Arithmetic with ratios of magnitudes.

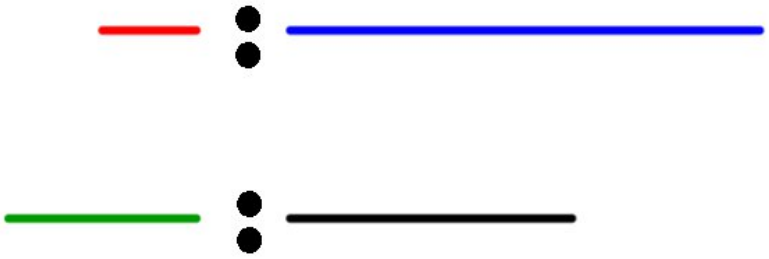
Some time ago I wrote an article which explained in great detail the theory of fraction arithmetic using only ratios. In this article I will explain these facts as if I am explaining to an elementary school student.

Ratio Addition:

To add the following ratios, we can proceed in either of two ways:

1. We can convert the red-blue ratio to an equivalent ratio that has the same consequent as the green-black ratio.
2. We can convert the green-black ratio to an equivalent ratio that has the same consequent as the red-blue ratio.

I will show you how to do (1).



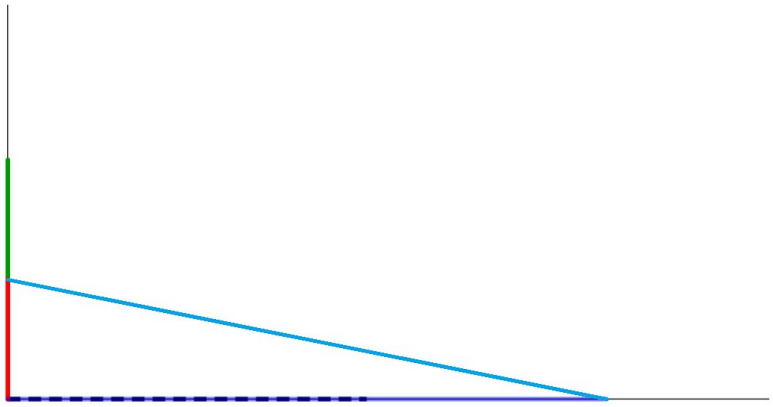
Step 1.

Using your compass or divider, plot the antecedent line segments (parts before colon) on a vertical line and the consequent line segments (parts after colon) on a horizontal line segment like this:

I have puposely drawn the black line segment in strips so you can see where it begins and ends.

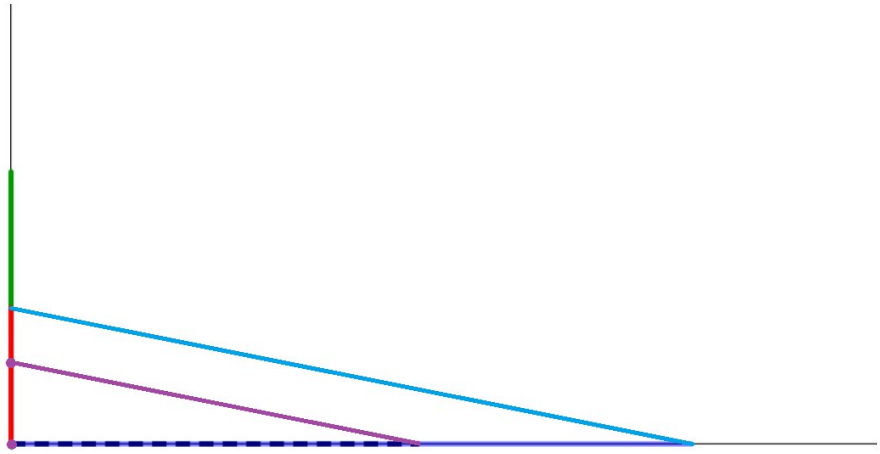
Step 2.

Join the endpoints of the red and blue line segments as follows:

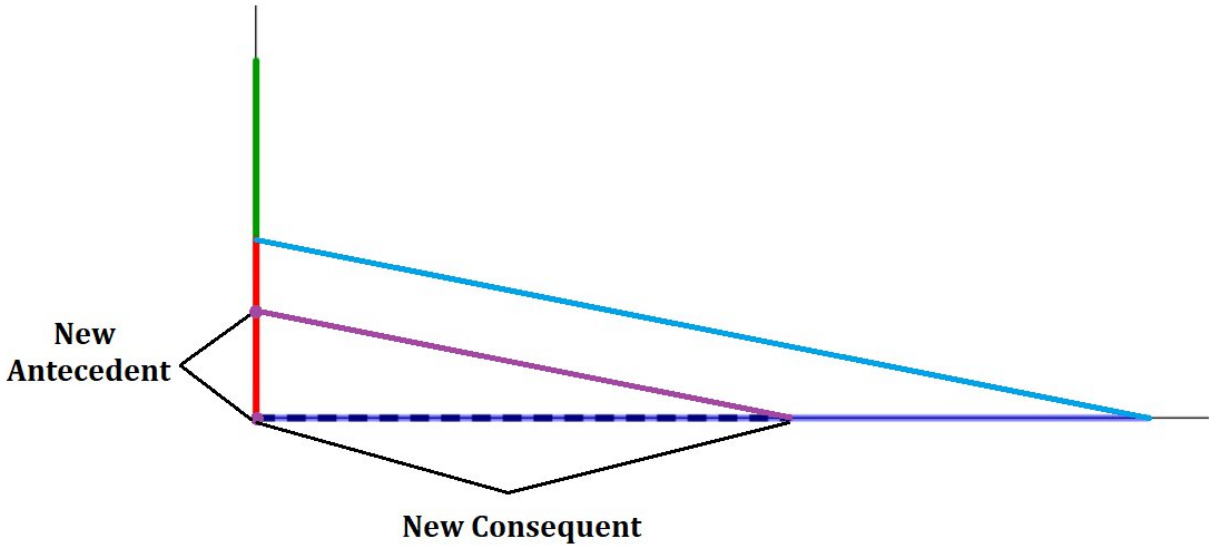


Step 3.

