Structural Health Monitoring
2021, Vol. 0(0) 1–27

DOI: 10.1177/14759217211028439
journals.sagepub.com/home/shm

An alternative quantification of the value of information in structural health monitoring

Mayank Chadha[1], Zhen Hu[2] and Michael D Todd[1]

Abstract

Analogous to an experiment, a structural health monitoring (SHM) system may be thought of as an information-gathering mechanism. Gathering the information that is representative of the structural state and correctly inferring its meaning helps engineers (decision-makers) mitigate possible losses by taking appropriate actions (risk-informed decision-making). However, the design, research, development, installation, maintenance, and operation of an SHM system are an expensive endeavor. Therefore, the decision to invest in new information is rationally justified if the reduction in the expected losses by utilizing newly acquired information is more than the intrinsic cost of the information acquiring mechanism incurred over the lifespan of the structure. This article investigates the economic advantage of installing an SHM system for inference of the structural state, risk, and lifecycle management by using the value of information (VoI) analysis. Among many possible choices of SHM system designs (different information-gathering mechanisms), pre-posterior decision analysis can be used to select the most feasible design. Traditionally, the cost–benefit analysis of an SHM system is carried out through pre-posterior decision analysis that helps one evaluate the benefit of an experiment or an information-gathering mechanism using the expected value of information metric. This study proposes an alternate normalized metric that evaluates the expected reward ratio (benefit/gain of using an SHM system) relative to the investment risk (cost of SHM over the lifecycle). The analysis of evaluating the relative benefit of various SHM system designs is carried out by considering the concept of the VoI, by performing pre-posterior analysis, and the idea of a perfect experiment is discussed.

Keywords

Bayesian decision theory, value of information, pre-posterior analysis, behavioral psychology, structural health monitoring, machine learning, digital twins, miter gates

Introduction

Structural health monitoring (SHM)[1] aims to assess the current structural health state in such a way to enable stakeholders to make informed performance, maintenance, and/or repair decisions, based on appropriate analyses of in situ measured data. This goal is achieved by a modern paradigm involving periodically spaced or continuous data acquisition, extraction of relevant features to establish the damage detection or classification hypothesis, and evaluating the hypothesis under all the sources of uncertainty and variability that inevitably corrupt the monitoring and inference process. Structural health monitoring is integrated with predictive degradation/failure and demand/loads models ("prognosis") into a decision-making framework for optimal life-cycle management of the structure.[2] However, the benefits related to improved decision-making that an SHM system is expected to bring to life-cycle management are balanced by the costs that it incurs. Fundamentally, the evaluation of an SHM system essentially depends on its design; at the core of any well-designed SHM system is a data acquisition system that relies on (usually an array of) deployed sensors to initiate the information workflow from which ultimate decisions about operations, maintenance, and other life-cycle actions will be made. Therefore, an SHM system can be thought of as an information-gathering mechanism, yet there are costs to design, research, develop, install, maintain, and operate the SHM system. Therefore, along the lines of the discussion by Howard,[3] agreeing to

[1]Department of Structural Engineering, University of California, San Diego, La Jolla, CA, USA
[2]University of Michigan Dearborn, Dearborn, MI, USA

Corresponding author:
Michael D Todd, Department of Structural Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0085, USA.
Email: mdtodd@eng.ucsd.edu

invest in new information is rationally justified if the reduction in the expected losses by utilizing newly acquired information is more than the intrinsic cost of the information acquiring mechanism incurred over the lifespan of the structure. Along the lines of this logic, a lot of focus has been on evaluating the value of SHM by using Bayesian pre-posterior decision analysis and *value of information* (VoI) as a metric to evaluate the benefit of an SHM system in various application problems.[4–10]

One of the primary objectives of this study is to evaluate the economic advantage of using a particular SHM strategy at two levels. First, we quantify whether the SHM strategy leads to an *expected reward* to *investment risk* ratio greater than unity for an instance of decision-making, considering a fixed time occurrence. For this scenario, we consider an initial one-time cost of designing and installing an SHM system as the *investment risk* (money spent). As a consequence of acquiring new information from the SHM system that ostensibly informs an updated understanding of the structural state, we expect to make better maintenance decisions at a given instance in time. We expect that the decision supported by newly acquired data is better than the decision made without any data, and it leads to a higher average cost-saving (or the *expected reward*) than the money spent on the SHM system itself (the *investment risk*). Second, we consider the lifecycle of the structure where the state of the structure evolves with time, and maintenance decisions are to be made over time. In this scenario, in addition to the initial design and installation costs, we also consider the cost of maintaining and operating an SHM system over the lifespan of the structure. Making optimal data-informed (obtained from an SHM system) maintenance decisions over the lifespan of the structure is expected to lead to relative cost-savings as compared to making maintenance decisions not backed by continuously updated data. A feasible SHM system design is the one that leads to higher cost savings as a consequence of better decision-making than the cost of design, installation, maintenance, and operation of the SHM system over the lifespan of the structure. In other words, an economically beneficial SHM system is the one that yields an *expected reward* to *investment risk* ratio greater than unity. This article is based on this key philosophy. We propose an *expected reward* to *investment risk* ratio as an alternative metric to the *expected value of information* (EVoI) traditionally used in pre-posterior decision analysis.

To evaluate the *expected reward* to *investment risk* ratio, we need a decision-making framework. In this article, we use *Expected Utility Theory*,[11,12] further including the behavioral risk profile of the decision-maker modeled using an individual's utility versus loss/wealth function as proposed in Chadha et al.[13] We consider the case where the degree of damage in the structure is completely characterized by a continuous state parameter. Due to numerous real-world uncertainties, it is often challenging to accurately estimate the unknown state parameter. Therefore, the state parameter is assumed unknown and not directly measurable, and hence is probabilistically inferred from observable sensor data. We also assume a specific set of maintenance strategies are already predefined for the structure of interest and have base consequence costs predefined by the stakeholder (organization or individual) for making an assessment for every possibility of the true structural state. We include the stakeholder's risk perception in the decision-making process, that is, the valuation about the outcome of an action, using risk profiling. The utility of a decision-maker is subjective and hence considers the fact that different decision-makers mentally assign a different importance factor (or in economic terms, the utility or risk intensity) to the seriousness/urgency in taking necessary actions with the increasing intensity of structural damage. An individual's utility versus loss/wealth (the risk profile) may be used to obtain the *modified consequence-cost* of performing maintenance strategies. The approach herein incorporates a layer of human psychology on selecting appropriate maintenance strategies that not only depend on the posterior distribution of unmeasurable damage state but also consider the behavioral risk profile of the decision-maker.

Our goal is to evaluate the economic advantage of deploying an SHM system before actually installing it. In other words, we shall only deploy a feasible SHM system that leads to an *expected reward* to *investment risk* ratio greater than unity. This can also be formulated into a design optimization problem: *among all the possible SHM system designs, choose/decide the optimal SHM system design as the one that maximizes the expected reward to investment risk ratio*. This is a useful problem because we do not have the SHM data yet. To evaluate the benefit of an SHM system, we must consider all relevant uncertainties in the data and arrive at an expected value of the reward (averaged over all the possibilities of data, process, etc.). This type of analysis is called pre-posterior decision analysis. It is a decision-making framework that helps the decision-maker to analyze the potential benefit of gathering *additional information* without actually performing an experiment or installing an information-gathering system. This pre-posterior analysis helps us decide (i) if the investment should be made to gather additional information and (ii) if multiple mechanisms for acquiring the information are available, which source of the information is the best.

We consider the inland waterway navigation infrastructure as an application case. The locks and dams that comprise the inland waterway navigation infrastructure consist of multi-hundreds of billions of dollars of capital investment in a major economic transportation corridor.[14–16] The United States Army Corps of Engineers (USACE) spends further billions of dollars in maintaining and operating this infrastructure, where the unscheduled shutdown of these assets and dewatering for inspection or repair are very costly.[17–19] The need for SHM to help facilitate maintenance and operations appears strong, but highly constrained budgets suggest SHM system allocation efforts must be optimized to meet risk-based constraints and yield the maximum possible VoI. This serves as a strong motivation to apply pre-posterior decision theory to analyze the VoI acquired through an SHM system (or the *Value of SHM*). Given the maintenance and repair policies proposed by the organization, pre-posterior decision analysis can be used to arrive at an SHM system design that can be supported by the budgetary constraints of the organization while reducing total life-cycle cost.

Within a navigation lock system, miter gates are one of the most common locking gates used; their most common failure mechanisms include loss of load-transferring contact in the quoin block (boundary-related damage).[20] Loss of contact leads to the formation of a gap between the gate and the wall quoin blocks at the bottom of the gate. The amount (or length) of loss of contact at the bottom of the gate is referred to as *gap length*. We assume that the degree of damage of the miter gate is completely characterized by this gap length. A high fidelity finite element model (FEM) of the gate is used to infer the gap length using a network of strain gauges. To perform the pre-posterior decision analysis, we have to consider all the possibilities of upstream and downstream loads, gap length evolution with time, and the uncertainties in strain gauge readings. This would require us to run the FEM numerous times which is computationally prohibitive. We address this problem by using a Gaussian Process Regression (GPR) machine learning model of the gate that is trained using the strain gauge measurements obtained from limited FEM runs. Finally, we demonstrate an example where we compare three strain gauge network designs including two randomly placed sensors with different numbers of sensors and one KL divergence-based optimized sensor network design (proposed in Ref. 21). We also investigate the impact of various risk profiles of decision-makers on the *value of SHM*.

Six new contributions to this field are discussed in this article: (1) we introduce a normalized measure of quantifying the value of SHM using *expected reward* to *investment risk ratio*; (2) we consider the benefit of SHM at two levels: (i) cost-saved as a consequence of better decision-making at any instance of time and (ii) net cost saved over the lifespan of SHM system usage; (3) along with the impact of the design of an SHM system towards the value it creates, we have also considered the influence of behavioral tendencies (biases and heuristics) of the decision-makers on the value of SHM; (4) two equivalent approaches to quantify the benefit of an SHM system are detailed, first using EVoI or equivalently using *expected reward* to *investment risk ratio*, and secondly using the idea of a perfect experiment; (5) we propose a *relative risk-adjusted reward* metric to quantify the relative benefit of one feasible design relative to another feasible design; and (6) the elucidated framework is applied to a real-world case study involving SHM and maintenance of a miter-gate.

The rest of the article is arranged as follows. *Decision-making framework* briefs the concepts of the expected utility theory, prior and posterior decision theory, and pre-posterior decision analysis. *Value of information considering an instance of decision-making* details the *Value of SHM* considering an instance of decision-making. *Demonstration problem description* describes the demonstration problem and models the risk profile of the decision-maker. *Value of information for life-cycle cost analysis* details the Value of SHM for lifecycle cost analysis considering the entire lifespan of the structure. *Numerical simulation* demonstrates a numerical example that applies the decision-making theoretical framework to the miter gate problem. Finally, *Conclusion* concludes the article.

Decision-making framework

To evaluate the economic VoI and worthiness of an information-gathering SHM system, we first need to detail the following:

1. A decision-making and VoI analysis framework.
2. Description of the structure, the information-gathering system, and the data to be acquired.

Once these items are defined, the following questions are investigated:

1. Is installing an SHM system beneficial relative to the absence of any SHM system?
2. Which SHM system yields the maximum expected reward to investment risk among the given set of available SHM system designs?
3. Is an information theory based KullbackLeibler (KL) divergence (or any other objective function)based optimal sensor design economically better than a random sensor network design in terms of life-cycle cost management?
4. What is the impact of the decision-maker's risk profile on the value of SHM system?
5. Given an SHM system design, the base consequence cost of making maintenance decisions, the risk profile of the decision-makers, what is the possible range of SHM system cost for it to be feasible?

Prior and posterior decision analysis

We begin by describing the decision-making framework. In previous work,[13] we detailed an *Expected Utility Theory* based decision-making workflow to select an optimal action (specifically to choose a maintenance strategy) for a predefined set of choices considering the decision-maker's behavioral risk profile modeled by an individual's *utility versus loss* function. The same framework is adapted as the decision-making model for this article. Therefore, we abundantly borrow the results from Chadha et al.[13] for this section. We briefly describe the behavioral psychology weighed decision-making framework in a form suitable for the current application. We start by presenting some preliminary definitions and notations. The real number space in d dimensions is represented by $\mathbb{R}^d$, with $\mathbb{R}^1 \equiv \mathbb{R}$. A random variable Y is a real-valued function defined on a discrete or a continuous sample space S_Y and is assumed to take values in a measurement space $\Omega_Y \in \mathbb{R}^d$, such that $Y : S_Y \rightarrow \Omega_Y \in \mathbb{R}^d$. Lower case letters y represent realizations of the random variable Y, such that $y \in \Omega_Y$. The probability density function is represented by $f_Y(y)$. For a random variable Y following a Gaussian distribution, with the mean μ_y and the standard deviation σ_y, we write the following

$$f_Y(y) = \frac{1}{\sigma_y}\phi\left(\frac{y-\mu_y}{\sigma_y}\right) = \frac{1}{\sigma_y\sqrt{2\pi}}e^{\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_y}\right)^2}; \quad Y \sim N\left(\mu_y, \sigma_y^2\right). \tag{1}$$

The expected value of a function $g(y)$ with respect to random variable Y is denoted by $E_Y[g(y)]$. The random variable can be any order tensor. No symbolic distinction is made for different dimensions d of the measurement space and the random variable. The vector-dimensionality of a random variable is contextual and is defined as needed.

Consider an SHM-based decision-making problem (like choosing a maintenance action) that depends on the state parameter(s) (defined later). Let Θ denote a random variable that represents the uncertain state parameter with the state parameter space Ω_Θ, such that $\theta \in \Omega_\Theta$ is a realization of Θ. The *decision space* (e.g., set of different maintenance actions, or equivalently, the set of the corresponding damage labels) is represented by Ω_D such that $\Omega_D = \{d_0, d_1, \cdots, d_n\}$ and $\theta \in \Omega_\Theta$. Here, the elements of Ω_D, that is, $d_i \in \Omega_D$, represent a damage label that has a corresponding maintenance action associated with (or designed for) it. The decision-maker attempts to answer the question: *For a given probability distribution of the state parameter, what damage rating must be assigned to the structure that leads to an optimal maintenance strategy?*

To answer this question, we first define the uncertainty in the state parameter θ by its probability density function $f_\Theta(\theta)$. Let θ_{true} represent the *true value of the state parameter*, and we assume that it cannot be measured. The numerical value of θ_{true} falls in the domain Ω_Θ. To predict the optimal decision, we need to minimize the average loss or the expected risk (also called the Bayes risk functional) arising as a consequence of making the decision. To arrive at the Bayes risk, we define the *loss/cost function* $L(d_i, \theta_{\text{true}})$ that defines the total loss or regret as a consequence of making the decision d_i considering all the possible values of the true state parameter $\theta_{\text{true}} \in \Omega_\Theta$. It gives an *extrinsic cost* involved with decision-making. The expected loss or the Bayes risk $\Psi_{\text{prior}}(d_i)$ is then defined as

$$\Psi_{\text{prior}}(d_i) = E_\Theta[L(d_i, \theta_{\text{true}} = \theta)] = \int_{\Omega_\Theta} L(d_i, \theta_{\text{true}} = \theta) f_\Theta(\theta)\, \mathrm{d}\theta \tag{2}$$

We understand that in the definition of the Bayes risk in equation (2), we consider all the possible values of the true gap length. From here on, unlike equation (2), we omit writing $\theta_{\text{true}} = \theta$ in the argument of the cost function while evaluating the expected value over the state-parameter space Ω_Θ. The optimal decision, denoted by $\mathscr{d}_{\text{prior}} \in \Omega_D$, is the one that minimizes the Bayes risk, or

$$\mathscr{d}_{\text{prior}} = \underset{d_i}{\operatorname{argmin}} \Psi_{\text{prior}}(d_i) \tag{3}$$

The prior distribution $f_\Theta(\theta)$ embeds our prior knowledge of the state parameter θ before any additional information is available. Obtaining the optimal decision using equation (3) is called a *prior decision analysis*.

We now consider a scenario where additional information is available. For sake of argument, we assume that the new information is obtained by a mechanism z (e.g., an SHM system). Let Ω_{X_z} represent the continuous measurement (or additional information) space, such that $x_z \in \Omega_{X_z}$. Let X_z denote the random variable representing the new/additional measurement/information obtained by the mechanism z. Designing and installing the information-gathering system incurs an *intrinsic cost* $C(z)$. Therefore, the sum total of the extrinsic and the intrinsic cost functions $(C(z) + L(d_i, \theta_{\text{true}}))$ is used for the further decision analysis. With the availability of additional information, we define our *Bayes conditional risk* $R_z(d_i; x_z)$ (conditioned on the new information/data x_z) and obtain the optimal decision $\mathscr{d}_z$ as

$$\begin{aligned} R_z(d_i; x_z) &= E_{\Theta|X_z}[L(d_i, \theta) + C(z)] \\ &= \int_{\Omega_\Theta} (L(d_i, \theta) + C(z)) f_{\Theta|X_z}(\theta|x_z)\, \mathrm{d}\theta \\ \mathscr{d}_z(x_z) &= \arg\min_{d_i} R_z(d_i; x_z) \end{aligned} \tag{4}$$

In the equation above, the quantity $R_z(d_i; x_z)$ represents *Bayes conditional risk* defining the expected value of loss as a consequence of making a decision d_i considering the posterior distribution of the state parameter $f_{\Theta|X_z}(\theta|x_z)$ (*conditioned* on the new information obtained by the mechanism z). If the new information is representative of the current state of the structure, the decision obtained using equation (4) is anticipated to be better than the decision obtained by the prior analysis using equation (3) because additional information x_z reduces the uncertainty and updates the decision-makers' understanding of the true state parameter through inference. Utilizing equation set (4) to obtain the optimal decision is referred to as *posterior decision analysis*. The subscript $(.)_{\text{prior}}$ and $(.)_z$ in Bayes risk and the optimal decision are meant for the prior analysis and the posterior decision analysis, respectively (considering the new information obtained by the mechanism z). We realize that the posterior $f_{\Theta|X_z}(\theta|x_z)$ is non-causal. The state parameter can be thought of as a *cause* with measurement being its *effect*. In this regard, inferring the state parameter (cause) given the measurement (effect) is non-causal.

