# My historic geometric theorem of January 2020.

# Introduction:

My theorem is to calculus as the theorem of Pythagoras is to all of mathematics. The theorem was inspired by the first rigorous formulation of calculus in human history - the New Calculus.

Using the identity, one can well define both the derivative and the definite integral using only sound geometry. This theorem produces identical results to the flawed mainstream formulation of calculus, that is, it allows for derivatives at points of inflection whereas the rigorous New Calculus does not. Students can now learn calculus rigorously in high school without the screeds of rot required in university courses.

# Theorem:

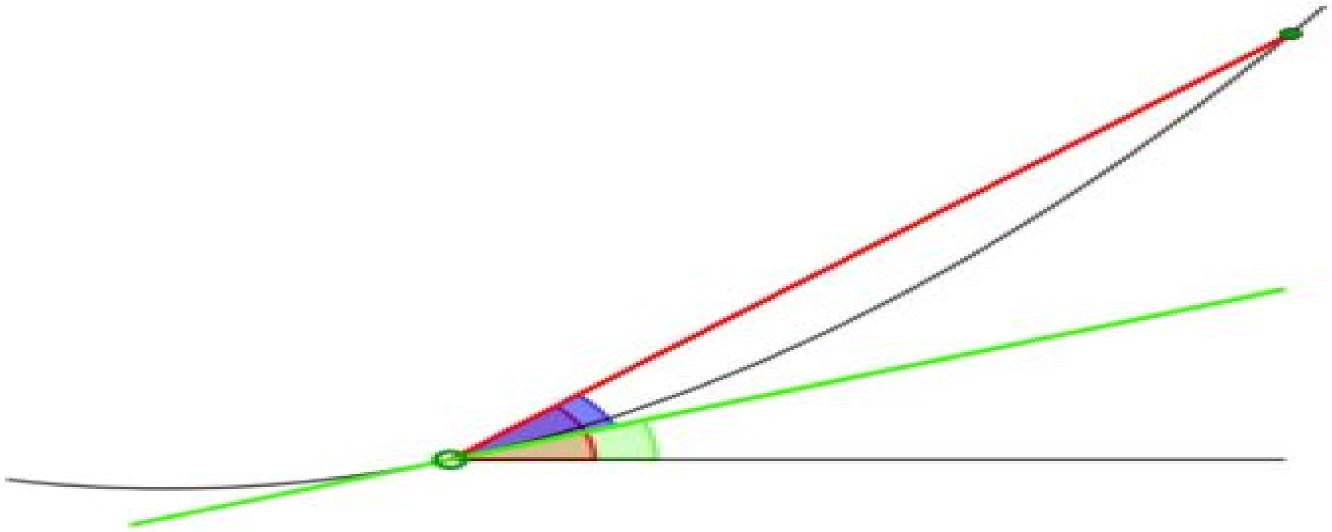
**If any function $f$ is smooth on an interval $(x, x+h)$, it is true that given any non-parallel secant line with endpoints $(x, f(x))$ and $(x+h, f(x+h))$, then the difference in slope between the non-parallel secant $\frac{f(x+h)-f(x)}{h}$ line and the tangent line $f'(x)$, is given by $Q(x,h)$ (an expression in $h$ that may also include $x$), which is never equal to zero unless $f$ is a linear function, that is,**

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$$

The above theorem allows for a rigorous formulation of calculus without any use of ill-formed concepts such as infinity, infinitesimals and the circular rot of limit theory.

In Fig. 1 of the next slide, each of the slopes (in terms of angles) are colour-coded and illustrate the relationship seen in the historic identity:

