), for this factor, We want a high confidence level, but not so high as to produce such a wide interval as to be useless. That's why 95% is the most common confidence level used.

Example

Given: n=20, 95% confidence, $\sigma = 0.8$. Compute for the length of interval.
Solution:

$$L = 2z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$
$$= 2\,(1.96)\left(\frac{0.8}{\sqrt{20}}\right) = 0.70125$$

EXERCISES

Compute the length of Interval for each of the following:

1. n=15, 90% confidence, $\sigma = 2$.
2. n=25, 95% confidence, $\sigma = 5$.
3. n=30, 99% confidence, $\sigma = 6$.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for an Appropriate Sample Size Using the Length of the Interval		
Lesson Competency :computes for an appropriate sample size using the length of the interval and solves problems involving sample size determination.**M11/12SP-IIIj-3 , M11/12SP-IIIj-4**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 3.32

CONCEPT NOTES

A. Compute sample size using length of the interval.
   Recall the formula for the length of interval, L= $2z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)$, we can derive the formula for finding the sample size, n = $4(\frac{(z_{a/2})(\sigma)}{L})^2$

B. Computing sample size using the margin of error, E= $z_{a/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

$$n = (\frac{(z_{a/2})(\sigma)}{E})^2$$

Examples

1. Given: L= 0.62, 95% confidence, $\sigma = 2$, Compute for the minimum sample size.

Solution

$$n = 4(\frac{(z_{a/2})(\sigma)}{L})^2$$

$$n = 4\left[\frac{(1.96)(2)}{0.62}\right]^2$$

n = 159.9 say 160

2. In a certain village, Leony wants to estimate the mean weight μ, in kilograms, of all six-year old children to be included in the feeding program. She wants to be 99% confident that the estimate of μ is accurate to within 0.06 kg. Suppose from a previous study, the standard deviation of the weights of the target population was 0.5 kg, what should the sample size be?

solution

Given the confidence 99%, then a= 0.01 and $z_{a/2} = 2.58$, E=0.06 and $\sigma = 0.5kg$.
Substituting:

$$n = (\frac{(z_{a/2})(\sigma)}{E})^2 = \left[\frac{(2.58)(0.5)}{0.06}\right]^2 = (\frac{1.29}{0.06})^2 = 462.25 \text{ say } 463$$

EXERCISES

Jonathan wants to replicate a study where the lowest observed value is 10.4 while the highest is 10.8. He wants to estimate the population mean μ to within an error of 0.022 of its true value. Using 99% confidence level, find the sample size n that he needs.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating Null and Alternative Hypothesis		
Lesson Competency : To illustrate null hypothesis and alternative hypothesis (M11/12SP-IVa-1)		
References : Statistics and Probability pages 216-222		LAS No.: 4.1

CONCEPT NOTES

Hypothesis testing is a decision-making process for evaluating claims about a population based on the characteristics of a sample purportedly coming from that population. The decision is whether the characteristic is acceptable or not. It involves making a decision between the two opposing hypotheses which are the null hypothesis and alternative hypothesis.

The **null hypothesis**, denoted by H_0, is a statement that there is no difference between a parameter and a specific value, or that there is no difference between two parameters.

The **alternative hypothesis**, denoted by H_1, is a statement that there is a difference between a parameter and a specific value, or that there is a difference between two parameters. When the alternative hypothesis utilizes the ≠ symbol, the test is said to be **non-directional**. When the alternative hypothesis utilizes the > or < symbol, the test is said to be **directional.** In problems that involve hypothesis testing, there are words like *greater, efficient, improves, effective, increases* and so on suggest a **right-tailed direction**. Words like *decrease, less than, smaller,* and the like suggest a **left-tailed direction**.

EXAMPLES:

Formulate a null hypothesis and its alternative hypothesis and identify whether the alternative hypothesis is directional or non-directional for each of the following:

1. The average TV viewing time of all five-year old children is 4 hours daily.

Solution:

Null Hypothesis: The average TV viewing of all five-year old children is 4 hours daily. (This is the claim.) In symbol, $H_0: \mu = 4$.

Alternative Hypothesis: The average TV viewing of all five-year old children is not 4 hours daily. (This is the opposite of the claim.) In symbol, $H_1 = \mu \neq 4$.

EXERCISES:

Formulate a null hypothesis and its alternative hypothesis for each of the following and identify whether the alternative hypothesis is directional or non-directional:

1. A college librarian claims that 20 storybooks on the average are borrowed daily.
2. The mean performance of all grade 6 leavers of a school in the NAT is 35.
3. The investor of a new kind of light bulb claims that all such bulbs last as long as 3000 hours.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating Type of Errors in Hypothesis Testing		
Lesson Competency : To illustrate types of errors in hypothesis testing (M11/12SP-IVa-1)		
References : Statistics and Probability pages 226-229		LAS No.: 4.2

CONCEPT NOTES

In hypothesis testing, we make decisions about null hypothesis. When we conduct a hypothesis test, there are four possible outcomes. The following decision grid shows the four outcomes.

		Decisions about the H_0	
		Reject	Do not Reject (or Accept H_0)
Reality	H_0 is true.	Type I error	Correct Decision
	H_1 is false.	Correct Decision	Type II error

If the null hypothesis is true and accepted, or if it is false and rejected, the decision is correct. If the null hypothesis is true and rejected, the decision is incorrect and this is a **Type I error.** If the null hypothesis is false and accepted, the decision is incorrect and this is a **Type II error**. In an ideal situation, there is no error when we accept the truth and reject what is false.

EXAMPLES:

1. Stephen says that he is not bald. His hairline is just receding. Is he committing an error? If so, what type of error?

Answer: Yes. A receding hairline indicates balding. This is a Type I error.

2. A man plans to go hunting the Philippine monkey-eating eagle believing that it is a proof of his mettle. What type of error is this?

Answer: Hunting the Philippine eagle is prohibited by law but still the man plans to go hunting. In this case, he committed a Type II error since he accepts a false reality.

EXERCISES:

1. Maria insists that she is 30 years old when, in fact, she is 32 years old. What error is Mary committing?

2. Suppose it is Christmas season and Jose thinks it is the month of January , what error is he committing?

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Calculating the Probabilities of Committing a Type I and Type II Error		
Lesson Competency : To calculate the probabilities of committing a Type I and Type II error (M11/12SP-IVa-2) To illustrate level of significance and rejection region (M11/12SP-IVa-1)		
References : Statistics and Probability pages 226-229		LAS No.: 4.3

CONCEPT NOTES

The probability of committing a Type I error is denoted by the Greek letter α (alpha) while the probability of committing a Type II error is denoted by β (beta).

Error in the Decision	Type	Probability	Correct Decision	Type	Probability
Reject a true H_0	I	α	Accept a true H_0	A	1- α
Accept a false H_0	II	β	Reject a false H_0	B	1- β

The most frequently used probability values for α and β are 0.05 and 0.01.