Pre-posterior decision analysis

As was discussed in the previous section, obtaining good quality new information about the system is consequential in making a better decision. However, acquiring information/data bears a cost. The pre-posterior decision analysis is the framework that helps the decision-maker to analyze the potential benefit of gathering *additional information* without actually performing an experiment or installing an information-gathering system. The decision-maker can pay to obtain the measurement made by the SHM system, or carrying out an inspection, or observe the outcome of an experiment. However, carrying any of these activities bears cost (like the cost to design the SHM system, sensor and maintenance costs, labor costs to carry out inspection, and testing costs). Acquiring the new information, regardless of mechanism, is meaningful and economical if and only if the additional cost required to gather the information is outweighed by the reduction in the expected losses evaluated by considering the additional information. This observation can be thought of as an *asset integration* tenet of expected utility theory as noted in Ref. 22; in other words, the prospect of paying to acquire new information is acceptable if and only if the utility resulting from experimenting exceeds the utility evaluated without it. Thus, pre-posterior analysis helps us decide if the price should be paid to gather additional information, and if multiple mechanisms for acquiring the information are available, which source of the information is the best.

To describe the framework, let Ω_Z represent the space of all the possible information acquiring mechanisms or systems (synonymously called experiments from here on), such that $\Omega_Z = \{z_0, z_1, z_2, \ldots, z_m\}$. Let X_{z_i} represent the random variable denoting the outcome/measurement of the data obtained by carrying out the experiment z_i, such that $x_{z_i} \in \Omega_{X_{z_i}}$. Here, z_0 represents the null case of carrying out *no experiment*, such that $X_{z_0} = \{\phi\}$. Let $C(z_i)$ represent the *intrinsic cost* of conducting the experiment z_i, with $C(z_0) = 0$. For the experiment z_i, the cost of making a decision d_j for a given state parameter θ is given by the sum of extrinsic and intrinsic costs $(L(d_j, \theta_{\text{true}}) + C(z_i))$. The total cost $(L(d_j, \theta_{\text{true}}) + C(z_i))$ is independent of the outcome x_{z_i} because the cost of conducting the experiment does not change with the outcome. Given the experiment-outcome pair (z_i, x_{z_i}), the remaining calculation is the same as *posterior decision analysis* discussed in *Prior and posterior decision analysis*, with an exception of using the cost $(L(d_j, \theta_{\text{true}}) + C(z_i))$ in place of the cost $L(d_j, \theta_{\text{true}})$ in the definition of Bayes conditional risk as defined in equation (4). Therefore, the most optimum decision for a given experiment-outcome pair (z_i, x_{z_i}) is then given as

$$\mathscr{d}_{\varkappa_i}(x_{z_i}) = \arg\min_{d_j} R_{z_i}(d_j; x_{z_i}), \text{ where} \tag{5a}$$

$$R_{z_i}(d_j; x_{z_i}) = E_{\Theta|X_{z_i}}\left[L(d_j, \theta) + C(z_i)\right] \tag{5b}$$

In the equation above, $R_{z_i}(d_j; x_{z_i})$ represents the Bayes conditional risk for the experiment-outcome pair (z_i, x_{z_i}) and the decision outcome d_j. The discussion so far is exactly the same as the posterior decision analysis. However, our goal for the pre-posterior analysis is to decide if and which experiment must be performed such that the new information obtained adds to the value of decision-making. We note that the experiment is actually not carried out during this phase of decision analysis, and therefore, all the possible measurement outcomes must be considered. Since the measurements depend on the state of the structure, the probability distribution of the measurements depends on three quantities: (1) the prior distribution of the state parameter at an instance of time; (2) the simulation or physics-based model (like FEM) that establishes a map between the state parameter and the measurements; and (3) a reasonably assumed noise structure. Since the measurements are uncertain and are quantified by their probability distribution function, the quantity that interests us is the expected value of the *minimum Bayes conditional risk* $R_{z_i}(d_j; x_{z_i})$ weighted over all the possible outcomes $x_{z_i} \in \Omega_{X_{z_i}}$ conditioned upon the prior distribution of the state parameter. It is defined by the Bayes risk $\Psi_{\text{avg}}(z_i)$ for an experiment z_i, such that

$$\Psi_{\text{avg}}(z_i) = E_{X_{z_i}}\left[\min_{d_j} R_{z_i}(d_j; x_{z_i})\right] = E_{X_{z_i}}[R_{z_i}(\mathscr{d}_{\varkappa_i}(x_{z_i}); x_{z_i})] \tag{6a}$$

$$\varkappa = \arg\min_{\varkappa_i} \Psi_{\text{avg}}(\varkappa_i) \tag{6b}$$

Here, $\varkappa \in \Omega_{\mathscr{Z}}$ represents the optimal experiment. We observe that if no additional information is acquired, as is the case with z_0, the decision-making process reduces to prior analysis, such that

$$\Psi_{\text{avg}}(z_0) = \min_{d_i} \Psi_{\text{prior}}(d_i) = \Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right) \tag{7}$$

One of the quantities required to evaluate the Bayes conditional risk $R_{z_i}(d_j; x_{z_i})$ is the posterior $f_{\Theta|X_{z_i}}(\theta|x_{z_i})$. However, unlike the posterior analysis, we do not have the measurement/outcome data x_{z_i} because no experiment has yet been performed. As the experiment is not performed and the measurements are not available during this phase of decision analysis, we consider all possible measurements that are simulated for a given prior distribution of the state parameter. Therefore, this analysis is called *pre-posterior decision analysis*. To evaluate the posterior $f_{\Theta|X_{z_i}}(\theta|x_{z_i})$, we use the Bayes theorem, which in turn requires obtaining the likelihood $f_{X_{z_i}|\Theta}(x_{z_i}|\theta)$. Since there is no availability of the data, the likelihood is assumed/modeled by the decision-maker based on past observation, physics-based simulations, or a reasonable assumption.

Value of information considering an instance of decision-making

Conditional value of information

Consider the prior and posterior decision analysis detailed in *Prior and posterior decision analysis*. We restate our assumption that the new information is of good quality in a way that it helps the decision-maker to have a better understanding of the state parameter in comparison to the decision-maker's prior knowledge. It is expected that with additional information, the expected loss should reduce. Acquiring new information via the system $z \in \Omega_Z$ helps us make better decisions as it reduces uncertainties in the state parameter, that is, the distribution $f_{\Theta|X_z}(\theta|x_z)$ has lower variance than $f_\Theta(\theta)$. This brings us to the definition of *conditional value of information* $\text{CVoI}(x_z)$ as

$$\text{CVoI}(x_z) = \Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right) - R_{z_i}\left(\mathscr{d}_{x_i}(x_{z_i}); x_{z_i}\right) \tag{8}$$

Note that the quantity $R_{z_i}(\mathscr{d}_{x_i}(x_{z_i}); x_{z_i})$ defined in equation (5b) takes into account the additional cost $C(z)$ of acquiring the information. Therefore, obtaining the new information $x_z \in \Omega_{X_z}$ is advantageous if the expected loss $R_{z_i}(\mathscr{d}_{x_i}(x_{z_i}); x_{z_i}) < \Psi_{\text{prior}}(\mathscr{d}_{\text{prior}})$, or equivalently, $\text{CVoI}(x_z) > 0$. However, it is more reasonable and desirable to define a quantity that measures the average VoI considering all the possible measurements as discussed in next section (since the measurements/data are generically stochastic).

Expected value of information or the value of experiment

When we perform a pre-posterior analysis for the experiment (or the information-gathering system) $z_i \in \Omega_Z$, all the possible values of the measurement $x_{z_i} \in \Omega_{X_{z_i}}$ corresponding to the experiment z_i are to be considered. Therefore, the *expected value of information* $\text{EVoI}(z_i)$ (also called the *expected value of experiment*) is then defined as the expected value of the $\text{CVoI}(x_{z_i})$ averaged over the entire measurement space X_{z_i}, yielding

$$\begin{aligned}\text{EVoI}(z_i) &= E_{X_{z_i}}[\text{CVoI}(x_{z_i})] = \Psi_{\text{avg}}(z_0) - \Psi_{\text{avg}}(z_i)\\ &= \Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right) - \Psi_{\text{avg}}(z_i)\end{aligned} \tag{9}$$

To understand the quantity $\text{EVoI}(z_i)$, we expand the expression in equation (9) using equations (2) and (6a) as

$$\begin{aligned}\text{EVoI}(z_i) &= \min_{d_j} E_\Theta\left[L\left(d_j,\theta\right)\right] - E_{X_{z_i}}\left[\min_{d_j} R_{z_i}\left(d_j; x_{z_i}\right)\right]\\ &= \min_{d_j} E_\Theta\left[L\left(d_j,\theta\right)\right] - E_{X_{z_i}}\left[\min_{d_j} E_{\Theta|X_{z_i}}\left[L\left(d_j,\theta\right)\right.\right.\\ &\quad \left.\left.+ C(z_i)\right]\right]\\ &= \min_{d_j} E_\Theta\left[L\left(d_j,\theta\right)\right] - E_{X_{z_i}}\left[\min_{d_j} E_{\Theta|X_{z_i}}\left[L\left(d_j,\theta\right)\right]\right]\\ &\quad - C(z_i) = C_{\text{save}}(z_i) - C(z_i)\end{aligned} \tag{10}$$

where

$$C_{\text{save}}(z_i) = \min_{d_j} E_\Theta\left[L\left(d_j,\theta\right)\right] - E_{X_{z_i}}\left[\min_{d_j} E_{\Theta|X_{z_i}}\left[L\left(d_j,\theta\right)\right]\right] \tag{11}$$

To proceed further, we note that the following identity holds

$$\begin{aligned}E_{X_{z_i}}\left[\min_{d_j} E_{\Theta|X_{z_i}}\left[L\left(d_j,\theta\right)\right]\right] &\leq \min_{d_j} E_{X_{z_i}}\left[E_{\Theta|X_{z_i}}\left[L\left(d_j,\theta\right)\right]\right]\\ &= \min_{d_j} E_\Theta\left[L\left(d_j,\theta\right)\right]\end{aligned} \tag{12}$$

Note that $E_{\Theta|X_{z_i}}[L(d_j,\theta)]$ is a function of (d_j,x_{z_i}) (state parameter θ is integrated out). Thus, it is easy to visualize the identity defined in equation (12) by defining the function $f(d_j,x_{z_i}) = E_{\Theta|X_{z_i}}[L(d_j,\theta)]$ and observing that

$$E_{X_{z_i}}\left[\min_{d_j}\left(f\left(d_j,x_{z_i}\right)\right)\right] \leq \min_{d_j} E_{X_{z_i}}\left[f\left(d_j,x_{z_i}\right)\right] \tag{13}$$

The identity (12) when applied to equation (11) leads to another important formula

$$C_{\text{save}}(z_i) \geq 0 \tag{14}$$

As mentioned in Ref. 6, the EVoI can be normalized by the minimum prior consequence cost $\Psi_{\text{prior}}(\mathscr{d}_{\text{prior}})$ to obtain the *relative value of information* of an experiment z (denoted by $\text{RVoI}(z)$) as

$$\text{RVoI}(z) = \frac{\Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right) - \Psi_{\text{avg}}(z)}{\Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right)} = \frac{\text{EVoI}(z)}{\Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}\right)} \tag{15}$$

The EVoI is impacted by two aspects. The first is gaining additional information that helps reduce the expected losses by an amount $C_{\text{save}}(z_i)$. Second, setting up an experiment z_i incurs additional intrinsic cost that leads to increase in the expected cost by an amount $C(z_i)$. Therefore, performing an experiment/inspection to gain new information is advantageous if and only if the cost of the experiment $C(z_i)$ is less than the reduction in losses $C_{\text{save}}(z_i)$, or if $C_{\text{save}}(z_i) - C(z_i) \geq 0$. In other words, it would be economical and rational to perform an experiment $z_i \in \Omega_Z$ if and only if $\text{EVoI}(z_i) \geq 0$. The value of experiment z_0 is $\text{EVoI}(z_0) = 0$ because no new information is gained (or $C_{\text{save}}(z_0) = 0$) and since there is no mechanism available to acquire new information, we have $C(z_0) = 0$.

Inspired from the Gambling theory[23] and the stock-market trading system design,[24,25] in the next section, we propose an alternate normalized metric to quantify the VoI that is inherently suitable for business-oriented decision-making in SHM.

Expected reward to investment risk ratio for an information-gathering system

We consider the concept of *reward to risk* ratio used in designing trading systems in the stock market. A technical analysis[26,27]-based trading system utilizes the newly acquired information on the price action of a stock to make trading decisions. Since the stock market is inherently uncertain and there is always a chance of a black-swan or a fat-tail event[28] (also known as a high-consequence low-probability event in the field of reliability analysis), a trading system must ensure that the trader gets out of a position taking a predefined loss instead of incurring severe portfolio-crippling loss. Therefore, a trading system is designed to expect a reward by risking a predefined monetary loss. This trading philosophy is popularly known as "*winning by losing*" in the trading world. Thus, among many possible trading-system designs, the one with a higher *expectancy* or *adjusted reward to risk ratio* (adjusted for the probability of winning and losing trades yielded by the trading system—also informally called Batting average) is the optimal. The goal is not to choose the system that gives maximum profit but may lead to severe draw-downs (hence can possibly lead to complete ruin—also called *gambler's ruin*), rather the goal is to design a system that yields maximum reward for a given risk (expecting a consistent long-term compounding of wealth). Similarly, Thorp[23–25] in his groundbreaking research developed a risk-adjusted reward-based betting system for Black Jack and Roulette (in collaboration with Claude Shannon) that decided on when and how much (optimal position sizing) to bet based on newly acquired information using card-counting for Black Jack and wearable-computer running a physics-based updating model for Roulette.

Along a similar line of reasoning, the reward obtained by newly acquired information/data from an SHM system (in terms of making optimal maintenance decisions and increasing the lifespan of the structure) must outweigh the financial resources risked for the design, installation, maintenance, and operation of an information-gathering system. We define the *expected reward* to *investment risk* ratio $\lambda(z)$ for an information-gathering system z as

$$\begin{aligned}\lambda(z) &= \frac{C_{\text{save}}(z)}{C(z)} = \frac{C_{\text{save}}(z)}{\text{EVoI}(z) + C_{\text{save}}(z)} \\ &= \frac{1}{C(z)}\left(\min_{d_j} E_\Theta\left[L_j(\theta)\right] - E_{X_{z_i}}\left[\min_{d_j} E_{\Theta|X_{z_i}}\left[L_j(\theta)\right]\right]\right)\end{aligned} \tag{16}$$

Therefore, the SHM system with $\lambda(z) \geq 1$ is a feasibly acceptable system with a positive risk-adjusted reward. The quantity $C_{\text{save}}(z)$ denotes the *expected reward* as a consequence of data-informed decision-making, and the quantity $C(z)$ can be thought of as an *investment risk* (money paid to design and install an SHM system). Among many possible information-gathering systems in Ω_Z, the most desirable system $\varkappa$ is the one that yields the maximum risk-adjusted reward, or

$$\varkappa = \arg\max_{\varkappa_i} \lambda(\varkappa) \tag{17}$$

Consider two feasible designs z_1 and z_2, such that $\lambda(z_1) > 1$ and $\lambda(z_2) > 1$. To quantify the relative benefit of one feasible design with respect to another feasible design in terms of their risk-adjusted reward, we propose *relative risk-adjusted reward* metric, denoted by $\chi(z_1, z_2)$, such that

$$\chi(z_1, z_2) = \frac{\lambda(z_1) - 1}{\lambda(z_2) - 1} \tag{18}$$

For two feasible designs z_1 and z_2, $\chi(z_1, z_2)$ quantifies the relative risk-adjusted reward of the design z_1 as compared to the design z_2. The value of $\chi(z_1, z_2)$ greater than one implies that the design z_1 leads to higher risk-adjusted savings as compared to the design z_2 by $\chi(z_1, z_2)$ times.