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$$



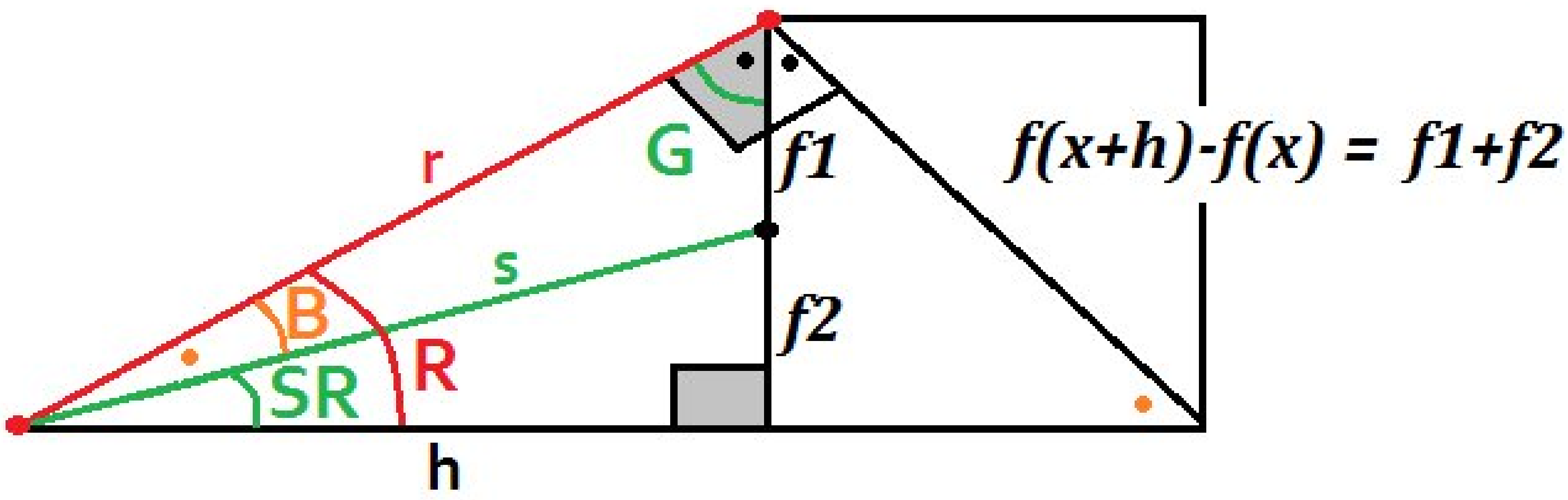
$\frac{f(x+h)-f(x)}{h}$ is the slope of the non-parallel secant line. [1]

$f'(x)$ is the slope of the tangent line. [2]

$Q(x,h)$ is the difference in slopes, [1]-[2]

# Fig. 1

$$\boxed{\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)}$$



In words, the identity $\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$ can be written as:

Slope of the non-parallel secant line (r)

= Slope of tangent line (s) + Difference in slopes ($Q(x,h)$) .

In geometry, the derivative $f'(x)$ is given by $\frac{f2}{h}$. The difference $Q(x,h)$ is given by $\frac{f1}{h}$ . Finally, the slope of the non-parallel secant line $\frac{f(x+h)-f(x)}{h}$ is given by $\frac{f1}{h} + \frac{f2}{h}$. We shall shortly look at the proof that is given by simple trigonometry, but let's see an example first.

# Example.

$$f(x) = x^2$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h} = 2x + h = f'(x) + Q(x,h)$$

If $x = 1$ and $h = 2$:

$$\frac{x^2+2xh+h^2-x^2}{h} = 2(1) + 2 = 4 \quad \leftrightarrow \quad f'(x) = 4 - 2 = 2$$

If $x = 1$ and $h = 1$:

$$\frac{x^2+2xh+h^2-x^2}{h} = 2(1) + 1 = 3 \quad \leftrightarrow \quad f'(x) = 3 - 1 = 2$$

As you can see, the value of the derivative or tangent line slope remains unchanged.

In other words, each non-parallel secant line slope has a unique $\boldsymbol{h}$ which never changes. No secant line can have more than one $\boldsymbol{h}$.

Now it's time for the proof of the theorem.

# Proof:

It's impossible to know using algebra alone where the tangent line meets the perpendicular that is dropped from the red endpoint of the non-parallel secant line. One has to start with trigonometry which is just a special kind of geometry, that is, ***circular geometry***.

In Fig. 1, $\boldsymbol{R = B + SR}$, so

$$\boldsymbol{\tan R = \tan(B + SR) = \frac{\tan B + \tan SR}{1 - \tan B \tan SR} = \frac{f(x+h) - f(x)}{h} = \frac{f1}{h} + \frac{f2}{h} = \frac{f1 + f2}{h}}$$

We know that $\tan SR$ is the tangent line slope given by $\frac{f2}{h}$ or $f'(x)$.