Using only compass and straight edge, construct a line segment (magenta) parallel to the light blue line segment in (2) so that it passes through the end point of the striped black line as follows:



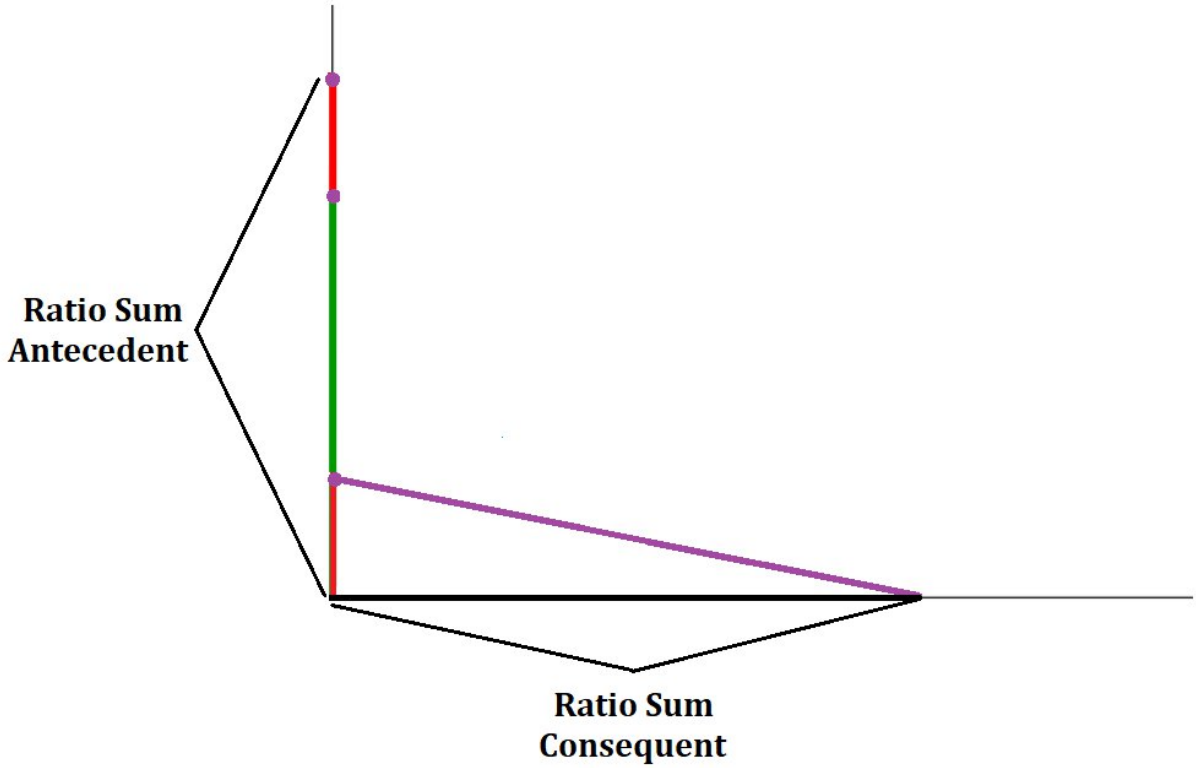
The distance from where the magenta line segment meets the vertical line to the vertex (where original vertical and horizontal lines meet) is the new antecedent line segment for the original red-blue ratio which now has the same consequent as the green-black ratio.



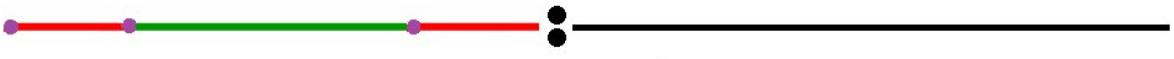
These steps are required before any ratio arithmetic can be performed.

Step 4.

Using your compass or divider, join the new antecedent from Step 3 to the green line segment. This new lines segment so formed is the Ratio Sum or Addition antecedent and the Ratio Sum or Addition consequent is the black line segment from the green-black ratio.



The Ratio Addition is shown below:



So, as you can see, we added two ratios geometrically without using any numbers whatsoever.

Do you think it would have mattered if one ratio represented a circle's circumference and the other ratio its diameter?

With the help of your teacher, show how you can perform subtraction (difference), division (quotient) and multiplication (product) using ratios of line segments, a compass or divider and a straight edge only. Your teacher can refer to my article for directions.

Exercises:

1. Let one ratio be a circle's circumference and the other ratio the diameter of the same circle. Find the quotient *circumference : diameter*.
2. Let one ratio be a square's diagonal and the other ratio the side of the samer square. Find the product *square diagonal : square side*.
3. Do you think it would make any difference if we used different circles and squares in (1) and (2) ?
4. Draw ratios with line segments in the following proportions: (a) 2:3 and (b) 1:5

Find the difference, sum, quotient and product using the geometric method described above and then verify the results using algebraic fractional arithmetic.

I am the great John Gabriel, discoverer of the New Calculus, the first rigorous formulation of calculus in human history. More advanced alien civilistions may already know of it. Learn also how I exposed the lie that mainstream calculus was made rigorous.