Department of Education
Region X - Northern Mindanao
DIVISION OF CAGAYAN DE ORO
Fr. William F. Masterson, SJ Avenue, Upper Balulang, Cagayan de Oro City

Learning Activity Sheets in General Mathematics

SHARED OPTIONS

Senior High Alternative Responsive Education Delivery

Competence.Dedication.Optimism

Preface

It has been elaborated in research and literature that the highest performing education systems are those that combine quality with equity. Quality education in the Department of Education (DepEd) is ensured by the learning standards in content and performance laid in the curriculum guide. Equity in education means that personal or social circumstances such as gender, ethnic origin or family background, are not obstacles to achieving educational potential and that inclusively, all individuals reach at least a basic minimum level of skills.

In these education systems, the vast majority of learners have the opportunity to attain high-level skills, regardless of their own personal and socio-economic circumstances. This corresponds to the aim of DepEd Cagayan de Oro City that no learner is left in the progression of learning. Through DepEd's flexible learning options (FLO), learners who have sought to continue their learning can still pursue in the Open High School Program (OHSP) or in the Alternative Learning System (ALS).

One of the most efficient educational strategies carried out by DepEd Cagayan de Oro City at the present is the investment in FLO all the way up to senior high school. Hence, Senior High School Alternative Responsive Education Delivery (SHARED) Options is operationalized as a brainchild of the Schools Division Superintendent, Jonathan S. Dela Peña, PhD.

Two secondary schools, Bulua National High School and Lapasan National High School, and two government facilities, Bureau of Jail Management and Penology-Cagayan de Oro City Jail and Department of Health-Treatment and Rehabilitation Center-Cagayan de Oro City, are implementing the SHARED Options.

To keep up with the student-centeredness of the K to 12 Basic Education Curriculum, SHARED Options facilitators are adopting the tenets of Dynamic Learning Program (DLP) that encourages responsible and accountable learning.

This compilation of DLP learning activity sheets is an instrument to achieve quality and equity in educating our learners in the second wind. This is a green light for SHARED Options and the DLP learning activity sheets will continually improve over the years.

Ray Butch D. Mahinay, PhD
Jean S. Macasero, PhD

Acknowledgment

The operation of the Senior High School Alternative Responsive Education Delivery (SHARED) Options took off with confidence that learners with limited opportunities to senior high school education can still pursue and complete it. With a pool of competent, dedicated, and optimistic Dynamic Learning Program (DLP) writers, validators, and consultants, the SHARED Options is in full swing.

Gratitude is due to the following:

- Schools Division Superintendent, Jonathan S. Dela Peña, PhD, Assistant Schools Division Superintendent Alicia E. Anghay, PhD, for authoring and buoying up this initiative to the fullest;
- CID Chief Lorebina C. Carrasco, and SGOD Chief Rosalio R. Vitorillo, for the consistent support to all activities in the SHARED Options;
- School principals and senior high school teachers from Bulua NHS, Lapasan NHS, Puerto NHS and Lumbia NHS, for the legwork that SHARED Options is always in vigor;
- Stakeholders who partnered in the launching and operation of SHARED Options, specifically to the Bureau of Jail Management and Penology-Cagayan de Oro City Jail and the Department of Health-Treatment and Rehabilitation Center-Cagayan de Oro City;

- Writers and validators of the DLP learning activity sheets, to which this compilation is heavily attributable to, for their expertise and time spent in the workshops;
- Alternative Learning System implementers, for the technical assistance given to the sessions; and
- To all who in one way or another have contributed to the undertakings of SHARED Options.

Mabuhay ang mga mag-aaral! Ito ay para sa kanila, para sa bayan!