Since $\frac{f(x+h)-f(x)}{h} = \frac{f1}{h} + f'(x)$, it follows that the difference

between $\frac{f(x+h)-f(x)}{h}$ and $f'(x)$ is equal to $\frac{f1}{h}$ or $Q(x,h)$.

$\frac{f2}{h}$ depends only on the value of $x$ because $f2$ is given by any of the $y$ ordinates of the tangent line less the perpendicular distance of the horizontal line to the $x$ axis.

On the other hand, $\frac{\boldsymbol{f1}}{\boldsymbol{h}}$ depends on the value of $\boldsymbol{x}$ and $\boldsymbol{h}$ because $\boldsymbol{f1 = f(x + h) - f2}$. Q.E.D

It is important to note that $\boldsymbol{h}$ is not relevant in the case of a linear function, that is, $\boldsymbol{Q(x,h) = 0}$, meaning this is the only case where the difference expression contains no $\boldsymbol{h}$. In every other case, the $\boldsymbol{Q(x,h)}$ difference will depend on $\boldsymbol{h}$ and may include $\boldsymbol{x}$ according to the given function.

Having proved the theorem, it is now easy to express the derivative using the aforementioned identity as:

$$f'(x) = \frac{f(x+h)-f(x)}{h} - Q(x,h)$$

The flawed mainstream definition $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ is equivalent to discarding the difference $Q(x,h)$, however, this ***limiting*** action would suggest that

$$f'(x) = \frac{f(x+h)-f(x)}{h}$$

which is false unless $f$ is a linear function.

The incorrigible fools of mainstream mathematics academia gave you this:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

instead of this

$$f'(x) = \frac{f(x+h)-f(x)}{h} - Q(x,h)$$

which is sound geometry and does not require you to learn any of the theory of limits with its warts and all.

The definite integral can be defined directly in terms of the same identity. I revealed (many years ago) that any definite integral is a product of two arithmetic means. The incorrigible morons of mainstream academia laughed me to scorn back then, but the vile, jealous scum are no longer laughing having found themselves on the wrong side of history.

To calculate an irregular bounded area between a curve and the $x$-axis, we need to find the arithmetic mean of all the $y$ ordinates of the function in the interval and then multiply it by the interval width – just as we would for a rectangle.

Let's see how we can find the definite integral of a function without the use of limit theory, in other words, the fundamental theorem of calculus which is directly obtained from the mean value theorem. You probably have never learned this before, so I'll quickly show you why it's true.

From the identity of the mean value theorem:

$$\frac{f(x+h)-f(x)}{h}=f'(c)$$

We obtain the fundamental theorem of calculus:

$$f(x+h)-f(x)=f'(c)\times h=\int_{x}^{x+h}f'(x)\,dx$$

The mean value theorem which is about an arithmetic mean, i.e. $f'(c)$, is easy to prove and you will see how it is done in the following proof.

We begin with an interval $(x, x+h)$ divided into $n$ equal parts as follows:

$$x \qquad x+\frac{h}{n} \qquad x+\frac{2h}{n} \qquad \ldots \qquad x+\frac{(n-1)h}{n} \qquad x+h$$

To find the arithmetic mean of all the $y$ ordinates of $\boldsymbol{f'(x)}$, we use my identity

$$f'(x) = \frac{f(x+h)-f(x)}{h} - Q(x,h)$$

and observe the following:

$$f'(x) + Q\left(x, \frac{h}{n}\right) = \frac{f\left(x+\frac{h}{n}\right)-f(x)}{\frac{h}{n}}$$

$$f'\left(x+\frac{h}{n}\right) + Q\left(x+\frac{h}{n}, \frac{h}{n}\right) = \frac{f\left(x+\frac{2h}{n}\right)-f\left(x+\frac{h}{n}\right)}{\frac{h}{n}}$$

...

$$f'\left(x+\frac{(n-1)h}{n}\right) + Q\left(x+\frac{(n-1)h}{n}, \frac{h}{n}\right) = \frac{f\left(x+\frac{(n-1)h}{n}+\frac{h}{n}\right)-f\left(x+\frac{(n-1)h}{n}\right)}{\frac{h}{n}}$$

Note that the right hand side sum telescopes, and all the purple terms cancel out to give $\frac{f(x+h)-f(x)}{\frac{h}{n}}$.