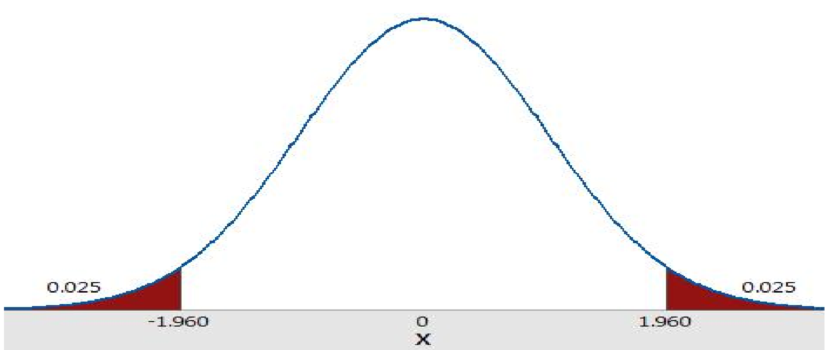
Under the normal curve, the **rejection region** refers to the region where the value of the test statistic lies for which we will **reject the null hypothesis**. The region is also called **critical region.**

EXAMPLES:

Recall that the critical values are the z-values associated with the probabilities at the tails of the normal curve.

1. For a 95% confidence level, find the critical values and the probability in committing error.

Solution: $\frac{0.95}{2}$ = 0. 4750 (expressed up to 4 decimal places so that we can identify an area in the normal curve table as close as to this value). In the normal curve table, the area 0.4750 corresponds to z =1.96. Thus, the critical values for the 95% confidence level are -1.96 and +1.96. And the probability in committing error is 0.025 both in the left and right tail.



EXERCISES:

1. Find the critical values and the probability in committing error for a 99% confidence level.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Solving Problems Involving Test of Hypothesis on the Population Mean		
Lesson Competency : **To solve problems involving test of hypothesis on the population mean. (M11/12SP-IVe-1)**		
References : Statistics and Probability pages 226-229		LAS No.: 4.4

CONCEPT NOTES

One of the processes in hypothesis testing is the calculation of the test statistic. A **test statistic** is a value used to determine the probability needed in decision-making. The decision that we make depends on the computed test statistic. The formula for computing the test statistic depends on the sample size. If the sample size is greater than 30, apply the Central Limit Theorem (CLT) using the **z-statistic** but if the sample size is less than 30, use the **t-statistic**.

EXAMPLES:

Problem: The owner of a factory that sells a particular bottled fruit juice claims that the average capacity of their product is 250 ml. To test the claim, a consumer group gets a sample of 100 such bottles, calculates the capacity of each bottle, and then finds the mean capacity to be 248 ml. The standard deviation s is 5ml. Is the claim true?

Solution:

Steps	Solution
1. Describe the population parameter of interest.	The parameter of interest is the mean μ of the population where the sample comes from.
2. Formulate the hypotheses; the null hypothesis and the alternative hypothesis	$H_0 : \mu = 250$ $H_1 : \mu < 250$
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem (CLT) to apply? • Are the samples selected randomly?	• Since n =100, by the Central Limit Theorem, the distribution is normally distributed. • Yes
4. Choose a significance level for α. • I the test two-tailed or one tailed? • Get the critical values from the test statistic table.	$\alpha = 0.05$ One tailed Critical values: -1.645

EXERCISES:

Solve the problem following the steps given above.

1. A researcher used a developed problem solving test to randomly select 50 Grade 6 pupils. In this sample, $\overline{X = 80}$ and s = 10. The mean μ and the standard deviation of the population used in the standardization of the test were 75 and 15, respectively. Use the 95% confidence level. Does the sample mean differ significantly from the population mean? Follow the steps 1-4.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Solving Problems Involving Test of Hypothesis on the Population Mean		
Lesson Competency : **To solve problems involving test of hypothesis on the population mean. (M11/12SP-IVe-1)**		
References : Statistics and Probability pages 226-229		LAS No.: 4.4

CONCEPT NOTES

As a continuation of the LAS 36,the following steps below in solving problems involving test of hypothesis on the population mean are need to be considered.

EXAMPLES:

Refer the sample problem from LAS 36.

Solution:

5. Select the appropriate test statistic. • Compute the test statistic	z statistic, s is the estimate of σ. $z = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ modified into $z = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ $z = \frac{248-250}{\frac{5}{\sqrt{100}}} = \frac{-2}{0.5} = -4$
6. State the decision rule.	Reject H_0 if the computed test statistic $\leq$ negative critical value or if the computed test statistic $\geq$ positive critical value. Otherwise, do not reject (or accept) H_0.
7. Compare the test statistic and the critical value. • Based on the decision rule, decide whether to reject or to accept H_0. • Interpret the result. • Take a course of action (optional)	$-4 < -1.645$ The null hypothesis is rejected Interpretation: There is enough evidence to warrant the rejection of the null hypothesis. The sample does not belong to the population whose mean μ is 250. So, there is a significant difference between the sample mean and the population mean. Therefore, the claim is not true.

EXERCISES:

Solve the problem following the steps given above.

1. A researcher used a developed problem solving test to randomly select 50 Grade 6 pupils. In this sample, $\overline{X} = 80$ and s = 10. The mean μ and the standard deviation of the population used in the standardization of the test were 75 and 15, respectively. Use the 95% confidence level. Does the sample mean differ significantly from the population mean? Follow the steps 5, 6 and 7 to solve the problem as the continuation of the steps from LAS 36.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Solving Problems Involving Test of Hypothesis on the Population Mean		
Lesson Competency : **To identify the parameter to be tested given a real-life problem (M11/12SP-IVa-3)** **To formulate the appropriate null and alternative hypotheses on a population mean. (M11/12SP-IVb-1)** **To identify the appropriate form of test statistic (M11/12SP-IVb-2)** **To identify the appropriate rejection region for a given level of significance. (M11/12SP-IVc-1)** **To compute for the test-statistic value (population mean). (M11/12SP-IVd-1)** **To draw conclusion about the population mean based on the test-statistic value and the rejection region. (M11/12SP-IVd-2)** **To solve problems involving test of hypothesis on the population mean. (M11/12SP-IVe-1)**		
References : Statistics and Probability pages 226-229		LAS No.: 4.5

CONCEPT NOTES

One of the processes in hypothesis testing is the calculation of the test statistic. A **test statistic** is a value used to determine the probability needed in decision-making. The decision that we make depends on the computed test statistic. The formula for computing the test statistic depends on the sample size. If the sample size is greater than 30, apply the Central Limit Theorem (CLT) using the **z-statistic** but if the sample size is less than 30, use the **t-statistic**.

EXAMPLES:

Problem: Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis $\mu < 6$. Use $\alpha = 0.05$.