Ray Butch D. Mahinay, PhD
Jean S. Macasero, PhD

Writers:

1. Concepcion L. Eduria — Tablon NHS
2. Evangeline M. Pailmao — Bulua NHS
3. Chona J. Valendez — Lapasan NHS

Editors/Validators:

1. Concepcion Eduria,PhD — Tablon NHS
2. Ray O. Maghuyop — EPS- Math, CID-Division Office

Reproduction (LRMDS)

1. Gemma P. Pajayon - Project Development Officer II
2. Lanie M. Signo - Librarian II

JEAN S. MACASERO, PhD
EPS- SHS Coordinator

RAY BUTCH D. MAHINAY, PhD
MT-1 ALS Coordinator

LOREBINA C. CARRASCO
Chief, CID

ALICIA E. ANGHAY, PhD
Assistant Schools Division Superintendent

JONATHAN S. DELA PEŇA, PhD., CESO V
Schools Division Superintendent

ACTIVITY NUMBER	LEARNING ACTIVITY TITLE	DATE	SCORE	ITEM
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2	Representation of Piece-Wise Functions			1
3	Evaluating Functions			2
4	Addition of Functions			2
5	Subtraction of Functions			2
6	Multiplication of Functions			2
7	Division of Functions			2
8	Composition of Functions			2
9	Solving Problem Involving Function			1
10	Representation of Rational Functional in Real Life Situation			1
11	Rational Functions, Rational Equation and Rational Inequality			5
12	Solving Rational Equations			2
13	Solving Rational Inequalities			2
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28	Represents Real Life Situation Involving Exponential Functions			3
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33	Exponential Functions Through Equations			3
34	Intercepts, Zeros and Asymptotes of Exponential Functions			2
35	Graph of Exponential Function (A)			1
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39	Solving Investment Problem Involving Exponential Functions			1
40	Introduction of Logarithms			6
41	Representation Real Life Situation Involving Logarithmic Functions			2
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43	Basic Properties of Logarithms			4
44	Laws of Logarithms			3
45	Logarithmic Equations			2
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48	Domain and Range of Logarithmic Functions			1
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50	Graphs of Logarithmic Functions			1
51	Solving Problems Involving Logarithmic Functions			1

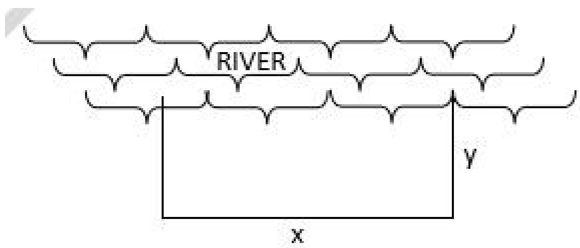
Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : REPRESENTATION OF FUNCTION		
Learning Competency : Represents real-life situations using functions. (M11GM-Ia-1)		
References : General Mathematics Teaching Guide,pp. 6-8		LAS No.: 1

CONCEPT NOTES

Function can be used to represent a model in real-life situations.

Example:

One hundred meters of fencing is available to enclose a rectangular area next to a river (see figure). Give a function A that can represent the area that can be enclosed, in terms of x.



Solution. The area of the rectangular enclosure is A = xy. We will write this as a function of x. Since only 100 m of fencing is available, then x + 2y = 100 or y = (100 – x)/2 = 50 – 0.5x. Thus, A(x) = x(50 – 0.5x) = 50x – 0.5x2.

Exercise: Answer the following problem:

Give a function C that can represent the cost of buying x meals, if one meal cost P40.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : REPRESENTATION OF FUNCTION (B)		
Learning Competency : Represents real-life situations using piecewise function (M11GM-Ia-1)		
References : General Mathematics Teaching Guide, pp 9-10		LAS No.: 2

CONCEPT NOTES

A piecewise function or a compound function defined by multiple subfunctions where each subfunction applies to a certain interval of the main function's domain.

Some situations can only be described by more than one formula, depending on the value of the independent variable.

Example:

A user is charged P300 monthly for a particular mobile plan, which includes 100 free text messages. Messages in excess of 100 are charged P1 each. Represent the monthly cost for text messaging using the function t(m), where m is the number of messages sent in a month.

Solution:

The cost of text messaging can be expressed by the piecewise function:

$$t(m) = \begin{cases} 300 & , if\ 0 < m \leq 100 \\ 300 + m, & if\ m > 100 \end{cases}$$

Exercises: Answer the following problem:

A jeepney ride costs P8.00 for the first 4 kilometers, and each additional integer kilometer adds P1.50 to the fare. Use a piecewise function to represent the jeepney fare in terms of the distance (d) in kilometers.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Evaluating Function.		
Learning Competency : Evaluates a function. (M11GM-Ia-2)		
References : General Mathematics Teaching Guide, pp 11-13		LAS No.: 3

CONCEPT NOTES

Evaluating a function means replacing the variable in the function, in this case x, with a value from the function's domain and computing for the result. To denote that we are evaluating f at a for some a in the domain of f, we write f(a).