Thus, summing the left hand side and the right hand side, we get:

$$\sum_{i=0}^{n-1}\left[f'\left(x+\frac{hi}{n}\right) + Q\left(x+\frac{hi}{n},\ \frac{h}{n}\right)\right]=\frac{f(x+h)-f(x)}{\frac{h}{n}}$$

Let $\boldsymbol{Q(x,h)} = \sum_{i=0}^{n-1}\left[\, Q\left(x+\frac{hi}{n},\ \frac{h}{n}\right)\right]$

$$\sum_{i=0}^{n-1}\left[f'\left(x+\frac{hi}{n}\right)\right] + Q(x,h) = \frac{f(x+h)-f(x)}{\frac{h}{n}}$$

Dividing by $n$ gives the **arithmetic mean**:

$$\frac{1}{n}\times\sum_{i=0}^{n-1}\left[f'\left(x+\frac{hi}{n}\right)\right] + \frac{Q(x,h)}{n} = \frac{f(x+h)-f(x)}{h}$$

Now we multiply by $\boldsymbol{h}$ to get the area:

[PC]

$$\frac{h}{n} \times \sum_{i=0}^{n-1} \left[ f'\left( x + \frac{hi}{n} \right) \right] + \frac{h \cdot Q(x,h)}{n} = f(x+h) - f(x)$$

OR

[MC]

$$\frac{h}{n} \times \sum_{i=0}^{n-1} \left[ f'\left( x + \frac{hi}{n} \right) \right] = f(x+h) - f(x) - \frac{h \cdot Q(x,h)}{n}$$

The above result [MC] is the **fundamental theorem of calculus.** Note that the result is obtained by a **FINITE number of steps**, that is, any integer value of $\boldsymbol{n > 0}$ will be sufficient to find the integral.

One might ask why we need to subtract $\frac{h \cdot Q(x,h)}{n}$ from $f(x+h) - f(x)$. The reason for this is immediately obvious from the my geometric identity:

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$$

If we want the arithmetic mean of all the $y$ ordinates of the function $\boldsymbol{f'}(\boldsymbol{x})$, then we must find the arithmetic mean in terms of $\boldsymbol{f}(\boldsymbol{x}+\boldsymbol{h})-\boldsymbol{f}(\boldsymbol{x}) - \frac{\boldsymbol{h}\cdot\boldsymbol{Q}(\boldsymbol{x},\boldsymbol{h})}{\boldsymbol{n}}$, for otherwise we are not considering $\boldsymbol{f'}(\boldsymbol{x})$, but the slopes of non-parallel secant lines given by $\frac{\boldsymbol{f}(\boldsymbol{x}+\boldsymbol{h})-\boldsymbol{f}(\boldsymbol{x})}{\boldsymbol{h}}$.

Example 1: Let $f(x) = x^n$

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$$

$$\frac{\left(\cancel{x^n} + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \ldots + h^n\right) - \cancel{x^n}}{h} = f'(x) + Q(x,h)$$

$$\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h + \ldots + h^{n-1} = f'(x) + Q(x,h)$$

Now subtract $Q(x,h)$ from both sides where $Q(x,h) = \binom{n}{2}x^{n-2}h + \ldots + h^{n-1}$

Thus,

$$\binom{n}{1}x^{n-1} = f'(x) \quad \text{or} \quad f'(x) = nx^{n-1} \text{ since } n = \binom{n}{1}$$

In some instances $\boldsymbol{Q(x,h)}$, may be very hard to find, but since we know that

$$\boldsymbol{Q(x,h)} = \frac{\boldsymbol{f(x+h)-f(x)}}{\boldsymbol{h}} - \boldsymbol{f'(x)}$$

This is not an impossible task.