Solution:

Steps	Solution
1. Describe the population parameter of interest.	The parameter of interest is the mean μ of the population where the sample comes from.
2. Formulate the hypotheses; the null hypothesis and the alternative hypothesis	$H_0 : \mu = 6$ $H_1 : \mu < 6$
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem (CLT) to apply? • Are the samples selected randomly?	• Since n =5, the Central Limit Theorem cannot be applied. • Yes

EXERCISE:

Solve the problem following the steps given above.

1. Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis $\mu \neq 6$. Use $\alpha = 0.05$. (Follow the first three steps.)

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Solving Problems Involving Test of Hypothesis on the Population Mean		
Lesson Competency : **To identify the parameter to be tested given a real-life problem (M11/12SP-IVa-3)** **To formulate the appropriate null and alternative hypotheses on a population mean. (M11/12SP-IVb-1)** **To identify the appropriate form of test statistic (M11/12SP-IVb-2)** **To identify the appropriate rejection region for a given level of significance. (M11/12SP-IVc-1)** **To compute for the test-statistic value (population mean). (M11/12SP-IVd-1)** **To draw conclusion about the population mean based on the test-statistic value and the rejection region. (M11/12SP-IVd-2)** **To solve problems involving test of hypothesis on the population mean. (M11/12SP-IVe-1)**		
References : Statistics and Probability pages 226-229		LAS No.: 4.5

CONCEPT NOTES

The different steps in solving problems involving test of hypothesis on the population mean are necessary to satisfy the given hypothesis. Steps 1-3 were already discussed from LAS 38.

EXAMPLES:

Problem: Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis μ < 6. Use $\alpha = 0.05$.

Solution:

Steps	Solution
4. Choose a significance level for α. • I the test two-tailed or one tailed? • Get the critical values from the test statistic table.	$\alpha = 0.05$ One tailed (left) From the t table, Df =5-1 =4, the critical value is -2.132.
5. Select the appropriate test statistic. • Compute the test statistic	z statistic, s is the estimate of σ. $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} = \frac{4.6-6}{\frac{1.5}{\sqrt{5}}}$ $t = \frac{-1.4}{0.669} = = -2.092$

EXERCISE:

Solve the problem following the steps 4 and 5 given above.

1. Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis μ ≠ 6. Use $\alpha = 0.05$

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Solving Problems Involving Test of Hypothesis on the Population Mean		
Lesson Competency : **To identify the parameter to be tested given a real-life problem (M11/12SP-IVa-3)** **To formulate the appropriate null and alternative hypotheses on a population mean. (M11/12SP-IVb-1)** **To identify the appropriate form of test statistic (M11/12SP-IVb-2)** **To identify the appropriate rejection region for a given level of significance. (M11/12SP-IVc-1)** **To compute for the test-statistic value (population mean). (M11/12SP-IVd-1)** **To draw conclusion about the population mean based on the test-statistic value and the rejection region. (M11/12SP-IVd-2)** **To solve problems involving test of hypothesis on the population mean. (M11/12SP-IVe-1)**		
References : Statistics and Probability pages 226-229		LAS No.: 4.5

CONCEPT NOTES

Steps 6 and 7, the decision rule and interpreting the result will be discussed as follow respectively.

EXAMPLES:

Problem: Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis $\mu < 6$. Use $\alpha = 0.05$.

Steps	Solution
6. State the decision rule.	Reject H_0 if the computed test statistic $\leq$ negative critical value or if the computed test statistic $\geq$ positive critical value. Otherwise, do not reject (or accept) H_0.
7. Compare the test statistic and the critical value. • Based on the decision rule, decide whether to reject or to accept H_0. • Interpret the result. • Take a course of action (optional)	$-2.092 > -2.132$, The null hypothesis is not rejected Interpretation: There is no enough evidence to warrant the rejection of the null hypothesis. The sample belongs to the population whose mean μ is 6. So, there is no significant difference between the sample mean and the population mean. Therefore, the claim is true.

EXERCISES:

Solve the problem following the steps 6 and 7 given above.
(Based your answers from the previous discussion.)

1. Test the null hypothesis that the mean of the population is 6 against the alternative hypothesis $\mu \neq 6$. Use $\alpha = 0.05$

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for the test-statistic of Population Proportion		
Lesson Competency : Computes for the test-statistic value(Population proportion) Draws conclusion about the population proportion based on the test-statistic value and the rejection region. Solves problems involving test of hypothesis on the population proportion.		
References : Statistics and Probability, pg.		LAS No.:

CONCEPT NOTES

The formula for the test statistic z for proportions is:

Test statistic $z = \frac{Sample\ proportion - null\ hypothesized\ proportion}{Standard\ deviation\ of\ sample\ proportion}$

that is,

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$

For a one- tailed test:

$$H_o: p = p_o$$

$H_1: p > p_o$ *and the rejection region is* $z > \ + z_\alpha$

Or ($H_1: p < p_o$) and the rejection region is $z > \ - z_\alpha$

For a two-tailed test:

$H_o: p = p_o$

$H_1: p \neq p_o$

The rejection region is $z < \ - z_{\frac{\alpha}{2}}$ or $z > z_{\frac{\alpha}{2}}$

Example:

Using the .05 level of significance, run a z-test given the following:

n= 74; $\hat{p}$= $\frac{5}{74}$; $p_o = 10\%$

Use both the traditional method and the p-value method.

Solution:

The key components of the hypothesis test, we have:

Steps	Solution
1. Describe the population parameter of interest.	The parameter of interest is the population proportion p.
2. Formulate the hypotheses: the null hypothesis and the alternative hypothesis.	$H_o: p = p_o$ $H_o: p = .10$ $H_1: p \neq .10$
3. Check the assumptions. • Is the sample size large enough for the Central Limit Theorem(CLT) to apply?	With n= 74, the central limit theorem applies.

Exercises:

Do the following: Do steps 1-3.

A school administrator claims that less than 50% of the students of the school are dissatisfied by the community cafeteria service. Test this claim by using sample data obtained from a survey of 500 students of the school where 54% indicated their dissatisfaction of the community cafeteria service. Use $\alpha = 0.05$.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for the test-statistic of Population Proportion		
Lesson Competency : Computes for the test-statistic value(Population proportion) Draws conclusion about the population proportion based on the test-statistic value and the rejection region. Solves problems involving test of hypothesis on the population proportion.		
References : Statistics and Probability, pg.		LAS No.:

CONCEPT NOTES

Using the .05 level of significance, run a z-test given the following: (Steps 4 and 5)

n= 74; $\hat{p}=\frac{5}{74}$; $p_o = 10\%$

Solution:

The key components of the hypothesis test, we have:

Steps	Solution
4. Choose a significance level size for α. Make a small when the consequences of rejecting a true H_o is severe. • Is the test two-tailed or one-tailed?	$\alpha = 0.05$ Two-tailed. (The problem does not suggest a direction)
5. Select the appropriate test statistic. • Compute the z statistic.	z- statistic $$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$$ $\hat{p}=\frac{5}{74} = 0.068$ p_0= population proportion (Given the null hypothesis) $q_0 = 1 - p_0 = 1 - 0.10 = 0.9$ $$z = \frac{0.068 - 0.1}{\sqrt{\frac{(0.1)(0.9)}{74}}}$$ $Z= \frac{-0.032}{\sqrt{\frac{(0.09)}{74}}}$ $Z= \frac{-0.032}{\sqrt{0.001}} = \frac{-0.032}{0.035} = -091$

Exercises:

Do the following: (Do steps 4 & 5)

A school administrator claims that less than 50% of the students of the school are dissatisfied by the community cafeteria service. Test this claim by using sample data obtained from a survey of 500 students of the school where 54% indicated their dissatisfaction of the community cafeteria service. Use $\alpha = 0.05$.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Computing for the test-statistic of Population Proportion		
Lesson Competency : Computes for the test-statistic value(Population proportion) Draws conclusion about the population proportion based on the test-statistic value and the rejection region. Solves problems involving test of hypothesis on the population proportion.		
References : Statistics and Probability, pg.		LAS No.:

CONCEPT NOTES

The z-test value obtained in the LAS 41 is -091. In the p- value approach we compute the probability value to the left of -0.91. That is, the area between z=0 and z=0.91 is given in the z-table is 0.3186. Therefore, the observed probability value is 0.5000- 0.3186= 0.1814. Since the test is two-tailed, the p-value is multiplied by 2. So, p-value=0.1814X2 = 0.3628

Example:

Using the .05 level of significance, run a z-test given the following:

n= 74; $\hat{p}$= $\frac{5}{74}$; $p_o = 10\%$

Use both the traditional method and the p-value method.

Solution:

The key components of the hypothesis test, we have these steps 6,7 and 8:

Steps	Solution
6. State the decision rule for rejecting or not rejecting the null hypothesis.	In the traditional method, $\alpha = 0.05$, reject if H_o if z value ≤ -1.96 or if the computed value is ≥ 1.96 Accept H_o if the computed z value > -1.96 or if the computed value is < 1.96. In the p-value method, reject H_o if the computed probability value is ≤ 0.05. AcceptH_o if the computed probability value > 0.05
7. Compare the computed values.	• Traditional method: We know that -0.91>-1.96. • P-value method: We know that 0.3628>0.05.
8. Interpret the result.	Based on the result, accept the null hypothesis H_o. There is no significant difference between the sample proportion and the population proportion.

Exercises:

Do the following. (Do the steps 6,7 and 8.)

A school administrator claims that less than 50% of the students of the school are dissatisfied by the community cafeteria service. Test this claim by using sample data obtained from a survey of 500 students of the school where 54% indicated their dissatisfaction of the community cafeteria service. Use $\alpha = 0.05$.

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Illustrating a Bivariate Data		
Lesson Competency :Illustrates the bivariate data.		
References : Statistics and Probability, pg.		LAS No.:44

CONCEPT NOTES:

Bivariate data are data that involve two variables as different from univariate data that involve only a single variable.

In bivariate data, the purpose of analysis is to describe the relationships where new statistical methods will be introduced. We will be describing relationships between related variables in terms of strength and direction.

Example:

1. IQ scores and math scores in a long exam
2. Rainfall amounts and plant growth
3. Exercise and cholesterol level of a group of people

Exercise:

Underline the bivariate data of the given statements.

1. An elastic spring can be subjected to varying stress which will be accompanied by a corresponding elongation.
2. We can collect from different cars their ages and their mileages.
3. A confined gas can have different volumes and corresponding pressure

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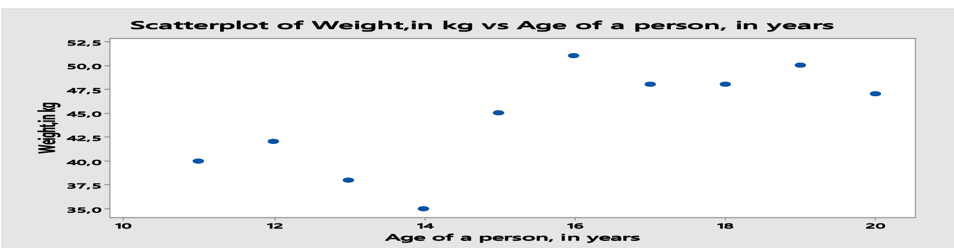
Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Constructing a Scatter Plot		
Lesson Competency :Constructs a scatter plot. Describes shape (form), trend (direction), and variation (strength) based on a scatter plot.		
References : Statistics and Probability, pg.		LAS No.:45

CONCEPT NOTES:
Scatter plot shows how the points collected from a set of bivariate data are scattered on the Cartesian plane. It gives a good visual picture of the two variables which helps in finding the relationship that exists between the two variables. A scatter plot is a graphical representation of the relationship between two variables.

Example:
Construct the scatterplot for the bivariate data. Describe trend (direction), and variation (strength) based on a scatter plot.

Age of a person, in years	11	12	13	14	15	16	17	18	19	20
Weights,in kg	40	42	38	35	45	51	48	48	50	47

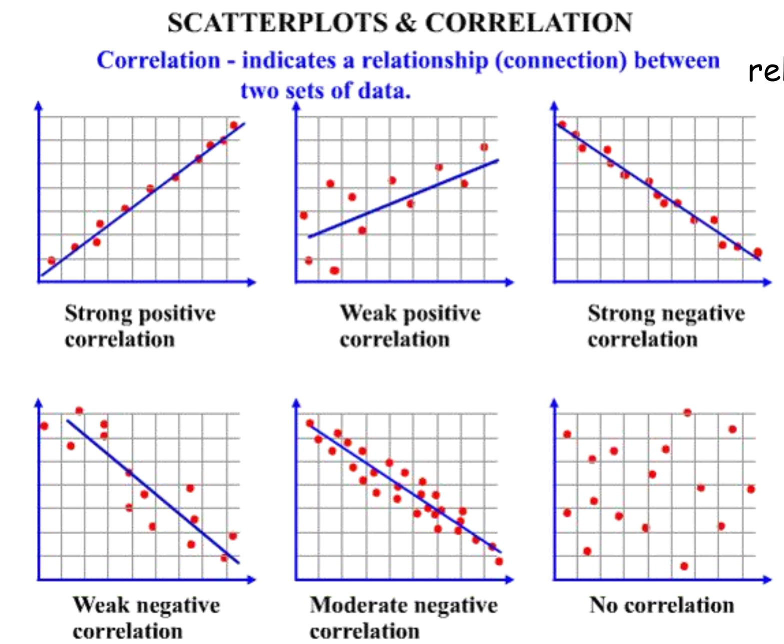


A positive relationship exists;
Exercise:
Construct the scatterplot for the bivariate data. Describe trend (direction), and variation (strength) based on a scatter plot.