Example:

Evaluate the function $f(x) = 2x + 1$ at $x = 1$

Solution:

$f(x) = 2x+1$
$f(1) = 2(1) + 1$
$f(1) = 2 + 1$
$f(1) = 3$

Exercises: Evaluate the following functions at x=3.

1. $f(x) = x-3$

2. $f(x)= 2x -7$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Operations and Functions		
Learning Competency : Perform addition on functions. (M11GM-Ia-3)		
References : General Mathematics Teaching Guide, pp 14-15		LAS No.: 4

CONCEPT NOTEs

Let f and g be any two functions. Their sum, denoted by f+g, is the function denoted by (f+g) (x) = f(x) + g(x).

Example1: Given : f(x) = x + 4 and g(x) = 2x + 1, find (f +g) (x).

Solution: (f+ g) (x) = f(x) + g(x)
= x + 4 + 2x + 1
= x + 2x +4 +1
= 3x + 5

Example 2: Given: f(x) = x – 3 and g (x) = 2x – 7, find (f+g) (x)

Solution: (f+g) (x) = f(x) + g(x)
= x – 3 + 2x – 7
= x + 2x -3 – 7
= 3x - 10

Exercises: Answer the following problems:

1. Given: f(x) = 3x -1 and g(x) = 2x + 3
2. Given : v(x) = x +7 and p(x) = x + 10

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Operations and Functions		
Learning Competency : Perform subtraction on functions. (M11GM-Ia-3)		
References : General Mathematics Teaching Guide, pp 14-15		LAS No.: 5

CONCEPT NOTES

Let f and g be any two functions. Their difference, denoted by f-g, is the function denoted by (f-g) (x) = f(x) - g(x).

Example 1: *Given* : f(x) = x -3 and g(x) = x + 4, find (f-g) (x).

Solution:

(f-g) (x) = f(x) – g(x)
= x -3 – (x + 4)
= x – 3 –x - 4
= x-x -3-4
= 0 -7
= -7

Example 2: *Given* : f(x) = 3x + 2 and g(x) = 4x -1, find (f-g)(x).

Solution:

(f-g) (x) = f(x) – g(x)
= 3x +2 – (4x – 1)
= 3x + 2 -4x +1
= 3x-4x +2 +1
= -x + 3

Exercises:

Find (f-g)(x), given the functions f and g:

1. f(x) = 3x + 4 and g (x) = 2x – 1
2. f(x) = x -1 and g(x) = x - 4

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Operations and Functions		
Learning Competency : Perform multiplication on functions. (M11GM-Ia-3)		
References : General Mathematics Teaching Guide, pp 15-17		LAS No.: 6

CONCEPT NOTES

Let f and g be any two functions. Their product, denoted by fg, is the function denoted by (fg) (x) = f(x) • g(x).

Example 1: Given: f(g) = x+ 1 and g(x) = x+ 2, find fg.

Solution:

(fg)(x) = f(x) • g(x)

= (x + 1)(x+2)

$=x^2 + 2x + x + 2$

$= x^2 + 3x + 2$

.

Example 2: Given : f(g) = x + 3 and g(x) = 2x – 1 , find fg.

Solution:

(fg)(x) = f(x) • g(x)

= (x + 3)(2x-1)

$=2x^2 - x + 6x - 3$

$= 2x^2 + 5x - 3$

Exercises:

Find (fg)(x), given the functions f and g:

1. f(x) = x + 4 and g (x) = x – 1
2. f(x) = 2x -1 and g(x) =3x - 4

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Operations and Functions		
Learning Competency : Perform division on functions. (M11GM-Ia-3)		
References : General Mathematics Teaching Guide, pp 17-18		LAS No.: 7

CONCEPT NOTES

Let f and g be any two functions. Their quotient, denoted by $\frac{f}{g}$, is the function denoted by $(\frac{f}{g})$ (x) = $\frac{f(x)}{g(x)}$, $g(x) \neq 0$

Example 1: Given f(x) = 3x + 2 and g(x) = x + 4, find $\frac{f}{g}$.