The previous example showed you how you can differentiate using my identity. In the next example, we shall see how you can understand the definite integral in terms of the same identity.

Let $\boldsymbol{f(x) = x^3}$, $\boldsymbol{h = 2}$, $\boldsymbol{n = 3}$. Find the area from $x = 1$ to $x = 3$.

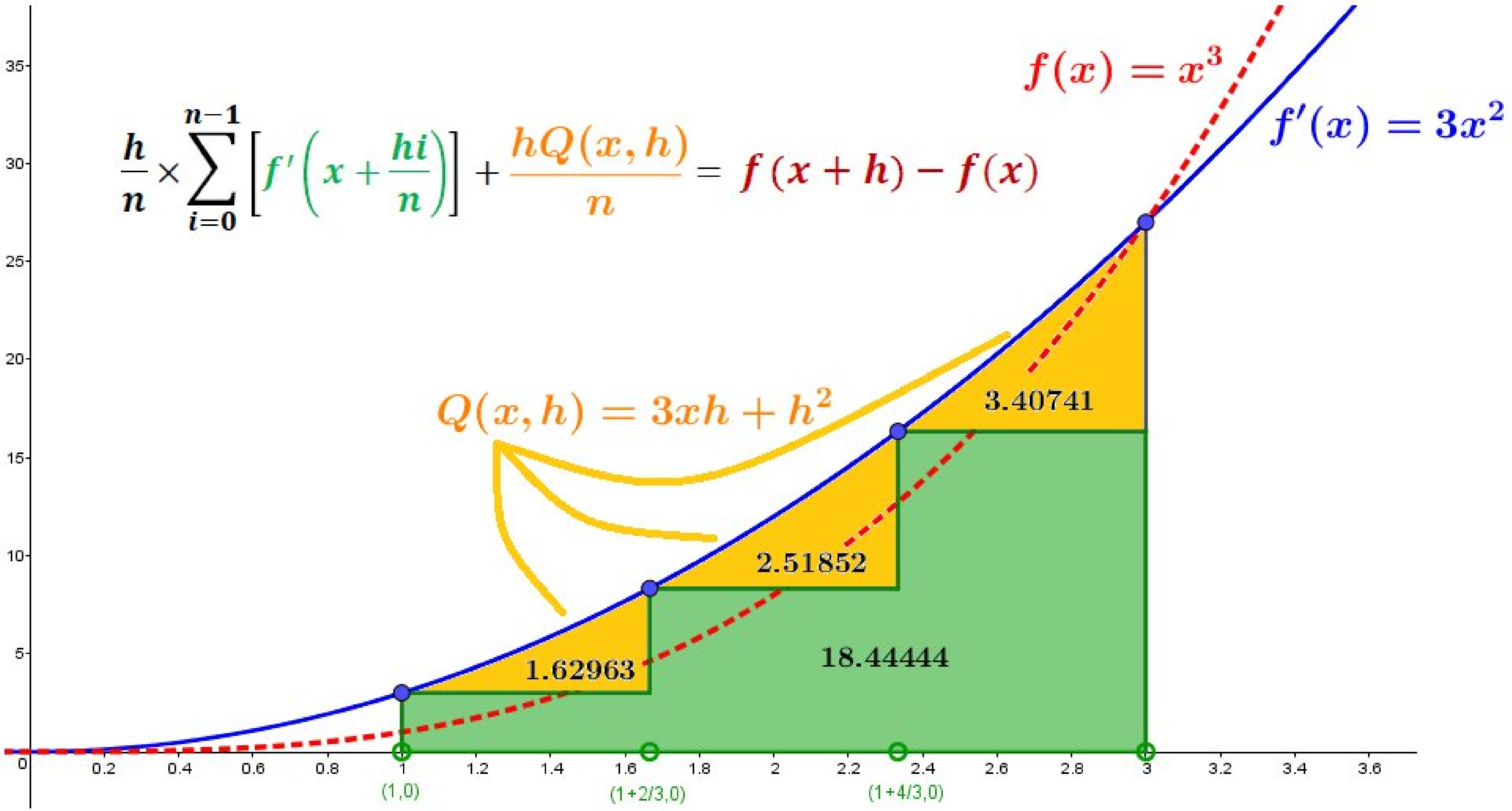
$$\frac{h}{n} \times \sum_{i=0}^{n-1} \left[ f'\left( x + \frac{hi}{n} \right) \right] + \frac{h \cdot Q(x,h)}{n} = f(x+h) - f(x)$$