Perfect experiment and perfect information

We now consider an experiment z with the cost $C(z)$ that gives the exact value (zero error) of the state parameter, such that $\theta = x_z$, where $x_z \in \Omega_{X_z}$. This implies

$$f_{X_z|\Theta}(x_z|\theta) = f_{\Theta|X_z}(\theta|x_z) = \delta(x_z - \theta) \tag{19a}$$

$$f_{X_z}(x_z) = f_\Theta(\theta) \tag{19b}$$

It might appear that the experiment z defined above involves directly measuring the state parameter. However, in most of the practical problems, the state parameter is not directly measured, and rather, it is inferred from some other attainable measurement. If that is the case, then we assume that the inferred state parameter gives the true value of the state parameter without any error. In that case, Ω_{X_z} would denote the space of the predicted/inferred gap length, such that equation (19) holds.

Using equations (5b) and (19), the Bayes conditional risk for the experiment z is written as

$$\begin{aligned} R_z(d_j; x_z) &= E_{\Theta|X_z}\left[\left(L(d_j,\theta) + C(z)\right)\right] \\ &= \int_\theta \left(L(d_j,\theta) + C(z)\right)\delta(\theta - x_z)\ \mathrm{d}\theta \\ &= L(d_j, \theta = x_z) + C(z) \end{aligned} \tag{20}$$

Using equations (6a) and (20), the average Bayes risk associated with the perfect experiment z is defined as

$$\begin{aligned} \Psi_{\text{avg}}(z) &= E_{X_z}\left[\min_{d_j} R_z(d_j, x_z)\right] \\ &= E_{X_z}\left[\min_{d_j}\left(L(d_j, \theta = x_z) + C(z)\right)\right]. \end{aligned} \tag{21}$$

As a consequence of equation (19b), we can write

$$\Psi_{\text{avg}}(z) = E_\Theta\left[\min_{d_j}\left(L(d_j,\theta) + C(z)\right)\right] \tag{22}$$

We call any experiment z that satisfies equation (19) as a *perfect experiment*. Since the cost of any experiment z is independent of its outcome x_z and the decision space Ω_D, equation (22) can be simplified as

$$\Psi_{\text{avg}}(z) = E_\Theta\left[\min_{d_j}\left(L(d_j, \theta_{\text{true}} = \theta)\right)\right] + C(z) \tag{23}$$

The *value of perfect experiment* is defined using equation (9) as

$$\text{EVoI}(z) = \Psi_{\text{prior}}(\mathscr{d}_{\text{prior}}) - \Psi_{\text{avg}}(z) \tag{24}$$

Using equations (2) and (3), we can write $\Psi_{\text{prior}}(\mathscr{d}_{\text{prior}}) = \min_{d_j} E_\Theta[L(d_j,\theta)]$. Therefore, using equations (23) and (24), we get

$$\text{EVoI}(z) = C_{\text{save}}(z) - C(z), \text{ where} \tag{25a}$$

$$C_{\text{save}}(z) = \min_{d_j} E_\Theta\left[L(d_j,\theta)\right] - E_\Theta\left[\min_{d_j}\left(L(d_j,\theta)\right)\right] \tag{25b}$$

We note that a *perfect experiment* yields an exact and error-free value of the state parameter. We now define the *value of perfect information* PVoI as the cost of the perfect experiment $\hat{z}$ for which the EVoI vanishes, or

$$\text{EVoI}(\hat{z}) = 0 \tag{26}$$

In other words, $\hat{z}$ is a fictitious experiment that is a special case of a perfect experiment in which the reduction in losses due to additional information $C_{\text{save}}(\hat{z})$ compensate with the cost of the experiment $C(\hat{z})$, or the risk-adjusted reward becomes unity: $\lambda(\hat{z}) = 1$. Hence, $\text{PVoI} = C(\hat{z})$ is the maximum expense that should be incurred out of pocket to acquire additional information. Experiment or SHM system z with the cost $C(z) > C(\hat{z})$ need not even be considered. Therefore, a rational decision-maker decides to perform an experiment z if either of the following five equivalent statements hold

$$\text{EVoI}(z) \geq 0 \tag{27a}$$

$$C_{\text{save}} \geq C(z) \tag{27b}$$

$$\text{RVoI}(z) \geq 0 \tag{27c}$$

$$\lambda(z) \geq 1 \tag{27d}$$

$$C(z) \leq C(\hat{z}) \tag{27e}$$

Equation set (27) gives the conditions satisfying the *asset integration* tenet of expected utility theory as mentioned in Ref. 22. In other words, the prospect of carrying out an experiment z is acceptable if the utility resulting from carrying out the experiment exceeds the utility evaluated without it, that is, $\text{EVoI}(z) \geq 0$. We reinforce the fact that the experiments z_0 and $\hat{z}$ have zero *value of experiment* because z_0 does not yield any new information and the experiment $\hat{z}$ yields the maximum possible information at the highest rationally payable cost $C(\hat{z})$.

Having discussed the decision-making and VoI framework, we detail a demonstration problem in the next section.

Demonstration problem description

Miter gate

To demonstrate the application of the concepts discussed so far, we consider an example problem of the Greenup miter gate maintained and managed by USACE located on the Ohio River, USA. Figure 1 shows the Greenup lock and the miter gate (image adapted from the USACE website and Eick et al.[29]). Loss of contact in the quoin blocks (boundary-related damage) is the most commonly observed damage mode in such systems.[19,30,20] Loss of contact leads to a formation of a gap between the gate and the wall quoin blocks at the bottom of the gate. The amount (or length) of loss of contact at the bottom of the gate is referred to as *gap length* in this article. Therefore, we consider the gap length as the continuous state parameter $\theta \in \Omega_\Theta$ (refer to Figure 2), such that $\Omega_\Theta = [\theta_{\min}, \theta_{\max}]$. Here, $\theta_{\min}$ is the lower bound of

Figure 1. Greenup locks and miter gate.

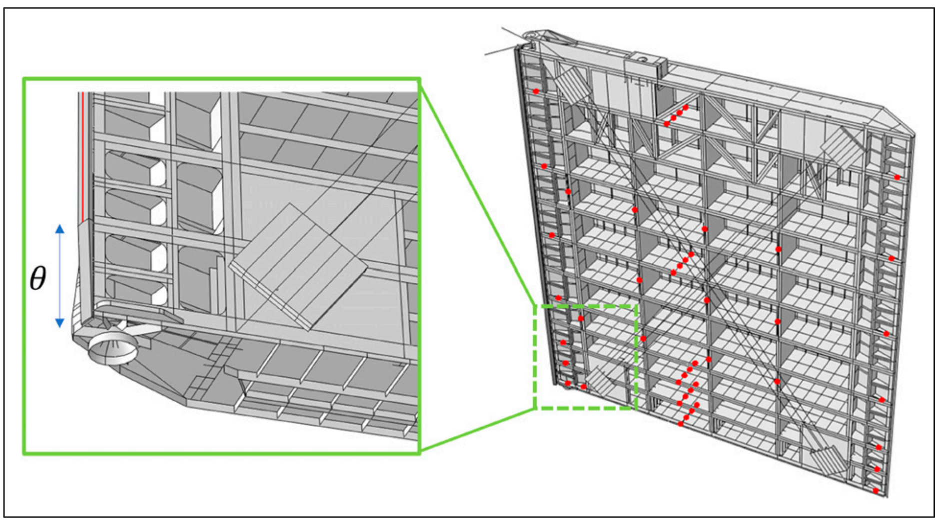

Figure 2. Physics-based model of miter gate and the bearing gap.

the gap length and θ_{max} is the upper bound of the gap length which indicates that the gate is critically damaged and the failure is to follow. This value is suggested by the USACE engineers based on their experience and past inspection data. In many cases, the data related to the failure of the structure may not be available because the decision-makers are risk-averse and they do not want to see a gap length to be large enough leading to failure. In such scenarios, a rigorous high-fidelity numerical simulation should be performed to estimate the θ_{max}. Based on feedback from the field engineers,[20] the upper bound of the gap length can be considered as $\theta_{max} = 180$ inches for the gates that have similar structural characteristics as the Greenup miter gate. If no value of θ_{min} is specified, it can be taken as 0 inches (indicating pristine state of the gate). Unlike non-binary rating protocols used by USACE, that is (*A, B, C, D, F,* and *CF*), to build our framework, we use a rather simplified binary labeling system that consists of two discrete damage labels/index of the miter gate, such that the decision space is $\Omega_D = \{d_0, d_1\}$, where the binary decisions are

$$
\begin{aligned}
d_0 &: \text{label indicating that the gate is undamaged with excellent operational capacity} \\
d_1 &: \text{label indicating that the gate is damaged and is not safely operational}
\end{aligned} \tag{28}
$$

The loss-of-contact part of the gate is always submerged in highly turbid water, and it consequently cannot be easily measured directly during normal operational conditions. Hence, gap length is an unknown parameter and must be inferred from indirect measurements. The Greenup miter gate is equipped with a strain gauge network illustrated by red dots in Figure 2. These strain gauge readings are recorded in real-time and are used as the observable set of measurements that will be used to infer the gap length. We simulate our data acquisition process using a high-fidelity FEM of the Greenup miter gate previously validated in the undamaged condition with the available strain sensor readings.[20] When the miter gate is first deployed, the gap length is reasonably assumed to be zero. An FEM of the pristine miter gate needs to be constantly updated as and when new information from the strain gauge sensor network is obtained. Because a very limited amount of data is available from Greenup, we turn to an FEM as the ground truth surrogate for data. In that regard, we assume that there is no measurement bias and the sensor readings are subject to random unbiased noise. As with any such model, its representative predictive value is only as good as its validation with regard to the real structure that it represents. In this case, the FEM was previously validated to the Greenup miter gate in the undamaged condition, as mentioned earlier, but the modeling of the damage itself could not be validated on actual data from the gate in a known damaged condition, so modeling bias error in the damage state could creep into the process. That does not change or otherwise invalidate the demonstration of the proposed approach or its utility, but rather it provides caution on interpreting the specific results for this case beyond the demonstration of the overall approach. The posterior distribution $f_{\Theta|X_z}(\theta|x_z)$ of the gap length given the strain sensors measurement is then obtained using Bayesian inference. Here, X_z denotes a random variable that represents the measurement obtained from the sensors deployed in the SHM system z, with Ω_{X_z} representing the space of those measurements. Figure 2 shows the physics-based FEM of the miter gate.

To simulate the strain gauge data, we rely on a GPR-based model trained using simulated observable strain values obtained from the validated FEM. Although there are infinite possible locations where strain gauges can be placed on a real miter gate, the FEM discretely covers the possible sensor locations using a countable number of strain gauges. The FEM itself is constructed using 3D quadrilateral and triangular shell elements in Abaqus and consists of a total of 64,919 elements. Every element has a local coordinate system $\{t_i\}$ defined in the underformed state and a global coordinate system $\{E_i\}$. The thickness of the element is in the direction t_3, and the top and bottom surface of the element is spanned by the vectors (t_1, t_2) as shown in Figure 3. The strain gauges are attached to the top and bottom surface of each element, measuring uniaxial strains along the direction t_1 and t_2. Each element is identified by its geometric centroid at the origin of the local coordinate system. Therefore, there are four possible arrangements of strain gauges on each element. These possibilities are identified using the following abbreviations

TH: top element, horizontal orientation along t_1
TV: top element, vertical orientation along t_2
BH: bottom element, horizontal orientation along t_1
BV: bottom element, vertical orientation along t_2
(29)

Based on the above abbreviations, for a typical element m, x^m_{TH} and x^m_{TV} in Figure 3 represent the measurement of strain from gauges attached to the top surface and oriented along t_1 and t_2, respectively. Similarly, x^m_{BH} and x^m_{BV} in Figure 3 represent the measurements of strain from gauges attached to the bottom surface and oriented along t_1 and t_2, respectively.

The gate is subjected to uncertain upstream and downstream hydrostatic loads quantified by the hydrostatic upstream and downstream heads; these are denoted by the random variables H_{up} and H_{down}, with realizations $h_{\text{up}} \in \Omega_{H_{\text{up}}}$ and $h_{\text{down}} \in \Omega_{H_{\text{down}}}$, respectively, where $\Omega_{H_{\text{up}}}$ and $\Omega_{H_{\text{down}}}$ represents space of all possible values of upstream and downstream head, respectively. The water heads over the

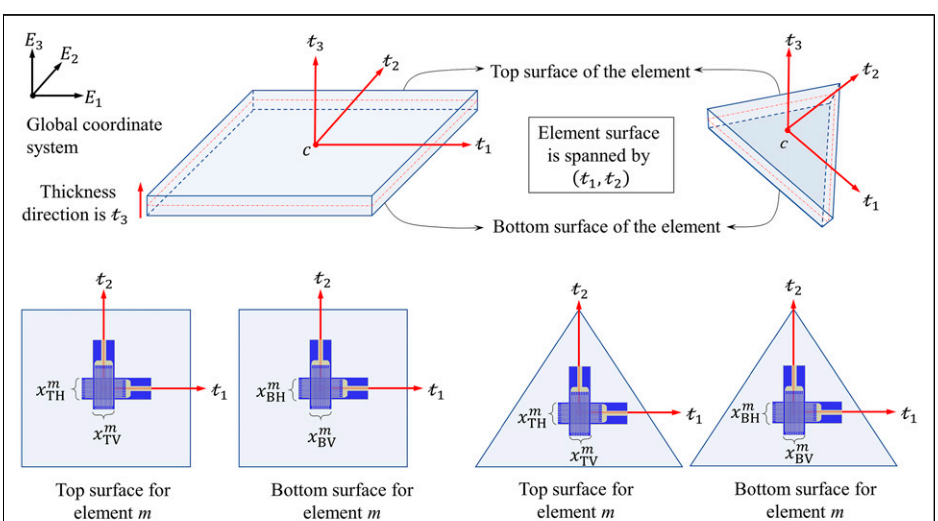

Figure 3. Orientation and the location of the strain gauge and different type of shell elements used in finite element model.

lifespan of the miter gate are modeled by time-series models using autoregressive moving average (ARMA) as follows

$$\begin{aligned} h_{\text{up}}(t_i) &= 172 + \varepsilon_i^{\text{up}} + 0.33h_{\text{up}}(t_{i-2}) + 0.35h_{\text{up}}(t_{i-2}) \\ &\quad + 0.52\varepsilon_{i-2}^{\text{up}} + 0.55\varepsilon_{i-1}^{\text{up}}, \text{ where, } \varepsilon_i^{\text{up}} \sim N\left(0, 2^2\right); \\ h_{\text{down}}(t_i) &= 95 + \varepsilon_i^{\text{down}} + 0.23h_{\text{down}}(t_{i-2}) + 0.25h_{\text{down}}(t_{i-2}) \\ &\quad + 0.52\varepsilon_{i-2}^{\text{down}} + 0.61\varepsilon_{i-1}^{\text{down}}, \text{ where, } \varepsilon_i^{\text{down}} \sim N\left(0, 2^2\right) \end{aligned} \tag{30}$$

Figure 4 illustrates one realization of the hydrostatic head time-series constructed using ARMA over a 60-month time period.

For an i^{th} strain gauge in the SHM system z, we assume an independent zero-mean additive Gaussian noise, denoted by a random variable ζ_{zi} with the realization ε_{zi}, is assumed for each strain gauge

$$\zeta_i \sim N\left(\mu_{\varepsilon_{zi}} = 0, \sigma_{\varepsilon_{zi}}^2\right) \tag{31}$$

The standard deviation of noise is assigned to be $\sigma_{\varepsilon_{zi}} = 5 \times 10^{-6}$ in accordance with reasonable commercial strain gauge performance.

Maintenance actions for the miter gate and the associated cost function

Let M_0 and M_1 represent the actions associated with the labels d_0 (rating the structure as undamaged) and d_1 (rating the structure as damaged), respectively. That is, if the structure is labeled/rated as d_i, with $i \in \{0, 1\}$, then we perform the maintenance M_i, such that

$$\begin{aligned} &M_0 : \text{Do nothing (continue operation)} \\ &M_1 : \text{Shut down, inspect, and repair or replace as required based on the inspection results} \end{aligned} \tag{32}$$

Choosing either M_0 or M_1 will have an associated consequence cost depending on what the true state of damage is. For instance, choosing M_0 for a newly constructed gate (with the true gap length value being zero or small) is obviously an optimal decision. On the other hand, the same maintenance action M_0 can lead to catastrophic consequences when the true value of gap length is close to $\theta_{\max}$ (implying a heavily damaged gate). Similarly, choosing M_1 for a pristine gate is unnecessary while it may be an optimal decision when the gate is approaching critical failure (with a larger value of true gap length). Therefore, to consider the economical consequence of deciding a maintenance action (or, equivalently choosing the state label), the organization needs to estimate the cost of performing maintenance for all the possible true degrees of damage defined by the state parameter gap length $\theta_{\text{true}} \in \Omega_\Theta$. The organization estimates this cost based on a detailed cost analysis of past maintenance data and/or their current maintenance policies.