Solution:

$$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$$

$$= \frac{3x+2}{x+4}$$

Example 2: Given f(x) = 3x + 4 and g(x) = 2x - 1, find $\frac{f}{g}$.

Solution:

$$(\frac{f}{g})(x) = \frac{f(x)}{g(x)}$$

$$= \frac{3x+4}{2x-1}$$

Exercises:

Find $(\frac{f}{g})$(x), given the functions f and g:

1. f(x) = x + 4 and g (x) = x – 1
2. f(x) = 2x -1 and g(x) =3x – 4

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Operations and Functions		
Learning Competency : Performs composition of functions. (M11GM-Ia-3)		
References : General Mathematics Teaching Guide, pp 20-21		LAS No.: 8

CONCEPT NOTES

Let f and g be functions. The composite function denoted by (f∘ g) is defined by (f∘ g)(x) = (f(g(x)). The process of obtaining a composite function is called function composition.

Example: Given f(x) = x + 1 and g (x) = 2x- 4, find (f∘ g) and (g∘ f).

Solution:

a. Because (f∘ g)(x) means (f(g(x)), we must replace each occurrence of x in the function f by g(x).
   f(x) = x +1
   (f∘ g)(x) = (f(g(x))
   = (g(x)) + 1
   = (2x-4) +1
   = 2x -4 +1
   =2x -3
   Thus, (f∘ g)(x) = 2x -3

b. (g∘ f)(x) means (g(f(x)). Hence we must replace each occurrence of x in the function g by f(x).
   g (x) = 2x – 4
   (g∘ f)(x) = (g(f(x))
   = 2(f(x)) - 4
   = 2(x+ 1) - 4
   = 2x +2 - 4
   = 2x -2
   Thus, (f∘ g)(x) = 2x -2

Exercise: Find (f∘ g) and (g∘ f).
Given f(x) = 3x + 1 and g (x) = x+ 7

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Problem Solving Involving Functions		
Learning Competency : Solves problems involving functions. (M11GM-Ia-4)		
References : General Mathematics Teaching Guide,		LAS No.: 9

CONCEPT NOTES

Functions can be used to model a problem in real-life situation.

Example:

Your Business Math teacher wants your class to run a T-Shirt Printing Business and has agreed to provide up to P 50,000 of his own money to help the class get started. To determine the profitability of the business, the class needs to know how much it will cost to produce the printed shirts and how many can the class expect to sell for a given price. The class has identified the following costs:

Heat press machine	P 24, 480
T-shirts in bulk	P 120 each
Transfers to press onto each shirt	P 40 each

The function of the cost C of producing x printed t-shirts as cost function, $C(x) = 160x + 24\ 480$. How much does it cost to produce 500 printed t-shirts?

Solution:

$C(x) = 160x + 24\ 480$

$C(500) = 160\ (500) + 24\ 480$

$= 75{,}000 + 24\ 480$

$= 99,\ 480$

Thus, the cost to produce 500 printed t-shirts is P 99,480.

Exercises: Solve the following problem:

Josh's group was selling suckers for fundraiser. They sold them for P25 a piece but had to pay shipping of 30. The function for their sucker sales is $P(s) = 25s - 30$. If they sold 150 suckers, how much money would they profit?

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Representing Real-Life Situation Using Rational Function		
Learning Competency : Represents real-life situations using rational functions. (M11GM-Ib-1)		
References : General Mathematics Teaching Guide, pp 24-25		LAS No.: 10

CONCEPT NOTES

-A polynomial function p of degree n is a function that can be written in the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots a_1 x + a_0$ where $a_0, a_1, \ldots a_n \epsilon R$, $a_n \neq 0$ and n is a positive integer. Each addend of the sum is a term of the polynomial function. The constants $a_0, a_1, a_2, \ldots a_n$ are the coefficients. The leading coefficient is a_n. . The leading term is $a_n x^n$, and the constant term is a_0.

-A rational function is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where p_x and q_x are polynomial functions and q_x is not the zero function (i.e., $q_x \neq 0$).