$$\frac{2}{3} \times \left[ f'(1) + f'\left(\frac{5}{3}\right) + f'\left(\frac{7}{3}\right) \right] + \frac{2 \cdot \left[ 3(1)\left(\frac{2}{3}\right) + 3\left(\frac{5}{3}\right)\left(\frac{2}{3}\right) + 3\left(\frac{7}{3}\right)\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 \right]}{3}$$

$$= f(3) - f(1) = \frac{2}{3} \times \left[ \frac{9}{3} + \frac{25}{3} + \frac{49}{3} \right] + \frac{204}{27} = 27 - 1$$

$$\frac{2}{3} \times \left[ \frac{83}{3} \right] + \frac{204}{27} = \frac{166}{9} + \frac{204}{27} = \frac{498}{27} + \frac{204}{27} = \frac{702}{27} = 26$$

The diagram below makes these facts clearer:



Notice that the orange areas (which are not triangles!) are given by

$$\boldsymbol{Q(x,h)} = \sum_{i=0}^{n-1} \left[ \boldsymbol{Q}\left( \boldsymbol{x} + \frac{\boldsymbol{hi}}{\boldsymbol{n}}, \ \frac{\boldsymbol{h}}{\boldsymbol{n}} \right) \right]$$

where $\boldsymbol{Q}\,(\boldsymbol{x}, \boldsymbol{h_i}) = \mathbf{3}\boldsymbol{xh} + \boldsymbol{h}^2$ for each $\boldsymbol{x} + \frac{\boldsymbol{hi}}{\boldsymbol{n}}$.

The green areas are given by

$$\frac{\boldsymbol{h}}{\boldsymbol{n}} \times \sum_{i=0}^{n-1} \left[ \boldsymbol{f}'\left( \boldsymbol{x} + \frac{\boldsymbol{hi}}{\boldsymbol{n}} \right) \right]$$

The sum of these orange and green areas is the area between the curve , the x-axis and the boundaries $x = 1$ and $x = 3$.

If you are wondering how the orange coloured regions are areas, then observe that

$$\frac{f(x+h)-f(x)}{h} = 3x^2 + 3xh + h^2 \quad \text{if} \quad f(x) = x^3$$

Since area is the product of two level magnitudes (aka incorrectly as arithmetic means), we have

$$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3 = \int_x^{x+h} 3x^2 dx$$

$$\frac{2}{3} \times \left[ f'(1) + f'\left(\frac{5}{3}\right) + f'\left(\frac{7}{3}\right) \right] + \frac{2 \cdot \left[ 3(1)\left(\frac{2}{3}\right) + 3\left(\frac{5}{3}\right)\left(\frac{2}{3}\right) + 3\left(\frac{7}{3}\right)\left(\frac{2}{3}\right) + 3\left(\frac{2}{3}\right)^2 \right]}{3}$$

$$= f(3) - f(1)$$

As you can see, there is no use of infinity, infinitesimals or the circular rot of limit theory, only sound geometry.

I solved the tangent line problem, not Newton or Leibniz or any of the mainstream mathematics idiots that followed them.

The aforementioned process is far simpler in the **New Calculus,** because parallel secant lines (as opposed to non-parallel secant lines ala Newton and Leibniz) are used and there is no extraneous term or expression $\boldsymbol{Q\,(x,h)}$ that led to the **behemoth** rot known as the ***theory of limits***. Limit theory is not required for either the derivative or integral.

Far too long the LIE (sincere or otherwise) that calculus requires limit theory has been propagated by the ignorant morons of the mainstream who are too stupid, too stubborn and too arrogant to accept correction.

Calculus does NOT require any LIMIT theory.

I am the great John Gabriel, discoverer of the first rigorous formulation of calculus in human history. More advanced alien civilisations may already know of it, because well-formed concepts are independent of the human mind or any other mind.