Age of a car, in years	0.5	1	1.5	2	3	4	4.5	5	6
Mileage, in km/liter	16	15	10	12	10	12	11	10	11

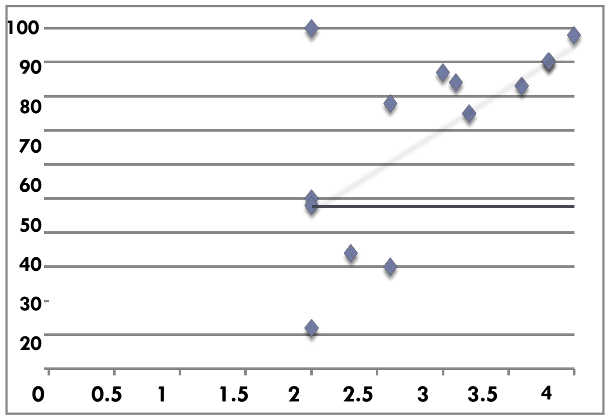
Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Estimating Strength of Association Between Variables		
Lesson Competency :Estimates strength of association between the variables based on a scatter plot. **M11/12SP-IVh-1**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 46

CONCEPT NOTES



Example

There ismoderate, positive, linear relationship between 2 variables.



EXERCISES

Given the figures below, identify the strength of their relationship.

Figure 1

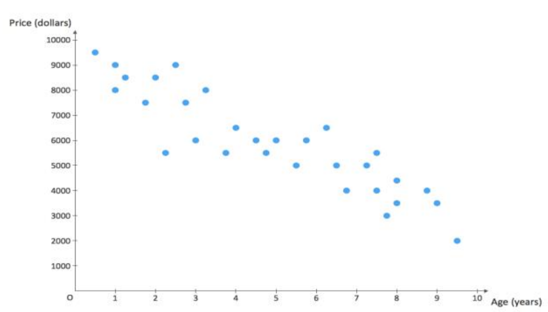


Figure 2

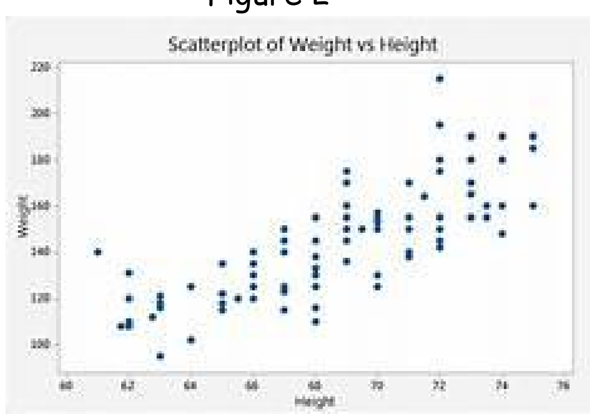


Figure 3

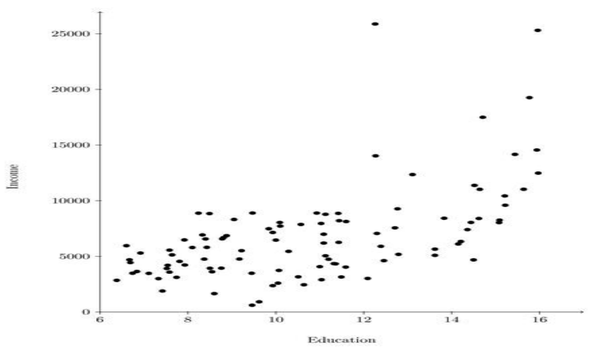
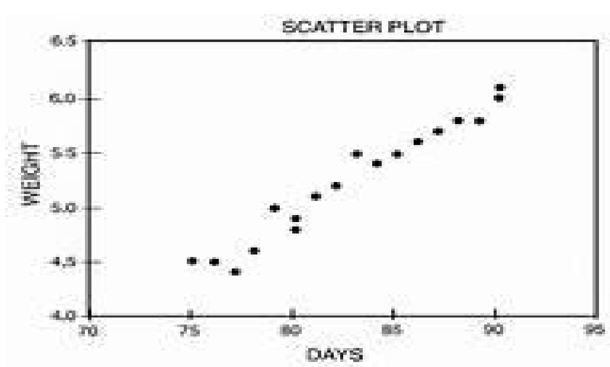


Figure 4



Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Calculating the Pearson's Sample Correlation Coefficient		
LearningCompetency :Calculates the Pearson's sample correlation coefficient and solves problems involving correlation analysis.**M11/12SP-IVh-2 , M11/12SP-IVh-3**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 47

CONCEPT NOTES

The most widely used correlation coefficient is the Pearson Product-Moment Correlation, r, often called, pearson r. The pearson r, can compare two sets of linear data that are of the ratio or interval type.

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{[\Sigma(x-x)^2\,\Sigma(y-y)^2]}}$$

Pearson r	Qualitative Description
± 1	Perfect
± 0.75 *to* $< \pm 1$	Very High
± 0.5 *to* ± 0.75	Moderately High
± 0.25 *to* < 0.5	Moderately Low
> 0 *to* $< \pm 0.25$	Very Low
0	No Correlation

Example:

A researcher wanted to determine if there is a relationship between the scores in Physics and Statistics. Interpret the result.

Student	Score in Statistics, x	Score in Physics, y
Al	3	5
Frans	9	8
Raf	10	10
Jam	12	9
Loi	7	8

Solution:

Steps	Student	x	y	x^2	y^2	xy
1. Square all entries in the x and y columns, and put them under the x^2 and y^2 columns respectively. 2. Multiply entries in x and y columns and put them under the xy column. 3. Get the sum of all entries in the x, y, x^2, y^2 and xy.	Al	3	5	9	25	15
	Frans	9	8	81	64	72
	Raf	10	10	100	100	100
	Jam	12	9	144	81	108
	Loi	7	8	49	64	56
	$\sum$	41	40	383	334	351

4. Substitute the values obtained from stem 3 in the formula, with n=5

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{[\Sigma(x-x)^2\,\Sigma(y-y)^2]}}$$

$$r = \frac{(5)(351)-(41)(40)}{\sqrt{[(5)(383)-(41)^2][(5)(334)-(40)^2]}} = 0.90$$

Interpretation: The computed r=0.90 indicates a Very High Positive Correlation. This means that the students who got high in Physics also high in Statistics.

EXERCISE: A researcher investigated the relationship between the height of a father and his eldest son, in inches.

Height of the Father	71	69	67	68	68	66	70	72	65	60
Height of the Son	71	69	69	65	66	63	68	70	60	58

Do the data support the hypothesis that height is hereditary? Explain. (hint: determine the relationship between the ht. of the father and his son.)

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Identifying the Independent and Dependent Variables		
LearningCompetency :. Identifies the independent and dependent variables.**M11/12SP-IVi-1**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 48

CONCEPT NOTES

In mathematical modeling, statistical modeling and experimental sciences, the values of dependent variables depend on the values of independent variables. The dependent variables represent the output or outcome whose variation is being studied.