Example 1. An object is to travel a distance of 10 meters. Express velocity v as a function of travel time t, in seconds.

Solution:

The following table of values show v for various values of t.

t (seconds)	1	2	4	5	10
v (meters per second)	10	5	2.5	2	1

The function $v(t) = \frac{10}{t}$ can represent v as a function of t.

Exercise: Solved the following problem:

The average cost per unit C(x), in dollars, to produce **x** units of toy cars is given by $S = \frac{8000}{x-50}$. What is the approximate cost per unit when 1250 toy cars are produced?

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Rational Functions, Rational Equation and Rational Inequality		
Learning Competency : Distinguishes rational function, rational equation, and rational inequality. (M11GM-Ib-2)		
References : General Mathematics Teaching Guide, pp 28-29		LAS No.: 11

CONCEPT NOTES

A rational expression is an expression that can be written as a ratio of two polynomials.

Examples of rational expressions are $\frac{2}{x}$, $\frac{x^2+2x+1}{x+1}$,, $\frac{5}{x-3}$.

The definitions of rational equation,inequalities and function are shown below.

	Rational Equation	Rational Inequality	Rational Function
Definition	An equation involving rational expressions.	An inequality involving rational expressions.	A function of the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions and q(x)≠0.
Examples:	$\frac{2}{x} - \frac{1}{2} = \frac{2}{3}$	$\frac{5}{x+1} < 1$	$f(x) = \frac{x+1}{2x+3}$

Exercises:

Determine whether the given is a rational function, a rational equation, a rational inequality or none of these.

1) $\frac{2x}{x+1} + \frac{1}{4} = \frac{2}{3}$

2) $\frac{3x}{x^2+2} - 1 \geq 0$

3) $f(x) = \frac{3x+5}{6x+3}$

4) $\frac{x}{1} + \frac{9x}{5} = \frac{2}{3}$

5) $5 > \frac{x+5}{x^2+2}$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Rational Equations		
Learning Competency : Solves rational equations. (M11GM-Ib-3)		
References : General Mathematics Teaching Guide, pp 31-33		LAS No.: 12

CONCEPT NOTES

To solve a rational equation:

(a) Eliminate denominators by multiplying each term of the equation by the least common denominator.

(b) Note that eliminating denominators may introduce extraneous solutions. Check the solutions of the transformed equations with the original equation.

Example 1. Solve for x. $\frac{2}{x} - \frac{3}{2x} = \frac{1}{5}$

Solution: The LCD of all the denominators is 10x. Multiply both sides of the equation by 10x and solve the resolving equation.

$$10x\left(\frac{2}{x}\right) - 10x\left(\frac{3}{2x}\right) = 10x\left(\frac{1}{5}\right)$$

$$20 - 15 = 2x$$

$$5 = 2x$$

$$\frac{5}{2} = x$$

Exercises:

Solve for x.

1) $\frac{x}{5} + \frac{1}{4} = \frac{x}{2}$

2) $\frac{1}{4} = \frac{3}{x} - \frac{1}{2}$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Rational Inequalities		
Learning Competency : Solves rational inequalities. (M11GM-Ib-3)		
References : General Mathematics Teaching Guide, pp 34-39		LAS No.: 13

CONCEPT NOTES

<table>
<tr><td>Steps in solving rational inequalities:</td><td></td></tr>
<tr><td>1. Rewrite the inequality as a single rational expression on one side of the inequality symbol and 0 on the other side.</td><td>$\frac{x+2}{x-3} \leq 0$</td></tr>
<tr><td>2. Get the meaningful numbers.
Set both the numerator and the denominator equal to zero. Then, solve. Mark these on the number line.Use a shaded circle for x= -2 (a solution) and unshaded circle for x = 3 (not a solution).</td><td>Numerator: x+ 2 = 0
X = -2
Denominator: x-3 =0
X = 3