For the sake of demonstration purposes, we adopt a linear cost function as discussed in Chadha et al.[13] This represents a case where the cost linearly increases with the true degree of damage. Another reasonable assumption would be a step function that assigns equal consequence cost to a range of true gap length values. We evaluate the consequence cost for both the maintenance strategies considering the extreme values of true gap length ($\theta_{\text{true}} = \theta_{\min} = 0$ inches and $\theta_{\text{true}} = \theta_{\max} = 180$ inches) and linearly interpolate the cost function for all the intermediate θ_{true} values. We do this because the extreme values of the gap have interpretable physical meaning. The value of $\theta_{\text{true}} = 0$ inches indicates that the gate is pristine, and the value of $\theta_{\text{true}} = 180$ inches indicates that the gate is severely damaged and a critical failure is expected. Under such damage conditions, the economical consequence of choosing a maintenance action can be reasonably evaluated since the consequences of decision-making are well-defined. However, we note that the organization can estimate/design the

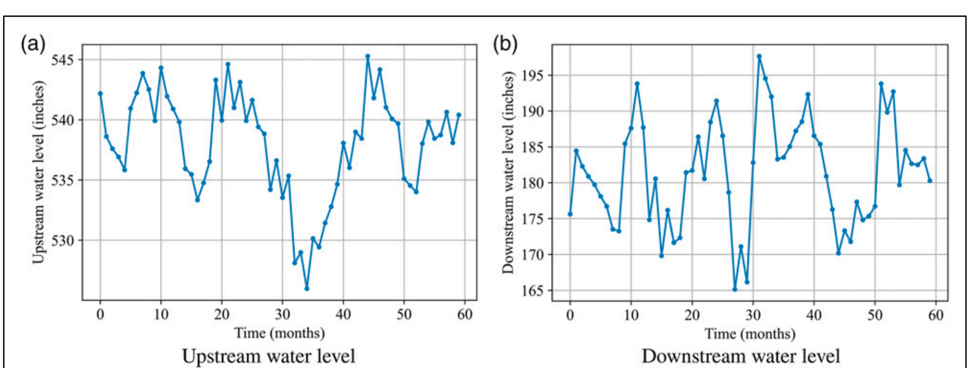

Figure 4. Realization of the hydrostatic water head time series.

cost functions as per their requirements and policies that need not necessarily be linear (another practical example can be a step-wise consequence cost). The framework developed here is generic and can support any such cost model. Let $L(d_0, \theta_{true})$ and $L(d_1, \theta_{true})$ denote the consequence costs of performing the maintenance actions M_0 and M_1, respectively, when the true degree of damage is defined by θ_{true}, such that

$$L(d_0,\theta_{true}) = \left(\frac{L(d_0,\theta_{max}) - L(d_0,0)}{\theta_{max} - \theta_{min}}\right)\theta_{true} + L(d_0,0)$$
$$L(d_1,\theta_{true}) = \left(\frac{L(d_1,\theta_{max}) - L(d_1,0)}{\theta_{max} - \theta_{min}}\right)\theta_{true} + L(d_1,0) \quad (33)$$

In the equation above, the extremes costs $L(d_i, 0)$ and $L(d_i, \theta_{max})$ are assumed to be known and fixed by the organization. Since $L(d_0, \theta_{max})$ is the maximum extreme cost, all other extreme costs can be expressed as a fraction of $L(d_0, \theta_{max})$. For the purposes of numerical simulation in this article, we assume $L(d_1, 0) = 0.15\ L(d_0, \theta_{max})$ and $L(d_1, \theta_{max}) = 0.4\ L(d_0, \theta_{max})$. We assign dollar value of US$1 million to $L(d_0, \theta_{max})$. Under this assignment, Figure 5 gives the cost functions $L(d_0, \theta_{true})$ and $L(d_1, \theta_{true})$.

The base cost functions $L(d_i, \theta_{true})$ are defined by the organization. Although the base cost is assumed to be linear in this article, it can bear any form (step-function, piecewise function, quadratic, etc.). In most cases, these costs are estimated based on the available data and are approximate. When it comes to maintenance decisions guided by the organization's maintenance policies or collective experience, we consider the real-world scenario where inspection engineers are authorized to execute those decisions. These decisions are subjective to the engineer's experience and their thought processes assumed commensurate with the broader policies or guidance provided by the organization. Therefore, the maintenance decisions may have slightly different cost consequences as defined by the base cost function. The risk profile of the decision-maker can be mathematically modeled by their utility versus wealth (or loss) function, or generally a utility function. An individual's utility gives their evaluation of the consequence/outcome of an action. The utility may be different from the real dollar cost (or value). Since a risk-averse decision-maker aims at losing less (or gaining more), his perceived value of cost/loss is higher than the real dollar cost. This leads to a concave-down utility function. On the other hand, a risk-seeker decision-maker is willing to risk more and hence assign a lower valuation to the real cost, leading to concave-up utility function. As discussed in Chadha et al.,[13] an individual's utility (or the risk profile) can be used to obtain the modified cost functions. This allows us to incorporate the risk-perception into the decision-making process. The risk-adjusted cost functions (distinguished by a hat $(\hat{\cdot})$) can be expressed as

$$\hat{L}(d_0,\theta_{true};\gamma,\xi) = a_0 \log\left(1 + b_0\left(\frac{L(d_0,\theta_{max}) - L(d_0,0)}{\theta_{max} - \theta_{min}}\right)\theta_{true}\right) + L(d_0,0)$$
$$\hat{L}(d_1,\theta_{true};\gamma,\xi) = a_1 \log\left(1 + b_1\left(\frac{L(d_1,\theta_{max}) - L(d_1,0)}{\theta_{max} - \theta_{min}}\right)\theta_{true}\right) + L(d_1,0) \quad (34)$$

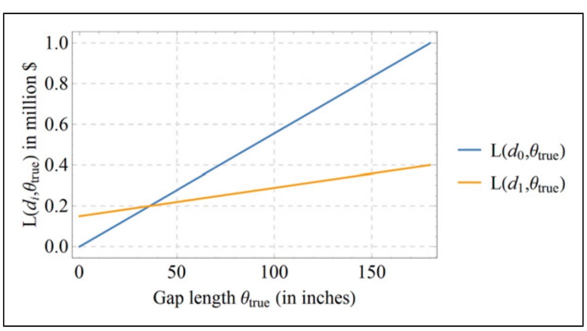

Figure 5. The cost function $L(d_i, \theta_{true})$.

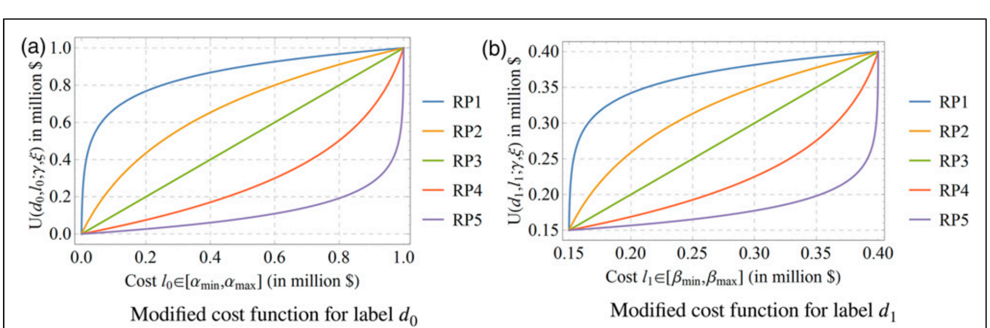

Figure 6. Risk intensitymodified cost function.

The risk profile is parameterized by (γ, ξ). For a given risk profile (γ, ξ), the constants a_0, a_1, b_0, and b_1 may be obtained by solving equation (16) of Chadha et al.[13] We note the following conditions defining the characteristics of the risk profile

$$\begin{aligned} \hat{L}(d_i, \theta_{\text{true}}; \gamma, \xi) > L(d_i, \theta_{\text{true}}) \text{ or } \xi < \gamma \\ : \text{ For risk} - \text{averse profile} \\ \hat{L}(d_i, \theta_{\text{true}}; \gamma, \xi) = L(d_i, \theta_{\text{true}}) \text{ or } \xi = \gamma \\ : \text{ For risk} - \text{neutral profile} \\ \hat{L}(d_i, \theta_{\text{true}}; \gamma, \xi) < L(d_i, \theta_{\text{true}}) \text{ or } \xi > \gamma \\ : \text{ For risk} - \text{seeker profile} \end{aligned} \tag{35}$$

For the purpose of simulation, we consider the 5 risk profiles mentioned in Table 1.

Figure 6 illustrates the modified cost function for various risk profiles listed in Table 1.

Empowered with the idea of risk profiles, we realize that risk profiles can be interpreted in 2 ways: (1) *Forward interpretation*: each risk profile represents individual decision-maker's behavior and (2) *Inverse interpretation*: each risk profile represents a risk intensity that the organization wants to include over the base cost to make a decision. A risk-averse profile demands a conservative decision, that is, a tendency to perform the maintenance M_1 at a relatively lower level of damage to avoid any disastrous and expensive consequence. On the other hand, a risk-seeker profile allows more flexible decision-making that would recommend the maintenance M_1 only when the degree of structural damage is approaching failure, that is, in a more risky state. To include the behavioral influence of the decision-maker into the decision-making framework discussed in *Decision-making framework and Value of information considering an instance of decision-making*, we simply replace the cost functions with the risk-modified cost functions, that is, $L(d_j, \theta_{\text{true}}) \rightarrow \hat{L}(d_j, \theta_{\text{true}}; \gamma, \xi)$.

Remark 1. For a multi-dimensional state parameter (unlike a scalar used in this article), the cost function would take a form of a hypersurface. The risk intensity of the decision-maker can then be introduced by defining an appropriate mapping function that takes the base hypersurface to a risk-modified hypersurface parameterized by appropriate risk-parameters with appropriate boundary restrictions. If the hypersurface is a Riemannian manifold, then the local risk intensity at a given state-vector can be defined by Riemannian curvature at that state.

Table 1. Examples of different risk profiles.

Risk profiles	ID	γ	ξ
Extreme risk-averter	RP1	0.8	0.25
Moderate risk-averter	RP2	0.8	0.6
Neutral risk bearer	RP3	0.8	0.8
Moderate risk-seeker	RP4	0.8	0.95
Extreme risk-seeker	RP5	0.8	0.999

A practical and simple example of pre-posterior decision analysis

As discussed in the introduction, the unscheduled shutdown of these navigation locks and inspecting them with divers or even dewatering them for inspection or repair is very costly to USACE. Consider the following simple information-gathering mechanism concerning the miter gate problem at hand:

1. z_0: No acquisition of data.
2. z_1: Send the diver to measure the gap length.
3. z_2: Dewater the gate and measure the exact gap length.

We analyze which option to choose among the above three choices constituting the inspection (or experiment) space $\Omega_Z = \{z_0, z_1, z_2\}$ using the *pre-posterior decision analysis*. We assume the cost of these inspections as $C(z_0)$ = US\$0, $C(z_1)$ = US\$0.02, and $C(z_2)$ = US\$0.2 (in millions). We also note that z_2 is a *perfect experiment/inspection* as it yields the exact value of the gap length. As such, with an aim of focusing on the pre-posterior analysis, we consider a prior distribution modeled by a Gaussian distribution with the mean μ_θ = 75 inches and standard deviation σ_θ = 20 inches, that is, $f_\Theta(\theta) = \frac{1}{20}\phi\left(\frac{\theta-75}{20}\right)$. We consider the linear cost function (or RP3) for the purpose of this example.

Since the information acquired through z_1 and z_2 directly give the gap length, $\Omega_{X_{z_i}}$ represents the space of gap length obtained from the experiment z_i such that $\Omega_{X_{z_i}} \equiv \Omega_\Theta$. Since we are not actually collecting new information (or conducting an experiment) for pre-posterior analysis, the likelihood $f_{X_{z_i}|\Theta}(x_{z_i}|\theta)$ is to be provided by the organization from the past data/experience/simulation. If no past data is available, a reasonable assumption of the likelihood must be made. Experiment z_1 involves sending a diver in. We assume that the gap length measured by the diver has some noise. We assume Gaussian noise of zero mean and a standard deviation of 3 inches leading to the likelihood $f_{X_{z_1}|\Theta}(x_{z_1}|\theta) = \frac{1}{3}\phi\left(\frac{x_{z_1}-\theta}{3}\right)$. Since z_2 is a perfect experiment, we have $f_{X_{z_2}|\Theta}(x_{z_2}|\theta) = \delta(x_{z_2} - \theta)$.

Given all the entities discussed above, by virtue of equivalence in the equations (27a) and (27e), there are two approaches to know if inspection must be conducted, and if yes, then which among the two z_1 and z_2 is most optimal. The first approach involves obtaining the expected value of information EVoI(z_i) and using equation (27a) to make a decision, whereas, the second approach involves obtaining the *value of perfect information* $\mathrm{PVoI} = C(\hat{z})$ and using equation (27b) to make a decision. Figures 7 and 8 illustrates the two approaches to decide on choosing the inspection strategies.

We can obtain the value of perfect information by solving for $C(\hat{z})$ in the slightly modified form of equation (26)

$$\mathrm{EVoI}(\hat{z}) = \min_{d_j} E_\Theta\left[\hat{L}\left(d_j, \theta_{\mathrm{true}}; \gamma, \xi\right)\right] - E_\Theta\left[\min_{d_j}\left(\hat{L}\left(d_j, \theta_{\mathrm{true}}; \gamma, \xi\right)\right)\right] - C(\hat{z}) = 0 \tag{36}$$

Solving the equation above for $C(\hat{z})$ for the current case, we obtain $C(\hat{z}) = 0.263$ million.

Approach 1 given in Figure 7 requires evaluation of the posterior and the evidence which can be involved and computationally expensive to obtain. On the other hand, approach 2 illustrated in Figure 8 demands an evaluation of $C(\hat{z})$ using equation (36) that if solved numerically would require using sample-based integration which is computationally cheaper to perform than to obtain the posterior and the evidence. We suggest readers use either approach 1 or 2 as it suits their problem.

For the given three inspection strategies z_i, Table 2 details the expected costs and the value of experiment/inspection $\mathrm{EVoI}(z_i)$. Table 2 clearly indicates that performing both the experiments z_1 and z_2 will be beneficial in accordance with all four conditions in equation set (27). However, although dewatering yields a higher net savings $C_{\mathrm{save}}(z_2) > C_{\mathrm{save}}(z_1)$, sending the diver in for taking measurement yields the best risk-adjusted reward, that is, $\lambda(z_1) > \lambda(z_2)$. The relative risk-adjusted reward of design z_1 relative to the design z_2 is $\chi(z_1, z_2) = 38.387$, that is, design z_1 leads to 38.387 times more risk-adjusted savings as compared to

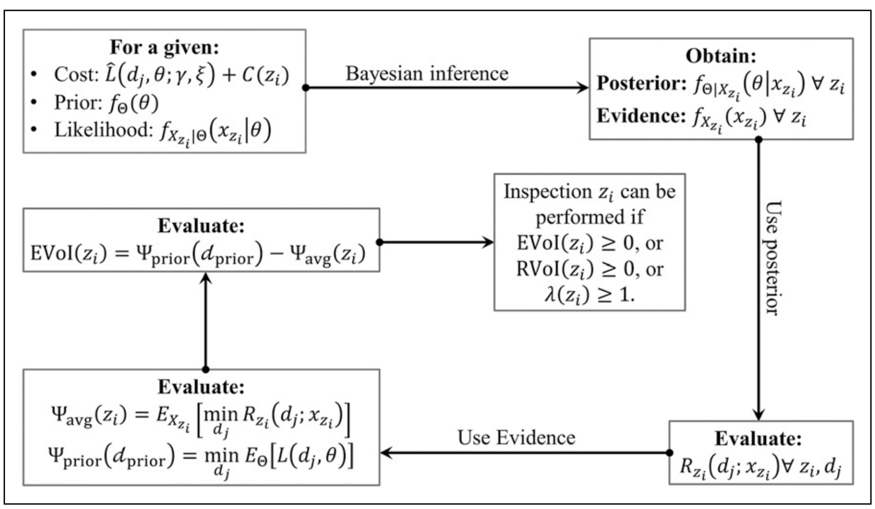

Figure 7. Approach 1 to make a decision on choosing the inspection strategies.

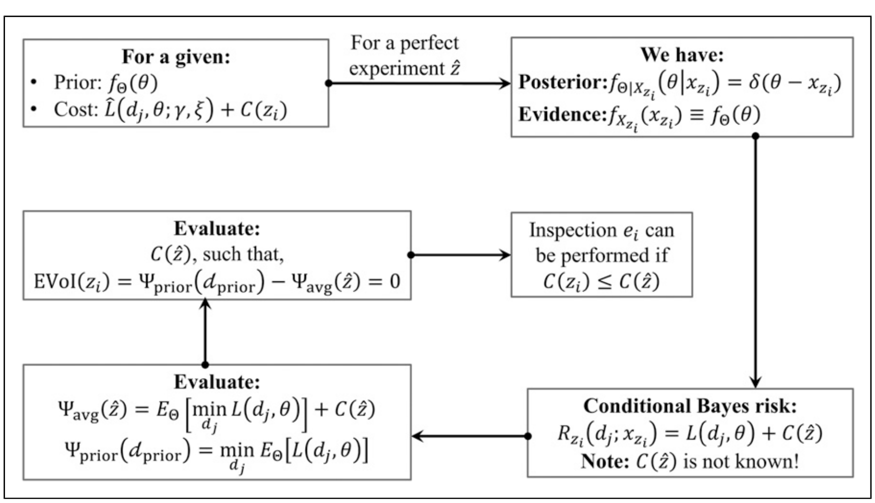

Figure 8. Approach 2 to make a decision on choosing the inspection strategies.

Table 2. Information-gathering mechanisms and their value of information.