EXAMPLES

1. **Effect of fertilizer on plant growth**

In a study measuring the influence of different quantities of fertilizer on plant growth, the independent variable would be the amount of fertilizer used. The dependent variable would be the growth in height or mass of the plant. The controlled variables would be the type of plant, the type of fertilizer, the amount of sunlight the plant gets, the size of the pots, etc.

2. **Effect of drug dosage on symptom severity**

In a study of how different doses of a drug affect the severity of symptoms, a researcher could compare the frequency and intensity of symptoms when different doses are administered. Here the independent variable is the dose and the dependent variable is the frequency/intensity of symptoms.

3. **Effect of temperature on pigmentation**

In measuring the amount of color removed from beetroot samples at different temperatures, temperature is the independent variable and amount of pigment removed is the dependent variable

EXERCISES

Identify the Dependent and Independent variables in each pair of the following variables. Place your answers in the succeeding table.

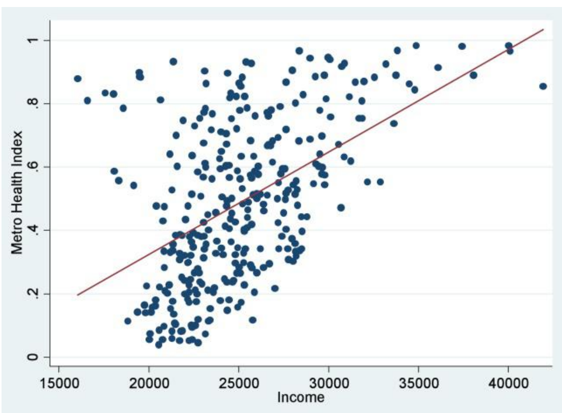
Pair	Dependent	Independent
Altitude and Acceleration due to gravity		
Price of goods and the Demand		
Monthly Salary and the Annual Income of a Worker		
IQ and Academic Performance of a Student		
Temperature and Volume of air in a Balloon		

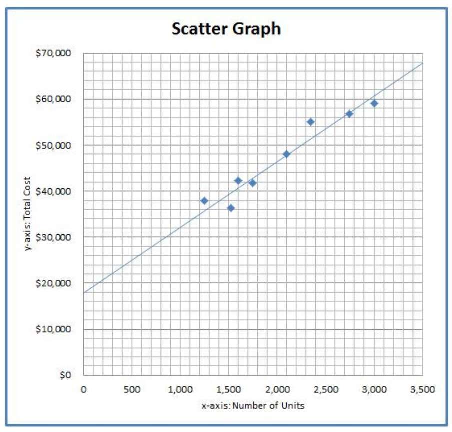
Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Best Fit on a Scatter Plot		
Lesson Competency :Draws the best-fit line on a scatter plot.**M11/12SP-IVi-2**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 49

CONCEPT NOTES

A best-fit line is meant to mimic the trend of the data. In many cases, the line may not pass through very many of the plotted points. Instead, the idea is to get a line that has equal numbers of points on either side. Most people start by eye-balling the data.

EXAMPLE





EXERCISES

Draw the Best-fit line on the scatter Plot of the figure given below.

Figure 1

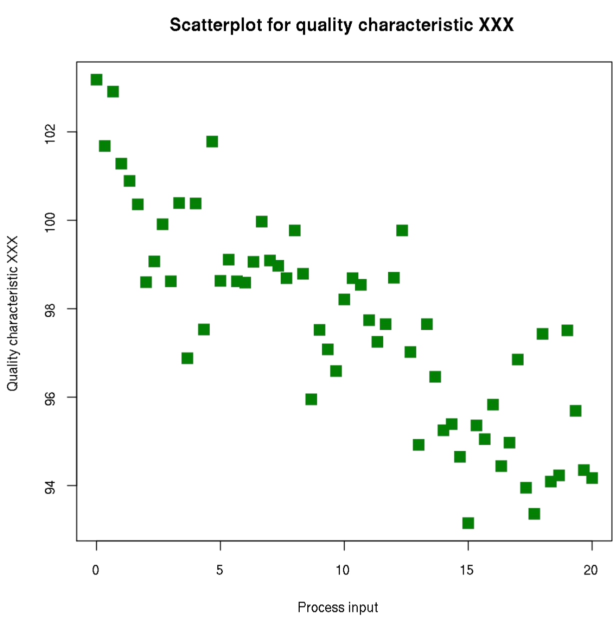
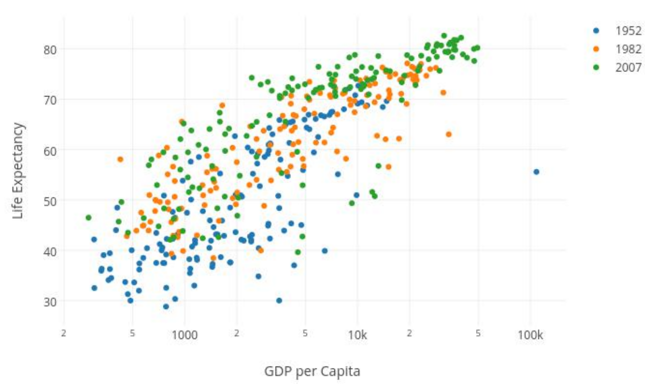


Figure 2



Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title :Calculating the Slope and Y-intercept of the Regression Line		
Lesson Competency :Calculates the slope and y-intercept of the regression line. **M11/12SP-IVi-3**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo		LAS No.: 50

CONCEPT NOTES

Regression analysis is widely used for prediction and forecasting, where its use has substantial overlap with the field of machine learning. Regression analysis is also used to understand which among the independent variables are related to the dependent variable, and to explore the forms of these relationships. In restricted circumstances, regression analysis can be used to infer causal relationships between the independent and dependent variables.

The Regression equation: $y' = bx + a$, where b is the slope and a is the y-intercept.

EXAMPLE

Compute slope and the y-intercept of the regression equation with the given data:

X values	1	3	4	2	5
Y values	2	3	3	4	5

Solution:

X	Y	x^2	y^2	xy
1	2	1	4	2
3	3	9	9	9
4	3	16	9	12
2	4	4	16	8
5	5	25	25	25
$\sum$ 15	17	55	63	56

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(17)(55)-(15)(56)}{5(55)-(15)^2} = 1.9$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{5(56)-(15)(17)}{5(55)-(15)^2} = 0.5$$

$$y' = bx + a$$

$$y' = 0.5x + 1.9$$

Therefore: Slope=0.5 and the y-intercept=1.9

EXERCISES

Compute slope and the y-intercept of the regression equation with the given data:

X values	2	3	3	4	5
Y values	3	5	7	7	8

Name:	Date:	Score:
Subject : Statistics and Probability		
Lesson Title : Interpreting the Calculated Slope and Y-intercept of the Regression Line		
Lesson Competency :interprets the calculated slope and y-intercept of the regression line. predicts the value of the dependent variable given the value of the independent variable. solves problems involving regression analysis. **M11/12SP-IVi-4 , M11/12SP-IVj-1 , M11/12SP-IVj-2**		
References : Statistics and Probability by R. Belecina, E. Baccay and E. Mateo	LAS No.: 51	

CONCEPT NOTES

The Regression equation: $y' = bx + a$, where b is the slope and a is the y-intercept.