-2 3</td></tr>
<tr><td>3. Construct a table of signs to determine the sign of the function in each interval determined by 0 and 2.
Note that $\frac{x+2}{x-3}$ is negative for any real values of x.</td><td>
<table>
<tr><td>interval</td><td>x ≤ -2</td><td>-2≤x <3</td><td>x>3</td></tr>
<tr><td>Test point</td><td>X=-3</td><td>X=-1</td><td>X=4</td></tr>
<tr><td>X+2</td><td>-</td><td>+</td><td>+</td></tr>
<tr><td>x-3</td><td>-</td><td>-</td><td>+</td></tr>
<tr><td>$\frac{x+2}{x-3}$</td><td>+</td><td>-</td><td>+</td></tr>
</table>
</td></tr>
<tr><td>4.Summarize the intervals satisfying the inequality. Plot these intervals on the number line.</td><td>$\{-2\} \cup (-2, 3) = [-2,3)$
</td></tr>
</table>

Exercises: Solve the following inequality:

1) $\frac{x+1}{x-5} > 0$ 2) $\frac{1}{x} < 4$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Representations of Rational Functions		
Learning Competency : Represents a rational function through its: table of values and graph . (M11GM-Ib-4)		
References : General Mathematics Teaching Guide, pp 41-43		LAS No.: 14

CONCEPT NOTES

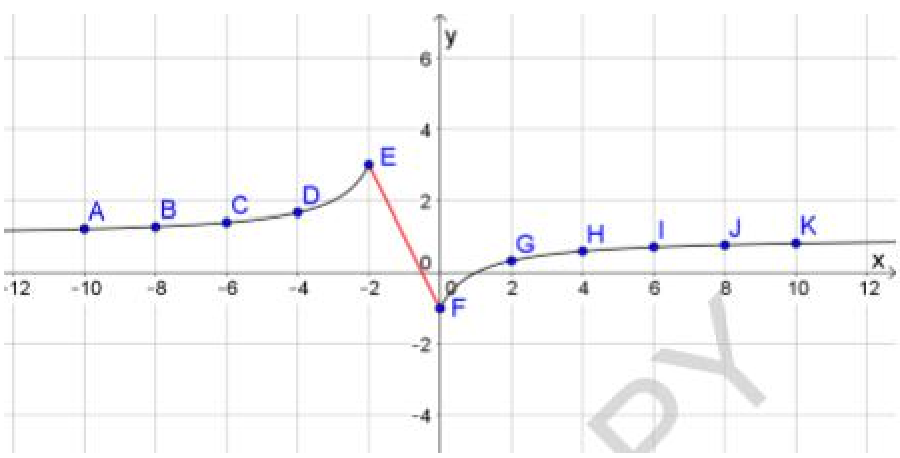
A rational function of the form $f(x) = \frac{p(x)}{q(x)}$ where p(x) and q(x) are polynomial functions and q(x)≠0. The domain of $f(x)$ is all values of x where q(x)≠0.

Example: Represent the rational function given by $f(x) = \frac{x-1}{x+1}$ using a table of values and plot a graph of the function by connecting points.

Solution: Construct a table of values for some x-values from -10 to 10.

x	-10	-8	-6	-4	-2	0	2	4	6	8	10
f(x)	1.22	1.29	1.4	1.67	3	-1	0.33	0.6	0.71	0.78	0.82

Plot the points on a Cartesian plane and connect the points



Exercise:

Represent the rational function $f(x) = \frac{x^2-3x-10}{x}$ using table of values. Sketch the graph of the function.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Domain and Range of a Rational Function		
Learning Competency : Finds the domain and range of a rational function. (M11GM-Ib-5)		
References : General Mathematics Teaching Guide,		LAS No.: 15

CONCEPT NOTES

The domain of a function is the set of all values that the variable x can take. The range of a function is the set of all values that f(x) will take.

Example: Determine the domain and range of the given rational function $f(x) = \frac{x-2}{x+2}$.

Solution:

a. Domain: Equate the denominator x+2 to 0.That is,

x+ 2= 0

x= -2

Observe that the function is undefined at x= -2. This means that x=-2 is not part of the domain of f(x). In addition, other values of x will make the function undefined.

Thus, the domain of f(x) is $\{x\epsilon R/x \neq -2\}$.

b. Range: Write the given function as an equation as follows

$$y = \frac{x-2}{x+2}$$

Solve the above equation for x.

$$y = \frac{x-2}{x+2}$$

y(x+2) = x -2

xy +2y = x-2

xy –x = -2y -2

x(y-1)= -2y -2

$$x = \frac{-2y-2}{y-1}$$

The above expression of x in terms of y shows that x is real for all real values of y except 1 since y = 1 will make the denominator y - 1 = 0.