Data-acquisition	$C(z_i)$ (in millions)	$\Psi_{avg}(z_i)$	$EVoI(z_i)$	$RVoI(z_i)$	$C_{save}(z_i)$ (in millions)	$\lambda(z_i)$
z_0	0	0.254	0	0	0	N/A
z_1	0.020	0.135	0.119	0.468	0.139	6.950
z_2	0.200	0.223	0.031	0.122	0.231	1.155

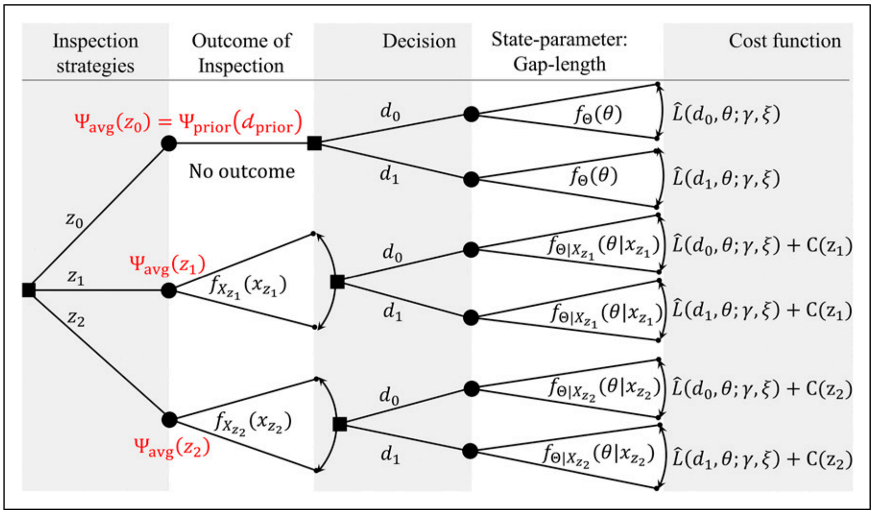

Figure 9. The decision tree for pre-posterior analysis.

the design z_1. Figure 9 shows the decision tree for pre-posterior analysis.

All the discussions carried so far, including the example in this section, consider only one instance of true gap length value at a fixed time (or the structural state at a fixed instance of time). The next section is about quantifying the VoI-gathering over time evolution of true gap length, or the life cycle of the structure.

Value of information for lifecycle cost analysis

Over the life cycle of the miter gate, the structural state evolves from *pristine condition* (defined by 0 gap length value) to *approaching critical failure* (defined by gap length being unacceptably high as θ_{max}). This evolution of state takes over the lifespan of the structure and is quantified by $\Omega_T = [0, t_{max}]$. We use *months* as the unit of time. Therefore, to evaluate the VoI gathered throughout the lifespan of the structure, we need a gap-growth or degradation model.

Gap-growth (degradation) model

Let Θ_t denote a random variable representing the gap length at any time $t \in \Omega_T$, such that its realization is denoted by $\theta(t)$

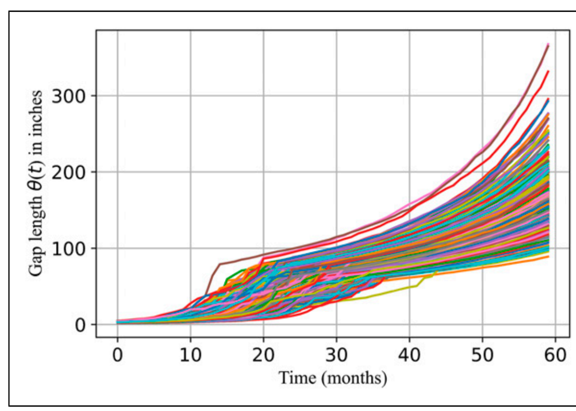

Figure 10. The time evolution of gap length.

$\in \Omega_{\Theta(t)}$, with $\Omega_{\Theta(t)} = \Omega_\Theta$. Since the time evolution of the gap length is not precisely known, we model it probabilistically (as shown in Figure 10), such that $f_{\Theta(t)}\ (\theta(t))$ denotes the prior distribution of gap length at time t.

The gap evolution over time is described by a piecewise multi-stage degradation model as follows

$$\theta(t_{k+1}) = \theta(t_k) + \theta(t_k)^{w(t_{k+1})}.Q(t_{k+1}).\exp(\sigma(t_{k+1}).U(t_{k+1})) \tag{37}$$

In the equation above, $\theta(t_{k+1})$ denotes the gap length at time step t_k; N_t is the total number of time steps; $U(t_{k+1})$ is a stationary Gaussian stochastic process; $\sigma_{t_{k+1}}$, $Q(t_{k+1})$, and $w(t_{k+1})$ are degradation state-dependent model parameters, which are given as follows

$$\begin{aligned} \sigma(t_{k+1}) &= \sigma_j \\ Q(t_{k+1}) &= Q_j \\ w(t_{k+1}) &= w_j \end{aligned} \tag{38}$$

where, the index j represents the degradation state, such that

$$j = h_s(\theta(t_k)) \tag{39}$$

The function $h_s(\cdot)$ maps the gap length to the degradation state j, such that

$$j = h_s(\theta(t)) = \begin{cases} 1, & \text{if } \theta(t) \in [0, e_1] \\ 2, & \text{if } \theta(t) \in [e_1, e_2] \\ \vdots & \\ N_d, & \text{if } \theta(t) \in [e_{N_d-1}, \infty] \end{cases} \tag{40}$$

where e_i for $i \in \{1, 2, \cdots, (N_d - 1)\}$ are the switching points that govern the transition between different degradation stages and N_d is the number of degradation stages. We assumed $N_d = 3$ for the current study. Since the switching points e_i in equation (40) are uncertain in nature, they are modeled by the Gaussian distribution as shown below

$$e_i \sim N\left(\mu_{e_i}, \sigma_{e_i}^2\right) \quad \forall \quad i \in \{1, 2, \cdots, N_d - 1\} \tag{41}$$

Inflation-adjusted cost function

Since we intend to do a lifecycle cost analysis that deals with decision-making at different time periods, we estimate all the costs at the current time. We do inflation adjustment to define the cost for any future time. The factor $(r(t) + 1)^t$ adjusts for the future inflation, where $r(t)$ is the assumed future monthly rate of inflation at time t (in months). We consider the following costs:

1. *Cost A*: The inflation-adjusted *consequence-cost of decision-making* at time t for the risk profile (γ, ξ), denoted by $\tilde{L}(d_j, \theta_{\text{true}}(t), t; \gamma, \xi)$, such that

$$\hat{L}\left(d_j, \theta_{\text{true}}(t), t; \gamma, \xi\right) = \hat{L}\left(d_j, \theta_{\text{true}}(t); \gamma, \xi\right).(r(t) + 1)^t \tag{42}$$

We note maintenance/repair/inspection decisions are made based on planned inspections/data-gathering at discrete time-steps. We assume a discrete time-space, denoted by $\Omega_{T_A} = \{t_{A_1}, t_{A_2}, \cdots, t_{A_{N_A}}\}$, containing N_A time-step (not necessarily uniform), such that $t_{A_{N_A}} \leq t_{\max}$.

2. *Cost B*: The *maintenance cost* of the information-gathering system, denoted by $C_M(t) = C_M\,(r(t) + 1)^t$. Here, C_M is the current estimated cost of maintenance for one instance of system maintenance. We assume that the maintenance of data-gathering system is done at discrete time-step, defined by the space $\Omega_{T_B} = \{t_{B_1}, t_{B_2}, \cdots, t_{B_{N_B}}\}$, containing N_B time step (not necessarily uniform), such that $t_{B_{N_B}} \leq t_{\max}$.
3. *Cost C*: The *operation cost* of the information-gathering system, denoted by $C_O(t) = C_O\,(r(t) + 1)^t$. Here, C_O is the current estimated operation-cost per month. We assume that the operational-cost is evaluated every month defined by the discrete time-space $\Omega_{T_C} = \{t_{C_1}, t_{C_2}, \cdots, t_{C_{N_C}}\}$ containing N_C time-step (not necessarily uniform), such that $t_{C_{N_C}} \leq t_{\max}$.
4. *Cost D*: The *cost of design and initial installation* of an information-gathering system $C(z)$. We assume this to be an initial cost and hence time-independent.

Expected value of information for lifecycle cost analysis

To start with, we consider the case when no new information is available. The expected consequence-cost at time t, denoted by $\Psi_{\text{prior}}(d_i, t)$, and the optimal decision, denoted by $\mathscr{d}_{\text{prior}}(t) \in \Omega_D$ is defined as

$$\begin{aligned} \Psi_{\text{prior}}(d_i, t) &= E_{\Theta(t)}\left[\hat{L}\left(d_j, \theta_{\text{true}}(t), t; \gamma, \xi\right)\right] \\ &= E_{\Theta(t)}\left[\hat{L}\left(d_j, \theta_{\text{true}}(t); \gamma, \xi\right).(r(t) + 1)^t\right] \\ \mathscr{d}_{\text{prior}}(t) &= \arg\min_{d_i} \Psi_{\text{prior}}(d_i, t) \end{aligned} \tag{43}$$

Assuming time-continuous decision-making, the total expected cost as a consequence of making optimal decisions over the structure's lifespan Ω_T, denoted by Ψ_{priorLC}, is obtained as

$$\begin{aligned} \Psi_{\text{priorLC}} &= \sum_{n=1}^{N_A} \min_{d_i} \Psi_{\text{prior}}(d_i, t_{A_n}) \\ &= \sum_{n=1}^{N_A} \Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}(t_{A_n}), t_{A_n}\right) \end{aligned} \tag{44}$$

The subscript LC in Ψ_{priorLC} and the following future notations represents *lifecycle*.

Now consider a situation when new information about the structure is available through the mechanism z (simulated for the purpose of preposterior analysis). Let $X_z(t)$ be the random variable representing the acquired (or simulated) data, such that its realization is denoted by $x_z(t) \in \Omega_{X_z(t)}$. Therefore, the most optimum decision at time t for a given experiment-outcome pair $(z, x_z(t))$ is then given as

$$\mathscr{d}_z(x_z, t) = \arg\min_{d_j} R_z\left(d_j; x_z(t)\right), \text{ where} \tag{45a}$$

$$R_z\left(d_j; x_z(t)\right) = E_{\Theta(t)|X_z(t)}\left[\widehat{\hat{L}}\left(d_j, \theta_{\text{true}}(t), t; \gamma, \xi\right) + C(z)\right]$$
$$= E_{\Theta(t)|X_z(t)}\left[\hat{L}\left(d_j, \theta_{\text{true}}(t); \gamma, \xi\right).(r(t)+1)^t\right]$$
$$+ C(z) \tag{45b}$$

The Bayes risk $\Psi_{\text{avg}}(z,\ t)$ is obtained by evaluating the expected value of the *conditional Bayes risk* $R_z(\mathscr{d}_{\mathscr{z}}(x_z, t); x_z(t))$ corresponding to optimal decision $\mathscr{d}_{\mathscr{z}}(x_z, t)$ for all the possible information/data (i.e., probabilistically defined by $f_{X_z(t)}(x_z(t))$ and is conditioned upon the prior distribution of the gap length at time t, or $f_{\Theta(t)}\ (\theta(t))$, which in turn is obtained from the prior gap-degradation model). We have

$$\Psi_{\text{avg}}(z, t) = E_{X_z(t)}\left[\min_{d_j} R_z\left(d_j; x_z(t)\right)\right]$$
$$= E_{X_z(t)}[R_z(\mathscr{d}_{\mathscr{z}}(x_z(t)); x_z(t))] \tag{46}$$

Considering decision-making at a fixed time t (as discussed in *Expected value of information or the value of experiment*), the EVoI for the experiment z at time t is given by

$$\text{EVoI}(z, t) = \Psi_{\text{prior}}\left(\mathscr{d}_{\text{prior}}(t), t\right) - \Psi_{\text{avg}}(z, t) \tag{47}$$

The quantity EVoI(z, t) is useful to evaluate the advantage of information-gathering as a consequence of decision-making using the acquired data through the experiment z at a fixed instance of time t (not the entire lifespan). Since EVoI(z, t) measures the benefit of SHM for decision-making at an instance of time, in the form presented in equation (47), it only considers the *cost A* at time t and *cost D* and ignores the cost of maintenance and operations. The expression of EVoI(z, t) in equation (47) is particularly desirable to understand how the value of acquiring information to make maintenance decisions evolves over time. Finally, equation 47 can be written in a more desirable form using equations (46) and (43) as

$$\text{EVoI}(z,t) = C_{\text{save}}(z,\ t) - C(z), \text{ where,}$$
$$C_{\text{save}}(z, t) = E_{X_z(t)}\left[\min_{d_j} \hat{L}\left(d_j, \theta_{\text{true}}(t), t; \gamma, \xi\right)\right]$$
$$- \min_{d_j} E_{\Theta(t)}\left[\hat{L}\left(d_j, \theta_{\text{true}}(t), t; \gamma, \xi\right)\right] \tag{48}$$

In the equation above, $C_{\text{save}}(z,\ t)$ gives the expected cost saved by virtue of making a better decision based on newly acquired data at time t through the mechanism/system z.

Finally, the total expected cost including the consequence cost of making optimal decisions based on newly available data, and also including the maintenance cost (*cost B*) and operational cost (*cost C*) of the information-gathering system over the structure's lifespan, denoted by Ψ_{avgLC} is obtained as

$$\Psi_{\text{avgLC}}(z) = C(z) + \sum_{n=1}^{N_A}\left(\Psi_{\text{avg}}(z, t_{A_n}) - C(z)\right)$$
$$+ \sum_{n=1}^{N_B} C_M.(r(t_{B_n})+1)^{t_{B_n}} + \sum_{n=1}^{N_C} C_0.(r(t_{C_n})+1)^{t_{C_n}} \tag{49}$$

Note that, for null experiment z_0, we have $\Psi_{\text{avgLC}}(z_0) = \Psi_{\text{priorLC}}$. Finally, the *expected value of information over the lifecycle* $\text{EVoI}_{\text{LC}}(z)$ of the information-gathering mechanism z is defined as

$$\text{EVoI}_{\text{LC}}(z) = \Psi_{\text{priorLC}} - \Psi_{\text{avgLC}}(z)$$
$$= C_{\text{saveLC}}(z) - (C(z) + C_{M\&O}(z)) \tag{50}$$

In the equation above, $C_{M\&O}(z)$ denotes the total cost of maintenance and operation of the SHM system over the lifespan of the structure, such that

$$C_{M\&O}(z) = \sum_{n=1}^{N_B} C_M.(r(t_{B_n})+1)^{t_{B_n}} + \sum_{n=1}^{N_C} C_0.(r(t_{C_n})+1)^{t_{C_n}} \tag{51}$$

The quantity $C_{\text{saveLC}}(z)$ in equation (50) denotes the expected savings over the lifecycle of the structure, such that

$$C_{\text{saveLC}}(z) = \sum_{n=1}^{N_A}\left(\min_{d_i} E_{\Theta(t_{A_n})}\left[\hat{L}\left(d_j, \theta_{\text{true}}(t_{A_n}), t_{A_n}; \gamma, \xi\right)\right]\right)$$
$$.(r(t_{A_n})+1)^{t_{A_n}}$$
$$- \sum_{n=1}^{N_A}\left(E_{X_z(t_{A_n})}\left[\min_{d_i} E_{\Theta(t_{A_n})|X_z(t_{A_n})}\right.\right.$$
$$\left.\left.\times\left[\hat{L}\left(d_j, \theta_{\text{true}}(t_{A_n}), t_{A_n}; \gamma, \xi\right)\right]\right]\right).(r(t_{A_n})+1)^{t_{A_n}} \tag{52}$$

The *relative value of information over the lifecycle* of an experiment z (denoted by RVoI(z)) can be obtained by normalizing $\text{EVoI}_{\text{LC}}(z)$ with respect to the prior lifecycle cost Ψ_{priorLC}

$$\text{RVoI}_{\text{LC}}(z) = \frac{\Psi_{\text{priorLC}} - \Psi_{\text{avgLC}}(z)}{\Psi_{\text{priorLC}}} = \frac{\text{EVoI}_{\text{LC}}(z)}{\Psi_{\text{priorLC}}} \tag{53}$$

Finally, the risk-adjusted return ratio for lifecycle cost analysis is defined as

$$\lambda_{\mathrm{LC}}(z) = \frac{C_{\mathrm{saveLC}}(z)}{\mathrm{EVoI}_{\mathrm{LC}}(z) + C_{\mathrm{saveLC}}(z)} \tag{54}$$

For two feasible designs z_1 and z_2, such that $\lambda_{\mathrm{LC}}(z_1) > 1$ and $\lambda_{\mathrm{LC}}(z_2) > 1$, the relative benefit of one feasible design with respect to another feasible design in terms of their risk-adjusted reward over the lifecycle of the structure is defined by $\chi_{\mathrm{LC}}(z_1, z_2)$, such that

$$\chi_{\mathrm{LC}}(z_1,z_2) = \frac{\lambda_{\mathrm{LC}}(z_1) - 1}{\lambda_{\mathrm{LC}}(z_2) - 1} \tag{55}$$

An SHM system z with $\mathrm{EVoI}_{\mathrm{LC}}(z) \leq 0$, or equivalently $\lambda_{\mathrm{LC}}(z) \geq 1$ leads to net cost-saving (in average sense) over the lifespan of the structure and hence is economically feasible. The next section deals with the application of the theoretical results discussed so far into a miter gate problem.