EXAMPLE

The following data pertains to the height of a father and his eldest son, in inches. Find the height of the son if the father's height is 78 inches.

Height of the Father 71 69 67 68 68 66 70 72 65 60
Height of the Son 71 69 69 65 66 63 68 70 60 58

Solution

x	y	x^2	y^2	xy
71	71	5041	5041	5041
69	69	4761	4761	4761
69	71	4761	5041	4899
65	68	4225	4624	4420
66	68	4356	4624	4488
63	66	3969	4356	4158
68	70	4624	4900	4760
70	72	4900	5184	5040
60	65	3600	4225	3900
58	60	3364	3600	3480
T=659	T=680	T=43601	T=46356	T=44947

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(680)(43601)-(659)(44947)}{10(43601)-(659)^2} = 16.55$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{10(44947)-(659)(680)}{10(43601)-(659)^2} = 0.78$$

$$y' = bx + a$$

$$y' = 0.78x + 16.55$$

Find the height of the son if the father's height is 78 inches.

Y'=0.78 (78)+16.55

Y'=77.39 say 77 inches

Interpretation:
The predicted height of the son whose father is 78 inches is 77 inches.

EXERCISE

Data show the population of the Philippines from 2005-2012. Find the predicted population in the year 2013.

Year ,x	2005	2006	2007	2008	2009	2010	2011	2012
Population, y (in Million)	85.26	86.97	88.71	90.46	91.02	92.6	94.18	95.77

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating Random Variable		
Lesson Competency : To illustrate a random variable (M11/12SP-IIIa-1) To find the possible values of a random variable (M11/12SP-IIIa-3)		
References : Statistics and Probability pages 2-5		LAS No.: 3.1

CONCEPT NOTES

A **random variable** is a function that associates a real number to each element in the sample space. It is a variable whose values are determined by chance. Use capital letters to denote or represent a variable.

The set of all possible outcomes of an experiment is called a **sample space.**

EXAMPLE:

Suppose three cell phones are tested at random. We want to find out the number of defective cell phones that occur.

Let D represent the defective cell phone and N represent the non-defective cell phone. If we let X be the random variable representing the number of defective cell phones, can you show the values of the random variable X?

Consider the table below.

Possible Outcomes/Sample Space	**Value of the Random Variable X (number of defective cell phones)**
NNN	0
NND	1
NDN	1
DNN	1
NDD	2
DND	2
DDN	2
DDD	3

The values of the random variable are 0, 1, 2, 3. This means that there could be a zero, one, two, or three numbers of defective cell phones in the possible outcomes of an experiment.

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the values of the random variable Y. Make a table to illustrate the situation.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Distinguishing Discrete from Continuous Random Variable		
Lesson Competency : To distinguish between a discrete and a continuous random variable. (M11/12SP-IIIa-2)		
References : Statistics and Probability pages 6-8		LAS No.: 3.2

CONCEPT NOTES

The random variables whose set of possible outcomes is countable are called **discrete random variables.** Mostly, discrete random variables represent count data, such as the number of defective chairs produced in a factory.

A random variable is a **continuous random variable** if it takes on values on a continuous scale. Often, continuous random variables represent measured data, such as heights, weights, and temperatures.

Example:

Classify the following random variables as discrete or continuous.

1. the number of defective computers produced by a manufacturer

Answer: Discrete because number of defective computers can be counted as a whole number

2. the weight of newborns each year in the hospital

Answer: Continuous because the weight of newborns cannot be counted as a whole number but it can be measured.

EXERCISES:

Classify the following random variables as discrete or continuous.

1. the number of siblings in a family of a region
2. the speed of a car
3. the number of female students
4. the number of presidents of the Philippines
5. the amount of sugar in a cup of coffee

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Illustrating a Probability Distribution for a Discrete Random Variable		
Lesson Competency : To illustrate a probability distribution for a discrete random variable and its properties. (M11/12SP-IIIa-4)		
References : Statistics and Probability pages 9, 15-16		LAS No.: 3.3

CONCEPT NOTES

A **discrete probability distribution** or a **probability mass function** consists of the values a random variable can assume and the corresponding probabilities of the values.

Properties of a Probability Distribution

1. The probability of each value of the random variable must be between or equal to 0 and 1. In symbol, we write it as $0 \leq P(X) \leq 1$.
2. The sum of the probabilities of all values of the random variable must be equal to 1. In symbol, we write it as $\Sigma\, P(X) = 1$.

Examples:

A. Find the probability of the following events:

1. Getting an even number in a single roll of a die.
Answer: $\frac{1}{2}$ since there are 3 even numbers in a die.
2. Getting a red ball from a box containing 3 red and 6 black balls.
Answer: $\frac{1}{3}$ since there are 3 out of 9 balls from a box

B. Determine whether the distribution represents a probability distribution.

X	1	5	8
P(X)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

Answer: It is a probability distribution because the total or the sum of P(X) is equal to 1.

EXERCISES:

A. Find the probability of the following events:

1. Getting an odd number in a single roll of a die.
2. Getting a tail in tossing a coin.

B. Determine whether the distribution represents a probability distribution.

X	1	3	4	6	8	9
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Probability Distribution for a Discrete Random Variable		
Lesson Competency : To construct the probability distribution of discrete random variable and its corresponding histogram (M11/12SP-IIIa-5) To compute probabilities corresponding to a given random variable (M11/12SP-IIIa-6)		
References : Statistics and Probability pages 9-15		LAS No.: 3.4

CONCEPT NOTES

Steps in constructing the probability distribution of discrete random variable:

1. Determine the sample space.
2. Count the number of the identified outcome in the sample space and assign number to this outcome.
3. Get the possible values of the random variable and compute the probability.
4. Make a table of the probability distribution.

Examples:

Suppose two coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 1 and 2.

Step 1. The sample space for this experiment is: S= {HH, HT, TH, TT}

Step 2. Count the number of tails in the sample space and assign number to this outcome.

Possible Outcomes/Sample Space	Value of the Random Variable Y (number of tails)
HH	0 (no tail)
HT	1 (only one tail)
TH	1 (only one tail)
TT	2 (two tails)

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 1 and 2.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Probability Distribution for a Discrete Random Variable		
Lesson Competency : To construct the probability distribution of discrete random variable and its corresponding histogram (M11/12SP-IIIa-5) To compute probabilities corresponding to a given random variable (M11/12SP-IIIa-6)		
References : Statistics and Probability pages 9-15		LAS No.: 3.5

CONCEPT NOTES

Steps in constructing the probability distribution of discrete random variable:

1. Determine the sample space.
2. Count the number of the identified outcome in the sample space and assign number to this outcome.
3. Get the possible values of the random variable and compute the probability.
4. Make a table of the probability distribution.