Hence the range of f, which is the set of all possible values of y, is given by $\{y\epsilon R/y \neq 1\}$.

Exercises:

Find the domain and range of the given rational function $f(x) = \frac{x+1}{2x\_-2}$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Intercepts of Rational Function		
Learning Competency : Determines the intercepst of a rational function. (M11GM-Ic-1)		
References : General Mathematics Teaching Guide,		LAS No.: 16

CONCEPT NOTES

The x- intercept of the graph of the function are also zeroes of the given function.

The y –intercept is the function value when x = 0.

Example: Consider the function $f(x) = \frac{x-2}{x+2}$.

a. Find the x- intercept.

Recall that the x-intercepts of a rational function are the values of x that will make the function zero. A rational function will be zero if its numerator is zero. The numerator of the given function is x- 2.

Solve for x:

x-2 = 0

x= 2

The x –intercept of the function is 2.

b. Find the y-intercept.

Let x=0

Substituting,

f(0) = $\frac{0-2}{0+2}$

= $\frac{-2}{2}$

= -1

The y-intercept of a function is -1.

Exercise:

Find the intercepts the given rational function $f(x) = \frac{x+1}{2x\_-2}$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Asymptotes of Rational Function		
Learning Competency : Determines the vertical asymptote of a rational function. (M11GM-Ic-1)		
References : General Mathematics Teaching Guide,		LAS No.: 17

CONCEPT NOTES

An asymptote is a line (or a curve) that the graph of a function gets close to but does not touch.

The vertical line x = a is a vertical asymptote of a function f if the graph of f either increases or decreases without bound as the x -values approach a from the right or left.

Finding the Vertical Asymptotes of a Rational Function

- Find the values of a where the denominator is zero.
- If this value of a does not make the numerator zero, then the line x = a is a vertical asymptote.

Example: Find the vertical asymptote of the graph of the given rational function $f(x) = \frac{1}{x+1}$.

Solution:

- Set the denominator equal to 0 and solve for x.

  X+ 1 = 0

  X=-1

  The graph has the line x=-1 as vertical asymptote.

Exercise:

Find the vertical asymptote of the graph of the given rational function $f(x) = \frac{4x}{2x+1}$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : A Horizontal Asymptote of a Rational Function		
Learning Competency : Determines the vertical asymptote of a rational function. (M11GM-Ic-1)		
References : General Mathematics Teaching Guide,		LAS No.: 18

CONCEPT NOTES

The horizontal asymptote is determined by comparing the degrees of N(x) and

D(x) in $f(x) = \frac{a_n x^n + a_{n-1} a^{n-1} + \ldots a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots b_1 x + b_0}$.

a. If n<m, the graph of f has the line y =0 as a horizontal asymptote.
b. If n=m, the graph of f has the line $y = \frac{a_n}{b_m}$ as a horizontal asymptote where a_n and b_m or the leading coefficients of the numerator and denominator, respectively.
c. If n>m, the graph of f has no horizontal asymptote.

Example:

Find the horizontal asymptote of the graph of the following rational functions:

a) $f(x) = \frac{1}{x+1}$ b) $f(x) = \frac{4x}{2x+1}$

Solution:

a. Consider the numerator x+1. Since the degree of the numerator is less than the degree of the denominator, so the graph has the line y= 0 as a horizontal asymptote.
b. Since the degree of the numerator is equal to the degree of the denominator. The leading coefficient of the numerator is 4 and the leading coefficient of the denominator is 2. So, the graph has the line $y = \frac{4}{2}$ or y =2 as a horizontal asymptote.

Exercises: Find the horizontal asymptote of the graph of the rational functions.