Remark 2. Note that $\mathrm{EVoI}_{\mathrm{LC}}(z)$ is a differential measure. On the other hand, the expected reward to investment risk $\lambda_{\mathrm{LC}}(z)$ is a normalized metric. Therefore, $\mathrm{EVoI}_{\mathrm{LC}}(z)$ quantifies absolute gain in dollar over the lifecycle, whereas, $\lambda_{\mathrm{LC}}(z)$ quantifies the compounded gain in percentage over the lifecycle. In that regard, $\lambda_{\mathrm{LC}}(z)$ is a better metric than $\mathrm{EVoI}_{\mathrm{LC}}(z)$. For instance, consider two feasible SHM systems:

1. z_1: SHM system z_1 leads to an expected reward of US\$1100 on an investment of US\$1000 over the lifecycle of the structure
2. z_2: SHM system z_2 leads to an expected reward of US\$10,100 on an investment of US\$10,000 over the lifecycle of the structure

Both the systems have same expected value of information over the lifecycle, that is, $\mathrm{EVoI}_{\mathrm{LC}}(z_1) = \mathrm{EVoI}_{\mathrm{LC}}(z_2) =$ US\$100. Therefore, both the investment scenarios are equivalent as per the EVoI metric. However, it is clear that the first scenario lead to a percentage gain of 10% (or $\lambda_{\mathrm{LC}}(z_1) = 1.1$) and the second scenario leads to a net percentage gain of 1% (or $\lambda_{\mathrm{LC}}(z_2) = 1.01$). Therefore, clearly, SHM system z_1 is superior to the system z_2 which $\mathrm{EVoI}_{\mathrm{LC}}(z)$ fails to capture. The design z_1 leads to 10 times more risk-adjusted savings than the design z_2, that is, $\chi_{\mathrm{LC}}(z_1, z_2) = 10$. Therefore, an optimal design is the one that maximizes the risk-adjusted reward and not the one that maximizes absolute gain.

Similar to λ_{LC}, RVoI_{LC} is also a normalized measure. We note that, for two designs z_1 and z_2, the following condition holds

$$\frac{\mathrm{RVoI}_{\mathrm{LC}}(z_1)}{\mathrm{RVoI}_{\mathrm{LC}}(z_2)} = \frac{\mathrm{EVoI}_{\mathrm{LC}}(z_1)}{\mathrm{EVoI}_{\mathrm{LC}}(z_2)} \neq \frac{\lambda_{\mathrm{LC}}(z_1)}{\lambda_{\mathrm{LC}}(z_2)} \tag{56}$$

Since $\lambda_{\mathrm{LC}}(z)$ is expressed in terms of the expected reward and investment risk ratio (that are crucial in making business decisions), inherently by its very definition it is advantageous to use λ_{LC} for the scenario where the business decision on SHM are to be made.

Numerical simulation

We consider the miter gate structure with strain data acquired using strain gauge network. As such, consider the following four SHM system designs, including three strain gauge network designs and one null design:

1. z_0: No acquisition of data (null design).
2. z_1: Acquire data using Bayesian optimized strain gauge network containing $N_{\mathrm{sg}}(z_1) = 10$ number of strain gauges as detailed in Yang et al.[21] (optimized using risk-weighed KL divergence objective functional).
3. z_2: Acquire data using strain gauge network randomly distributed across the miter gate structure containing $N_{\mathrm{sg}}(z_2) = 10$ number of strain gauges.
4. z_3: Acquire data using strain gauge network randomly distributed across the miter gate structure containing $N_{\mathrm{sg}}(z_3) = 20$ number of strain gauges.

Let $X_{z_i}(t)$ be the random variable representing strain gauge readings at time t for the network design z_i with $i \in \{1, 2, 3\}$. Evaluating $\mathrm{EVoI}_{\mathrm{LC}}(z_i)$ and the reward to risk ratio $\lambda_{\mathrm{LC}}(z)$ over the lifecycle, requires obtaining the posterior distribution $f_{\Theta(t)|X_{z_i}(t)}(\theta(t)|x_{z_i}(t))$ using Bayesian inference as discussed in the next section.

Bayesian inference

For any given sensor-design z, a realization of the measurement vector $x_z \in \Omega_{X_z}$ (say at any fixed time t and z here represent any design z_i of interest) is the strain recorded at $N_{\mathrm{sg}}(z)$ number of strain gauge locations. The measurements obtained from the strain gauges are used to infer the gap length θ using the Bayes theorem. In the context of inferring θ, the evidence $f_{X_z}(x_z)$ is just a normalizing constant. Therefore, the Bayes theorem may be written as

$$f_{\Theta|X_z}(\theta|x_z) \propto f_{X_z|\Theta}(x_z|\theta) f_\Theta(\theta) \tag{57}$$

The prior distribution at any time t is obtained from the gap length evolution model discussed in *Gap-growth (degradation) model*. For the problem at hand, we estimate the likelihood $f_{X_z|\Theta}(x_z|\theta)$ using simulated data obtained through the FEM model or a digital twin. Let $g_z(\theta, h_{\text{up}}, h_{\text{down}})$ define the *true* strain response for the sensors included in design z obtained by the FEM or digital twin model, such that $g_z(\theta, h_{\text{up}}, h_{\text{down}}) = (g_{z1}(\theta, h_{\text{up}}, h_{\text{down}}), \cdots, g_{zN_{\text{sg}}(z)}(\theta, h_{\text{up}}, h_{\text{down}}))$. Similarly, $x_z = (x_{z1}, \cdots, x_{zN_{\text{sg}}(z)}) \in \Omega_{X_z}$ represent the *observed* strain readings. The measurement model for the strain gauges included in the design z is given by

$$x_z = g_z(\theta, h_{\text{up}}, h_{\text{down}}) + \varepsilon_z \tag{58}$$

In the equation above, x_z is one of the realizations of the random vector X_z. The vector ε_z is the realization of the random vector ζ_z with $\varepsilon_z = (\varepsilon_{z1}, \varepsilon_{z2}, \cdots, \varepsilon_{zN_{\text{sg}}(z)})$. It represents the measurement noise/error vector for the design z, where ε_{zi} denotes the error between the measurement output and FEM predicted response (assumed to be the true response) corresponding to the i^{th} strain gauge in the design z. Let ζ_{zi} (with ε_{zi} as its realization) denote the random variable for the noise in i^{th} strain gauge. We assume that ε_z follows a zero-mean Gaussian distribution with independent components, that is, the noise/error terms of all $N_{\text{sg}}(z)$ strain gauges are assumed to be statistically independent. In addition, we assume that each strain gauge has same standard deviation $\sigma_{\varepsilon_{zi}}$, such that

$$f_{\zeta_z}(\varepsilon_{z1}, \varepsilon_{z2}, \cdots, \varepsilon_{zN_{\text{sg}}(z)}) = \prod_{i=1}^{N_{\text{sg}}(z)} f_{\zeta_{zi}}(\varepsilon_{zi}) = \prod_{i=1}^{N_{\text{sg}}(e)} \frac{1}{\sigma_{\varepsilon_{zi}}} \phi\left(\frac{\varepsilon_{zi}}{\sigma_{\varepsilon_{zi}}}\right) \tag{59}$$

Using the measurement model defined in equation (58), and the description of noise in equation (59), the likelihood of observing the strain measurement $x_z \in \Omega_{X_z}$ for the gap length θ can be written as

$$f_{X_z|\Theta}(x_z|\theta) = \prod_{i=1}^{N_{\text{sg}}(z)} \frac{1}{\sigma_{\varepsilon_{zi}}} \phi\left(\frac{x_{zi} - g_{zi}(\theta, h_{\text{up}}, h_{\text{down}})}{\sigma_{\varepsilon_{zi}}}\right) \tag{60}$$

Since the relationship between the gap length θ and the strain data x_z is highly non-linear and complex, we numerically infer the posterior distribution by using particle filters (see Ref. 21, 31, 16). Evaluation of the likelihood $f_{X_z|\Theta}(x_z|\theta)$ at numerous values of θ at different time periods using the full FEM is exorbitantly expensive. Therefore, we use a digital twin modeled by GPR model to predict the true strain value $g_z(\theta, h_{\text{up}}, h_{\text{down}})$. To simulate the measurement data, we obtain the response of the digital surrogate $g_z(\theta_{\text{true}}, h_{\text{up-true}}, h_{\text{down-true}})$ parameterized by a chosen/fixed value of true gap length θ_{true} subjected to chosen/fixed input loading $(h_{\text{up-true}}, h_{\text{down-true}})$. This strain gauge response is now corrupted by Gaussian noise of standard deviation $\sigma_{\varepsilon_{zi}}$ to mimic the real-world measurement noise. This corrupted strain response is now used as the measurement/observed data $x_z \in \Omega_{X_z}$.

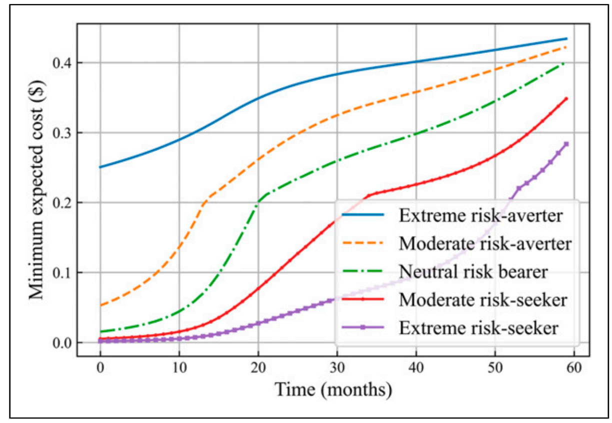

Figure 11. Design z_0: minimum prior expected Bayes risk or expected loss.

Numerical results

Considering no SHM system is installed: Null design z_0*.* For design z_o, there is no new information in the form of strain gauge measurements. Therefore, we use the prior gap length degradation model illustrated in Figure 10 to evaluate the minimum prior expected Bayes risk, that is, $\min_{d_i} \Psi_{\text{prior}}(d_i, t) = \Psi_{\text{prior}}(\mathscr{d}_{\text{prior}}, t)$, for various modified consequence cost or risk profiles (see Figure 11). This quantity provides a base relative to which the benefit of other SHM designs $\{z_1, z_2, z_3\}$ at making better (or worse) maintenance decisions is evaluated. We make the following observations:

1. The minimum expected cost is a continuously increasing function. This is expected as the assumed base cost function illustrated in Figure 5 increases as the degree of damage increases. It is obvious that the gap length would increase over time and hence the cost of maintenance would also increase accordingly.
2. For a fixed time, the minimum expected cost increases as the risk-aversion of the decision-maker increases (or equivalently, risk-seeker tendencies of the decision-maker decreases). A risk-averse profile demands a conservative decision, that is, a tendency to perform the maintenance M_1 (more expensive and conservative than the maintenance M_0) at a relatively lower level of damage to avoid any disastrous and expensive consequence. On the other hand, a risk-seeker profile allows more flexible decision-making

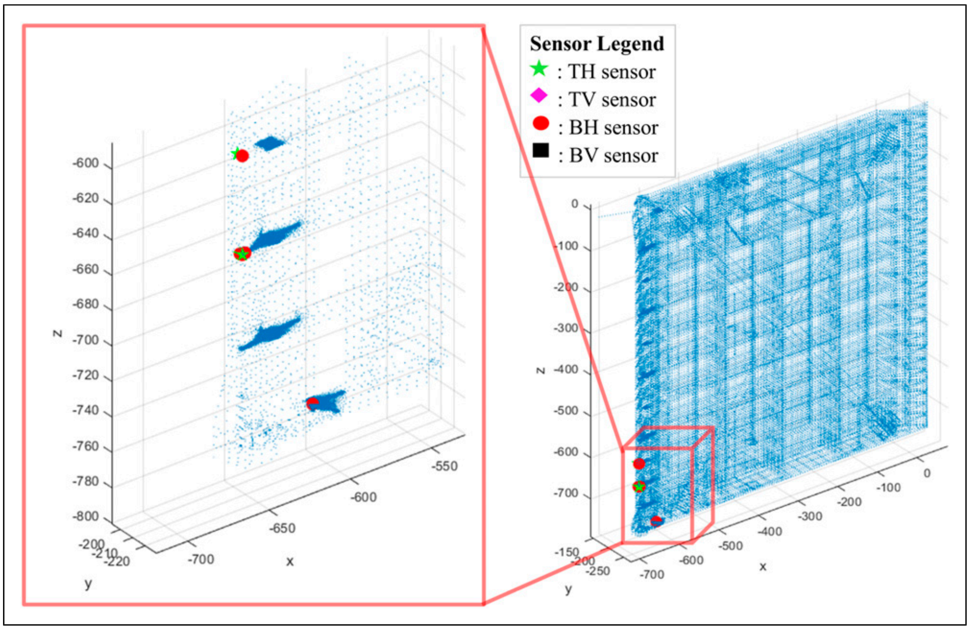

Figure 12. Design z_1: sensor network design.

that would recommend the maintenance M_1 only when the degree of structural damage is approaching failure, that is, in a more risky state.

Considering SHM system with 10 optimally designed sensors: Design $\mathbf{z}_1$. Sensor network design z_1 was arrived by maximizing the KL divergence that quantifies the relative gain in information contained in the posterior distribution of the gap length conditioned upon acquired (or simulated) strain data as compared to the information contained in the prior distribution of the gap length.[21] All the sensors in the design z_1 are closer to the gap and hence lead to better inference of the state (or the gap length). Figure 12 illustrates the sensor-network design z_1.

1. Figure 13 illustrates the inferred posterior gap-degradation model. We observe that the variability in the posterior distribution of the gap length at every time instance is smaller relative to the variability observed in the gap length in prior distribution shown in Figure 10. This is because the acquired sensor data helps us better understand (or infer) the current state of the structure (defined by gap length).
2. Figure 14 illustrates the cost saved as a consequence of choosing optimal maintenance strategy at the various instance of time based on newly acquired strain data for various risk profiles. As mentioned before, since $C_{\text{save}}(z_1, t)$ evaluates the advantage of SHM on decision-making relative to the null design z_0 at an instance of time, we do not consider the cost of maintenance and operation (cost B and C) of the SHM system in the evaluation of the quantity $C_{\text{save}}(z_1, t)$. For each of the risk profiles, we observe that the $C_{\text{save}}(z_1, t)$ is greater than or equal to zero (as per equation (14)) and it increases to a certain range of gap length following which it goes down. The value of $C_{\text{save}}(z_1, t)$ evaluates the economical benefit of arriving at a data-informed maintenance decision as a consequence of having an SHM system installed as compared to the decisions we would have made using null design z_0 (or by using our prior understanding of the state parameter). However, beyond a certain gap length value, the maintenance decision obtained using the posterior distribution of

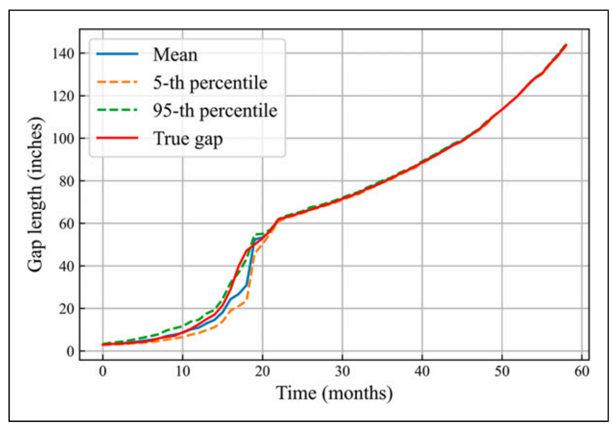

Figure 13. Design z_1: posterior gap-degradation model.

gap length is the same as the decision obtained using the prior gap length distribution. For instance, when the gap length is toward the higher end of the spectrum, say θ = 170 inches (closer to the critical failure), it is obvious that an engineer with any risk profile would choose to label the gate as damaged. In such obvious decisions, SHM is not necessarily useful at that instance of time. Similarly, it can be

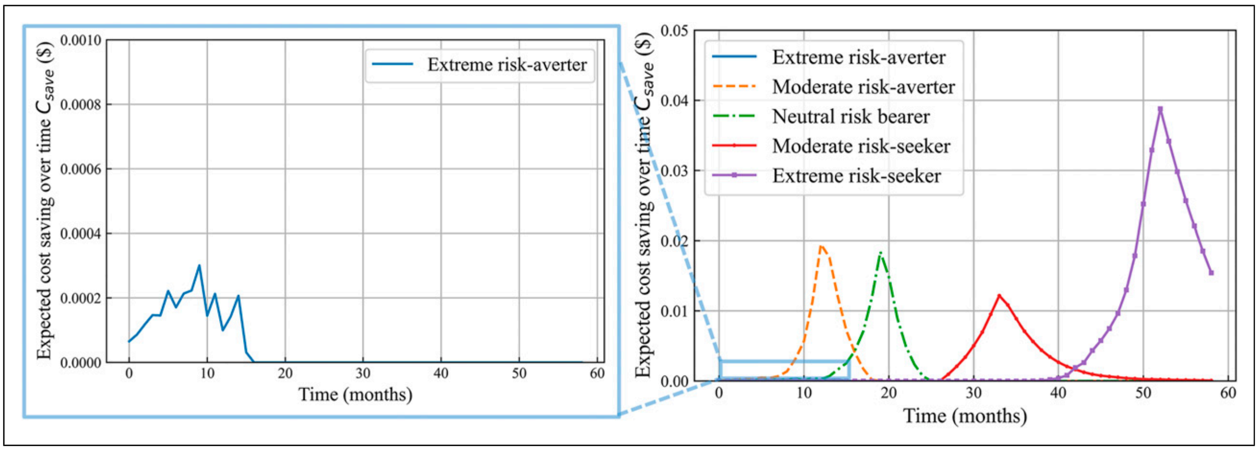

Figure 14. Design z_1: cost saving ($C_{save}(z_1, t)$) over time as a consequence of making better decisions.