Examples:

Suppose two coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 3 and 4.

Step 3. Get the possible values of the random variable and compute the probability.

Number of Tails Y	Probability P(Y)
0	$\frac{1}{4}$ since it occurs only once out of 4 outcomes
1	$\frac{2}{4}$ or $\frac{1}{2}$ since it occurs twice out of 4 outcomes
2	$\frac{1}{4}$ since it occurs once out of 4 outcomes

Step 4. Table1. The Probability Distribution or the Probability Mass Function of Discrete Random Variable Y

Number of Tails	0	1	2
Probability P(Y)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

EXERCISES:

1. Suppose three coins are tossed. Let Y be the random variable representing the number of tails that occur. Find the probability of each of the values of the random variable Y following the steps 3 and 4.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Mean of a Discrete Probability Distribution		
Lesson Competency : To illustrate the mean of a discrete random variable (M11/12SP-IIIb-1) To calculate the mean of a discrete random variable (M11/12SP-IIIb-2) To interpret the mean of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving mean of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 21-30		LAS No.: 3.6

CONCEPT NOTES

Steps in computing the mean of a discrete probability distribution:

1. Construct the probability distribution for the random variable.
2. Multiply the value of the random variable by the corresponding the probability.
3. Add the results obtained in Step 2.
4. Interpret the result.

Examples:

Consider rolling a die. What is the average number of spots that would appear?

Solution:

Step 1. Construct the probability distribution for the random variable.

Step 2. Multiply the value of the random variable by the corresponding the probability.

Number of Spots in a Die	Probability P(X)	X • P(X)
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$
Total		$\frac{21}{6}$ or 3.5

.

Step 3. Add the results obtained in Step 2 which is $\frac{21}{6}$ or 3.5

Step 4. Interpret the result.

The value obtained in Step 3 is called the mean of the random variable X or the mean of the probability distribution of X. The mean tells us the average number of spots that would appear in a roll of a die. So, the average number of spots that would appear is 3.5. Although the die will never show a number, which is 3.5, this implies that rolling the die many times, the theoretical mean would be 3.5.

EXERCISES:

The probabilities that a customer will buy 1, 2, 3, 4 or 5 items in a grocery store are $\frac{3}{10}, \frac{1}{10}, \frac{1}{10}, \frac{2}{10}, and\ \frac{3}{10}$, respectively. What is the average number of items that a customer will buy? Follow the steps in computing the mean of the discrete random variable.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.7

CONCEPT NOTES

The variance and standard deviation describe the amount of spread, dispersion, or variability of the items in a distribution.

Steps in Finding the Variance and Standard Deviation:

1. Find the mean of the probability distribution.
2. Subtract the mean from each value of the random variable X.
3. Square the results obtained in Step 2.
4. Multiply the results obtained in Step 3 by the corresponding probability.
5. Get the sum of the results obtained in Step 4 to obtain variance.
6. Get the square root of the variance to get the standard deviation.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 1 and 2.

Solution:

Step 1. Find the mean of the probability distribution in column 3.

Step 2. Subtract the mean from each value of the random variable X in column 4.

Number of Cars Sold (X)	Probability P(X)	X• P(X)	X -μ
0	$\frac{2}{10}$	0	0- 2.2 = -2.2
1	$\frac{2}{10}$	$\frac{2}{10}$	1-2.2 = -1.2
2	$\frac{3}{10}$	$\frac{6}{10}$	2-2.2 =-0.2
3	$\frac{2}{10}$	$\frac{6}{10}$	3-2.2 =0.8
4	$\frac{2}{10}$	$\frac{8}{10}$	4-2.2 =1.8
$\mu = \Sigma X \bullet P(X) = \frac{22}{10} = 2.2$			

EXERCISES:

When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 1 and 2.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.8

CONCEPT NOTES

The variance and standard deviation describe the amount of spread, dispersion, or variability of the items in a distribution.

Steps in Finding the Variance and Standard Deviation: (Continuation of LAS 3.7)

3. Square the results obtained in Step 2.
4. Multiply the results obtained in Step 3 by the corresponding probability.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 3 and 4.

Solution:

Step 3. Square the results obtained in Step 2 and write in column 5.

Step 4. Multiply the results obtained in Step 3 by the corresponding probability and write in column 6.

Number of Cars Sold (X)	Probability P(X)	X• P(X)	X -μ	$(X-\mu)^2$	$(X-\mu)^2 \cdot P(X)$
0	$\frac{2}{10}$	0	0- 2.2 = -2.2	4.84	0.484
1	$\frac{2}{10}$	$\frac{2}{10}$	1-2.2 = -1.2	1.44	0.288
2	$\frac{3}{10}$	$\frac{6}{10}$	2-2.2 =-0.2	0.04	0.012
3	$\frac{2}{10}$	$\frac{6}{10}$	3-2.2 =0.8	0.64	0.128
4	$\frac{2}{10}$	$\frac{8}{10}$	4-2.2 =1.8	3.24	0.648

EXERCISES:

When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 3 and 4.

Name:	Date:	Score:
Subject : STATISTICS AND PROBABILITY		
Lesson Title : Variance of a Discrete Probability Distribution		
Lesson Competency : To illustrate the variance of a discrete random variable (M11/12SP-IIIb-1) To calculate the variance of a discrete random variable (M11/12SP-IIIb-2) To interpret the variance of a discrete random variable (M11/12SP-IIIb-3) To solve problems involving variance of probability distribution (M11/12-IIIb-4)		
References : Statistics and Probability pages 31-45		LAS No.: 3.9

CONCEPT NOTES

Steps in Finding the Variance and Standard Deviation: (Continuation of LAS 3.8)
Step 5. Get the sum of the results obtained in Step 4 to obtain variance.
Step 6. Get the square root of the variance to get the standard deviation.

Examples:

The number of cars sold per day at a local car dealership, along with its corresponding probabilities, is shown in the table. Compute the variance and the standard deviation of the probability distribution following the steps 5 and 6.
Solution:

Step 5. Get the sum of the results obtained in Step 4 to obtain variance.
Answer: The variance is 1.56. In symbol, $\sigma^2 = \Sigma(\mathrm{X} - \mu)^2 \bullet \mathrm{P(X)} = 1.56$

Step 6. Get the square root of the variance to get the standard deviation.
The standard deviation is $\sigma = \sqrt[2]{1.56}$ = 1.25.

EXERCISES:

When three coins are tossed, the probability distribution for the random variable X representing the number of heads that occur is given below. Compute the variance and standard deviation of the probability distribution following the given steps 5 and 6.