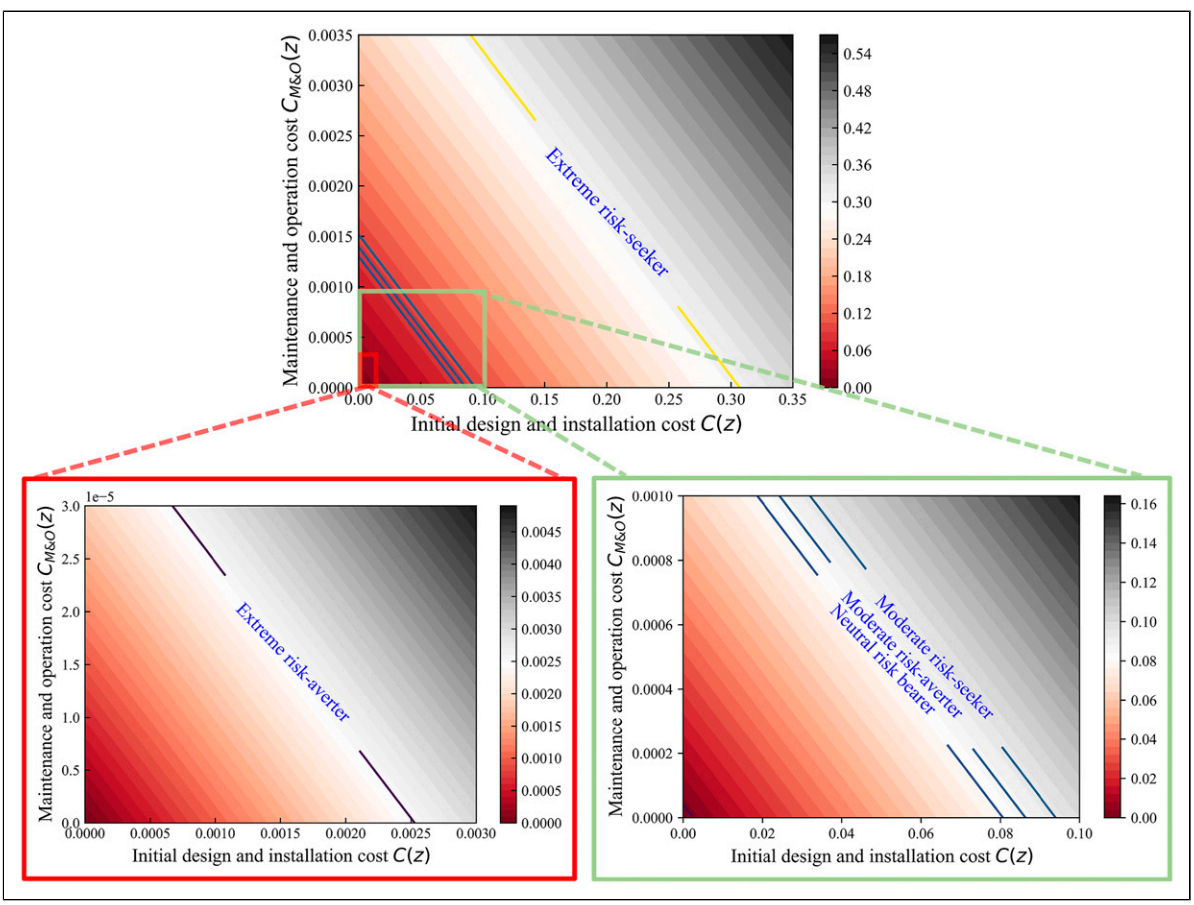

Figure 15. Design z_1: cost classifiers for different risk profiles.

seen that beyond 100 inches, the $C_{save}(z_1, t)$ for extreme risk seeker profile starts to decrease because the optimal decision-making considering the SHM system is converging towards the decision made using prior distribution (that does not have an SHM system installed).

3. As time passes, the gap value increases. We also observe a shift in the peaks of $C_{save}(z_1, t)$ towards the lower gap length as the risk aversion increases (or towards the higher gap length as intensity of risk-seeking increases). This is because the increase in risk-aversion of the decision-maker decreases the threshold of the gap beyond which it is obvious to her/him that the gate is damaged and the SHM system does not offer much benefit.
4. Figure 15 illustrates the cost classifier for various risk profiles that differentiates between a feasible and non-feasible SHM system considering the net benefit over the entire lifespan of the structure. It answers the following question: *given an SHM system design, the base consequence cost of making maintenance decisions, the risk profiles of the decision-maker, what is the range of SHM system cost for it to be feasible over the lifespan of its usage?* Every point on the plot gives a coordinate for the cost combination ($C(z)$, $C_{M\&O}(z)$). An SHM system with a cost combination of ($C(z)$, $C_{M\&O}(z)$) is feasible if it yields $\lambda_{LC} \geq 1$ or $\text{EVoI}_{\text{LC}} \geq 0$. A straight line classifier illustrated in different colors for various risk profiles is the locus of the cost coordinates ($C(z)$, $C_{M\&O}(z)$) for which $\lambda_{LC} = 1$ or $\text{EVoI}_{\text{LC}} = 0$. An SHM system with the cost coordinate ($C(z)$, $C_{M\&O}(z)$) belonging to the region below and including the classifier is a feasible design and leads to net cost saving over the lifespan of its usage. The converse holds for the region above the classifier. Finally, we observe that as the intensity of risk-aversion behavior increases, the flexibility to choose an SHM system decreases. This is because a risk-averse decision-maker makes more conservative and expensive decisions. For an SHM system to be feasible in the scenario where maintenance decisions are expensive, it must cost less.

Considering SHM system with 10 random sensors: Design z_2. The design z_2 illustrated in Figure 16 is obtained by randomly selecting 10 sensors using Latin Hypercube Sampling (LHS) that is subjected to a space-filling property.

1. Figure 17 illustrates the inferred posterior gap-degradation model for the design z_2. We observe that the variability in the posterior distribution of the gap length obtained using the design z_2 at every time instance is smaller relative to the variability observed in the gap length in prior distribution shown in Figure 10 but larger relative to the posterior gap-degradation model obtained using optimal design z_1 as shown in Figure 13. This is not surprising because the optimal sensors were designed to yield posterior with the maximum gain in information relative to the information contained in the prior. Just as seen in the design z_1, Figure 18 illustrates similar properties in $C_{save}(z_2, t)$ for various risk profile. The plots of $C_{save}(z_2, t)$ have similar properties of Figure 14 as discussed in *Considering SHM system with 10 optimally designed sensors: Design* z_1.

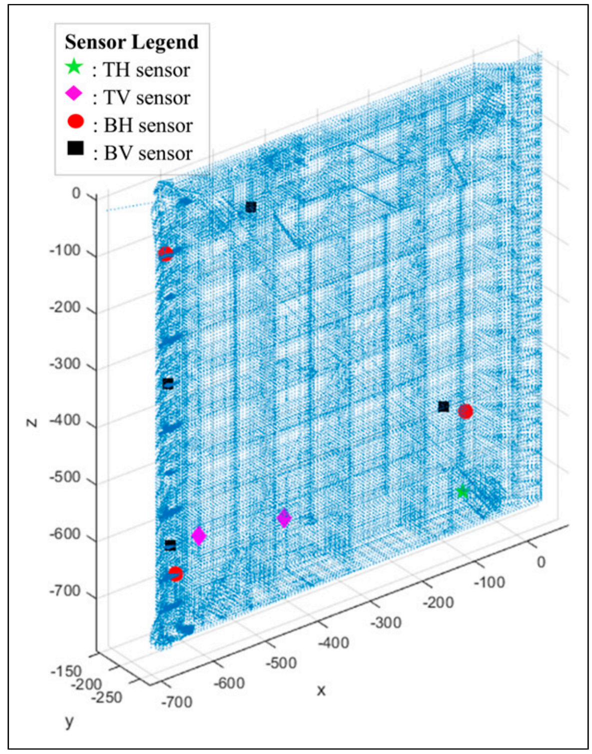

Figure 16. Design z_2: sensor network design.

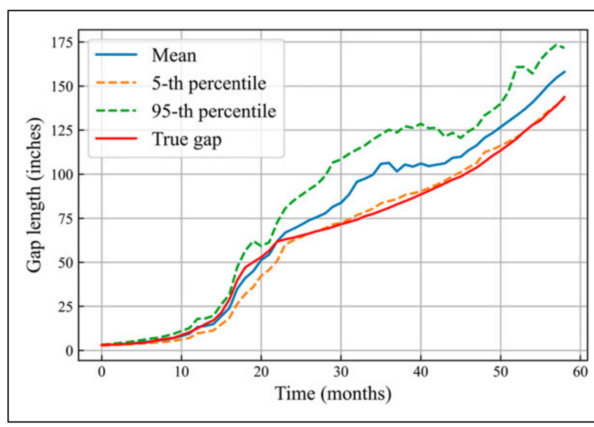

Figure 17. Design z_2: posterior gap-degradation model.

2. Figure 19 illustrates the cost classifier for the design z_2 considering the lifecycle of the structure. Design z_2 being a random design with merely 10 sensors combined with the fact that the cost functions corresponding to other relatively risk-averse behavior lead to conservative and expensive decisions budgetarily limits the choices of feasible SHM system z_2 as compared to the optimal design z_1.

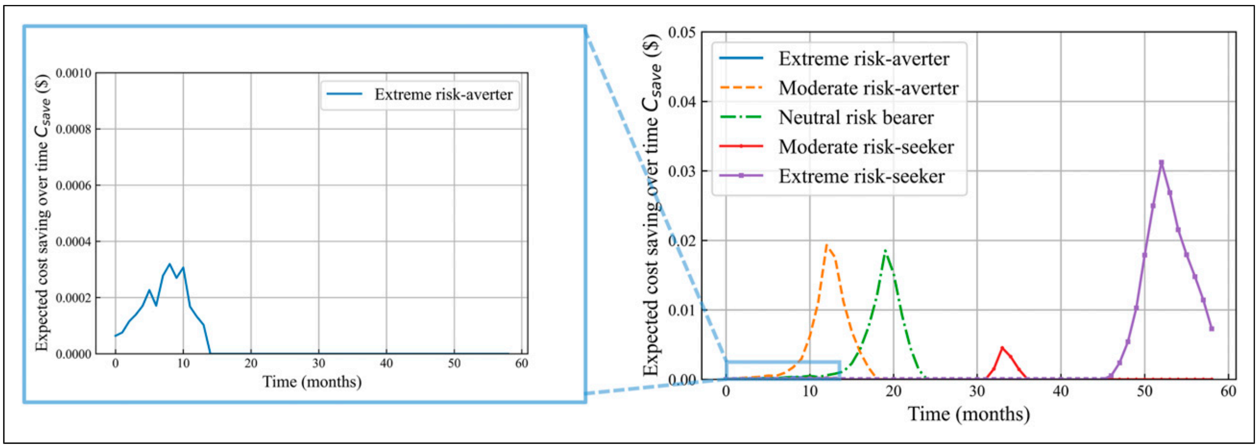

Figure 18. Design z_2: Cost saving ($C_{save}(z_2, t)$) over time as a consequence of making better decisions.

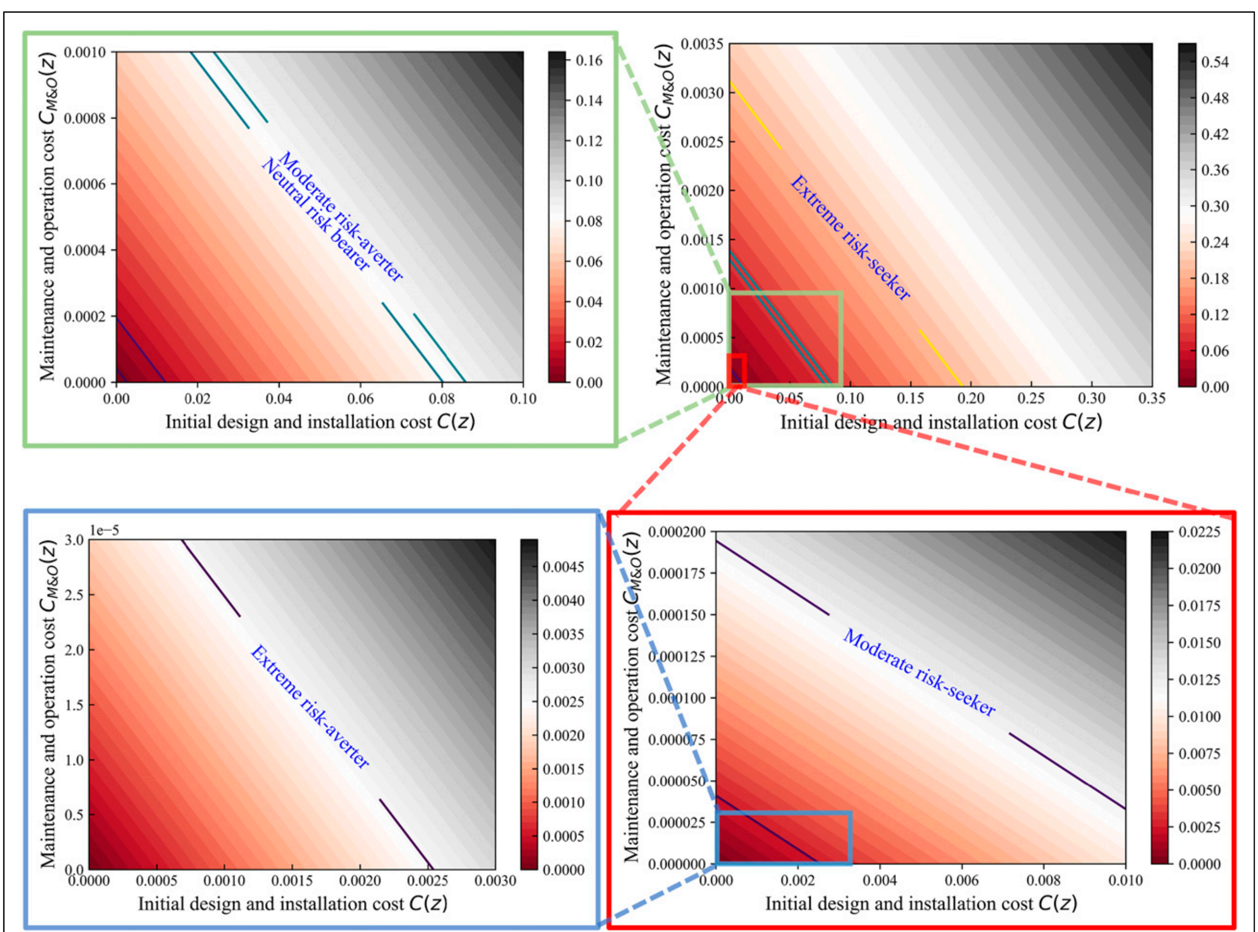

Figure 19. Design z_2: cost classifiers for different risk profiles.

This is evident from the fact that for a given risk profile, the classifier for the design z_2 shifts below the classifier for the design z_1. Hence, a random design with 10 sensors underperforms as compared to an optimal design with 10 sensors (as expected). However, we may anticipate competing results (relative to optimal KL divergence–based design) if we add more sensors (since the design is random). We consider such a design in the next section.

Considering SHM system with 20 random sensors: Design z_3. The design z_3 shown in Figure 20 is obtained by randomly selecting 20 sensors using Latin Hypercube Sampling (LHS).

1. As seen in Figure 21, design z_3 leads to better inference of gap length as compared to the design z_2. Figure 22 illustrates similar properties in $C_{save}(z_3, t)$ for various risk profile as observed for the design z_1 and z_2.
2. We observe that the resulting $C_{save}(z_3, t)$ is better than the results obtained for the design z_2 illustrated in Figure 18. This is not surprising since we have double the number of sensors and hence have superior edge due to additional data.
3. Figure 23 illustrates the cost classifier for design z_3 considering the lifecycle of the structure. Just like the designs z_1 and z_2, we observe that SHM strategy z_3 is feasible for all the risk profiles. Although design z_3 has 20 sensors, it leads to almost similar performance (or slight under performance) as the design z_1 for the following two reasons:
   - Design z_1 is optimal (optimized using KL divergence of the posterior relative to the prior gap-length distribution);
   - Design z_1 has half the number of sensors as design z_3. Therefore, it has lower intrinsic cost, or $C(z_1) < C(z_3)$.

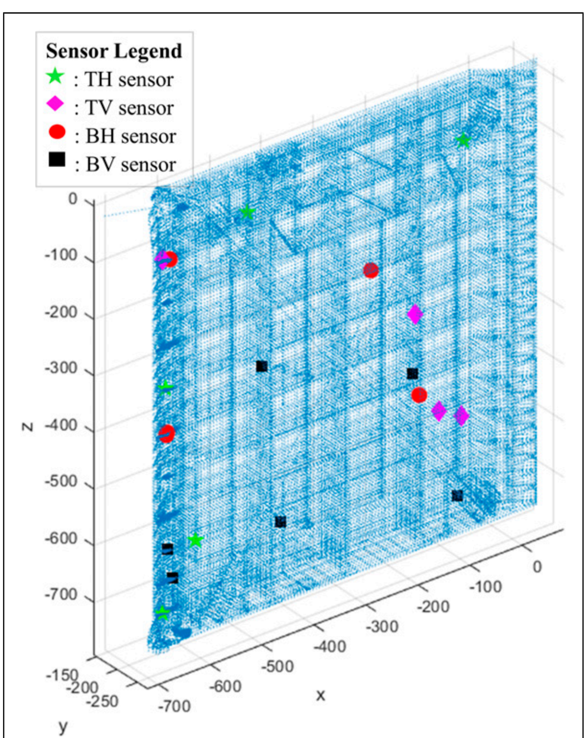

Figure 20. Design z_3: sensor network design.

Key observations. Table 3 presents the cost-savings over the lifecycle of the structure corresponding to various SHM designs for all five risk profiles.

Based on Table 3, following are the key observations:

1. Among the three SHM designs considered, KL divergence optimized design z_1 is the most valuable SHM system. Design z_1 outperforms the design z_2 for all the risk profiles, and it outperforms the design z_3 for risk-neutral, moderate risk-seeker, and extreme risk-seeker profiles. By outperformance, we mean that for a fixed $C(z)$ and $C_{M\&O}(z)$, the design z_1 maximizes the expected reward to investment risk ratio. This shows the importance of well-designed SHM systems.
2. The value of SHM not only depends on its design, but it is also impacted by the behavioral biases involved in the decision-making. As the intensity of risk-aversion behavior increases, the flexibility to choose a feasible SHM system decreases (as seen from the classifier of extreme risk-averter for all three designs). This is because a risk-averse decision-maker makes more conservative and expensive decisions. For an SHM system to be feasible in the scenario where maintenance decisions are expensive,

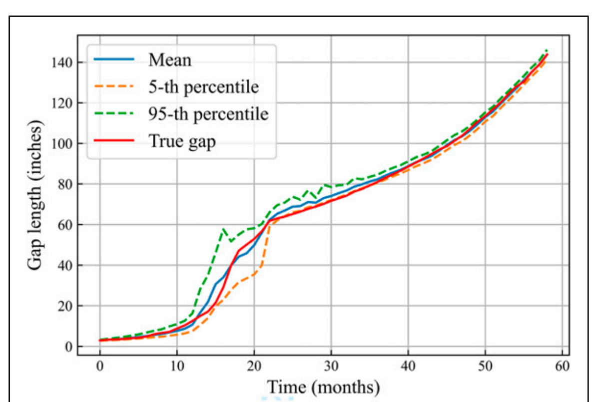

Figure 21. Design z_3: posterior gap-degradation model.

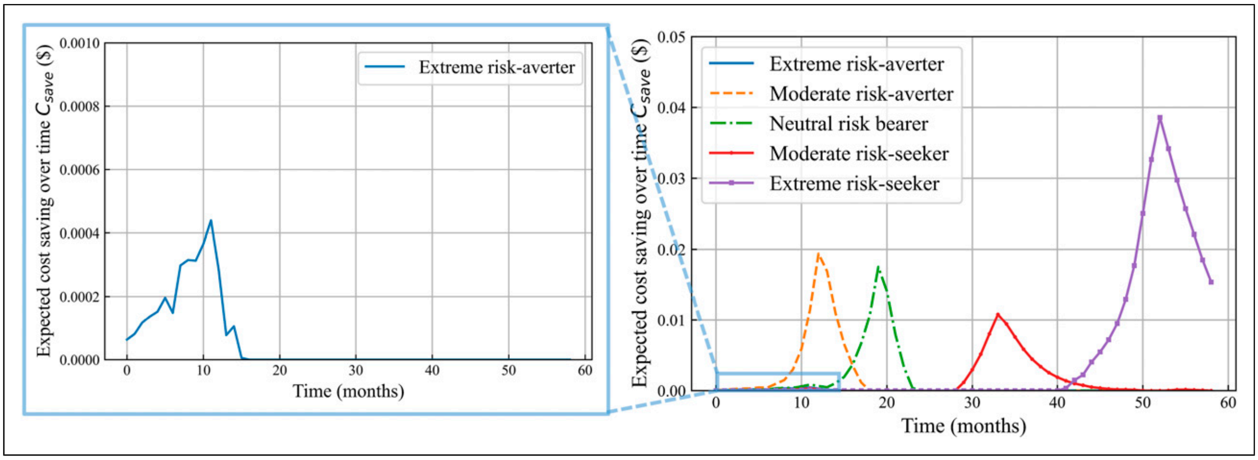

Figure 22. Design z_3: cost saving ($C_{save}(z_3, t)$) over time as a consequence of making better decisions.

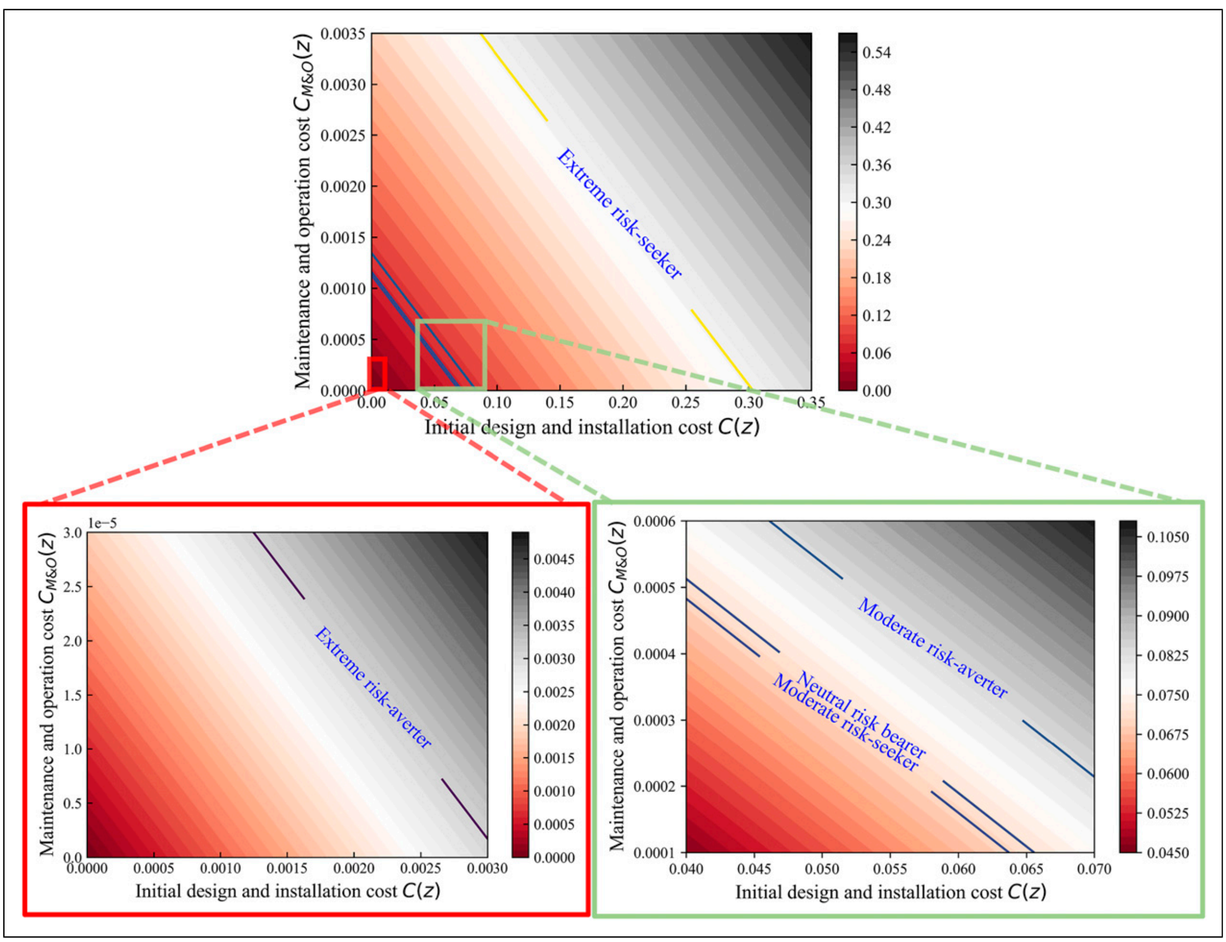

Figure 23. Design z_3: cost classifiers for different risk profiles.

Table 3. Expected reward or cost-saved over the lifecycle.

Risk profiles	Expected reward over the lifecycle C_{saveLC}		
	Design z_1	Design z_2	Design z_3
Extreme risk-averter	0.0025	0.0025	0.0031
Moderate risk-averter	0.0863	0.0859	0.0832
Neutral risk-bearer	0.0806	0.0802	0.0717
Moderate risk-seeker	0.0940	0.0120	0.0699
Extreme risk-seeker	0.3070	0.1930	0.3033

it must cost less. This budgetary constraint restricts our options for a feasible SHM system design.

Conclusion

This article utilizes pre-posterior decision analysis to evaluate the VoI acquired using an SHM system. An SHM system provides additional information from which the state of the structure can be inferred. However, it costs to design, install, maintain and operate an SHM system. Therefore, it is warranted to quantify the net benefit an SHM strategy would yield while in its design phase. Since there is no availability of real data (as the SHM system is in its design phase and has not yet been installed), the likelihood of observing the measurement data (like strain gauge reading) is assumed/modeled by the decision-maker based on the past observation, physics-based simulation, or a reasonable assumption. Since the SHM system is not installed yet and the measurements are not available, to evaluate the net benefit of an SHM system, it is necessary to consider all possible outcomes and uncertainties. This analysis is called *pre-posterior decision analysis*.

The benefit of an SHM system relative to the case where no new data is acquired is studied at 2 levels. Firstly, the benefit of an SHM strategy is investigated for an instance of decision-making at a fixed time occurrence. The SHM strategy leads to a greater than one *expected reward* to *investment risk* ratio for an instance of decision-making considering a fixed time occurrence. For this scenario, an initial one-time cost of designing and installing an SHM system is considered as the *investment risk* (money spent). Secondly, the net advantage of using an SHM system over the life cycle of the structure is evaluated. In this scenario, in addition to the initial design and installation costs, we also consider the cost of maintaining and operating an SHM system over the lifespan of the structure. A feasible SHM system is expected to yield relative cost-savings over the lifespan of the structure as compared to making maintenance decisions not backed by continuously updated data. An economically beneficial SHM system is the one that yields a greater than or equal to unity *expected reward* to *investment risk* ratio. This study proposes an *expected reward* to *investment risk* ratio as an alternative quantity to EVoI traditionally used in pre-posterior decision analysis. Unlike EVoI that is defined as the difference between *expected reward* and *investment risk* and hence a differential measure, an *expected reward* to *investment risk* ratio gives a normalized benefit of an SHM system.

It is observed that the benefit of an SHM system depends on two key factors: its design and risk profile of the decision-maker or equivalently, the risk intensity (just like a factor of safety) that an organization wants to impose on their base cost estimates. The approach is exemplified in a case study involving SHM and maintenance of a miter gate, part of a lock system enabling navigation of inland waterways.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: Funding for this work was provided by the US Army Corps of Engineers through the US Army Engineer Research and Development Center Research Cooperative Agreement, W912HZ-17-2-0024.

ORCID iD

Michael D Todd https://orcid.org/0000-0002-4492-5887

References

1. Farrar CR and Worden K. *Structural health monitoring: a machine learning perspective*. New Mexico, USA; John Wiley & Sons, 2012.
2. Vega MA, Hu Z and Todd MD. Optimal maintenance decisions for deteriorating components in miter gates subject to uncertainty in the condition rating protocol. *Reliab Eng Syst Safe* 2020; 204: 107147.
3. Howard R. Information value theory. *IEEE Trans Syst Sci cybernetics* 1966; 2(1): 22–26..
4. Raiffa H and Schlaifer R. *Applied Statistical Decision Theory.* Cambridge: Wiley, 1961.
5. Pozzi M and Der Kiureghian A. Assessing the value of information for long-term structural health monitoring. In: Health monitoring of structural and biological systems 2011, San Diego, CA: International Society for Optics and Photonics, 2011, p.79842W, vol. 7984.
6. Thöns S. On the value of monitoring information for the structural integrity and risk management. *Computer-Aided Civil Infrastructure Eng* 2018; 33(1): 79–94.
7. Konakli K and Faber MH. Value of information analysis in structural safety. In: Vulnerability, uncertainty, and risk: quantification, mitigation, and management, Liverpool, UK: ASCE, 2014, pp.1605–1614.
8. Zonta D, Glisic B and Adriaenssens S. Value of information: impact of monitoring on decision-making. *Struct Control Health Monit* 2014; 21(7): 1043–1056.
9. Klerk WJ, Schweckendiek T, den Heijer F, et al. Value of information of structural health monitoring in asset management of flood defences. *Infrastructures* 2019; 4(3): 56.

10. Straub D. Value of information analysis with structural reliability methods. *Struct Saf* 2014; 49: 75–85.
11. Von Neumann J, Morgenstern O and Kuhn HW. *Theory of games and economic behavior. commemorative edition*. Princeton, NJ: Princeton University Press, 2007.
12. Parmigiani G and Inoue L. *Decision theory: Principles and approaches*. Baltimore, USA: John Wiley & Sons, 2009, vol. 812.
13. Chadha M, Ramancha MK, Vega MA, et al. The Role of Risk Profile in State Determination of Structures. In Proceedings 10th International Conference on Structural Health Monitoring of Intelligent Infrastructure (SHMII-10), Porto, Portugal, June 30-July 2, 2021
14. Daniel RA. Mitre gates in some recent lock projects in the Netherlands (Stemmtore in einigen neuen Schleusenanlagen in den Niederlanden). *Stahlbau* 2000; 69(12): 952–964.
15. Richardson GC. Navigation locks: navigation lock gates and valves. *J Waterways Harbors Division* 1964; 90(1): 79–102.
16. Vega MA, Hu Z and Todd MD. Optimal maintenance decisions for deteriorating quoin blocks in miter gates subject to uncertainty in the condition rating protocol. *Reliability Eng Syst Saf* 2020; 204: 107147.
17. Parno M, O'Connor D and Smith M. High dimensional inference for the structural health monitoring of lock gates 2018. arXiv preprint.
18. Schwieterman JP, Field S, Fischer L, et al. *An analysis of the economic effects of terminating operations at the chicago river controlling works and o'brien locks on the chicago area waterway system*. Chicago, IL: DePaul University, 2010.
19. Foltz SD. Investigation of mechanical breakdowns leading to lock closures. Technical Report. Champaign, IL: ERDC-CERL CHAMPAIGN United States, 2017.
20. Eick BA, Treece ZR, Spencer BF Jr, et al. Automated damage detection in miter gates of navigation locks. *Struct Control Health Monit* 2018; 25(1): e2053.
21. Yang Y, Chadha M, Hu Z, et al. A probabilistic optimal sensor design approach for structural health monitoring using risk-weighed F-divergence. *Mech Sys Signal Process* 2021; 161: 107920.
22. Kahneman D and Tversky A. Prospect theory: an analysis of decision under risk. *Econometrica* 1979; 47(2): 263–291.
23. Thorp EO. *Beat the Dealer: a winning strategy for the game of twenty one*. New York: Vintage, 1966, vol. 310.
24. Thorp EO. The kelly criterion in blackjack sports betting, and the stock market. In *The Kelly capital growth investment criterion: theory and practice*. Singapore: World Scientific, 2011, pp.789–832.
25. Thorp EO. *A man for all markets- From Las Vegas to wall street, How I beat the dealer and the market*. New York: Random House, 2017.
26. Livermore JL. *How to trade in stocks: The livermore formula for combining time element and price*. Laurus-Lexecon Kft., 1940.
27. Murphy JJ. *Technical analysis of the financial markets: A comprehensive guide to trading methods and applications*. USA: Penguin, 1999.
28. Taleb NN. *The Black Swan: The Impact of the Highly Improbable*, vol. 2. New York: Random House, 2007.
29. Eick BA, Smith MD and Fillmore TB. Feasibility of retrofitting existing miter-type lock gates with discontinuous contact blocks. *J Struct Integrity Maintenance* 2019; 4(4): 179–194.
30. Vega MA and Todd MD. A variational bayesian neural network for structural health monitoring and cost-informed decision-making in miter gates. *Struct Health Monit*, 2020; doi: 10.1177/1475921720904543.
31. Ramancha M., Astroza R., Conte J. P., et al. Bayesian nonlinear finite element model updating of a full-scale bridge-column using sequential monte carlo. In: Proceedings of the 38th IMAC, a conference and exposition on structural dynamics 2020. Houston, USA. Springer, 2021, Vol. 3, pp. 389–397.