$a.) f(x) = \frac{x^2}{x-1}$ b) $f(x) = \frac{x-2}{x+2}$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Graphing Rational Function		
Learning Competency : Graphs rational function. (M11GM-Ic-2)		
References : General Mathematics Teaching Guide,		LAS No.: 19

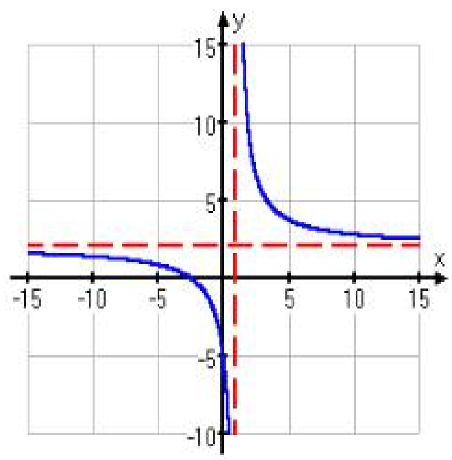
CONCEPT NOTES

To graph a rational function, you find the asymptotes and the intercepts, plot a few points, and then sketch in the graph.

Example: Sketch the graph of the rational function $f(x) = \frac{2x+5}{x-1}$.

a. Find the vertical asymptote.
   x- 1 = 0
   x= 1, vertical asymptote
   The dash line is the vertical asymptote.
b. Find the horizontal asymptote.
   $y = \frac{a_n}{b_m}$ = $\frac{2}{1}$ = 2, horizontal asymptote
c. Find the x and y – intercepts.
   X =0
   $y = \frac{0+5}{0-1} = \frac{5}{-1} = -5$

   f(x)=y=0
   $0 = \frac{2x+5}{x-1}$
   $0 = 2x + 5$
   -5 = 2x
   -2.5=x
   (0,-5) and (-2.5,0) are the intercepts..



d. Pick a few more x- values , compute the corresponding y-values then plot the points on a Cartesian plane.

x	-6	-1	2	3	6	8	15
y	1	-1.5	9	5.5	3.4	3	2.5

Exercise:

Sketch the rational function $f(x) = \frac{2}{x+1}$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving problems involving rational equation.		
Learning Competency : Solves problem involving rational equation. (M11GM-Ic-3)		
References : General Mathematics Teaching Guide,		LAS No.: 20

CONCEPT NOTES

Example: Bethany has scored **10** free throws out of **18** tries. She would really like to bring her free throw average up to at least **68%**. **How many consecutive free throws should she score in order to bring up her average to 68%?**

Solution:

Let's let x= the number of free throws that Bethany should score (in a row) in order to bring up her average. Again, we can use fractions, and this time they will represent the fraction of free throws that she scores. We'll start out with her current fraction (rate) of consecutive free throws, and then we'll add the number she needs to score to both the numerator (number she scores) and denominator (total number of throws): $\frac{10+x}{18+x}=\frac{68}{100}$.

$$\frac{10+x}{18+x}=\frac{68}{100}$$

$$\begin{aligned}(100)(10+x) &= (68)(18+x)\\ 1000+100x &= 1224+68x\\ 32x &= 224\\ x &= 7\end{aligned}$$

So Bethany needs **7 more consecutive free throws** to bring her free throw percentage up to **68%**.

Exercise: Solve the problem.

Two hoses are used to fill Maddie's neighborhood swimming pool. One hose alone can fill the pool in **10** hours; the second hose can fill it in **12** hours. The pool's drain can empty the pool in **8** hours. If the two hoses are working, and the drain is open (by mistake), how long will it take to fill the swimming pool?

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving problems involving rational equation.		
Learning Competency : Solves problem involving rational equation. (M11GM-Ic-3)		
References : General Mathematics Teaching Guide,		LAS No.: 21

CONCEPT NOTES

Example: Ten goats were set loose in an island and their population growth can be approximated by the function $P(t) = \frac{60(t+1)}{t+1}$. How many goats will there be after 5 years?

Solution: Evaluate the function for t=5.

$$P(t) = \frac{60(t+1)}{t+1}$$

$$P(5) = \frac{60(5+1)}{5+1}$$

$$= \frac{60(6)}{6}$$

$$= \frac{360}{6}$$

= 60

There will be 60 goats after 5 years.

Exercises:

In an interbarangay basketball league, the team from Barangay Culiat has won 12 out of 25 games, a winning percentage of 48%. We have seen that they need to win 8 games consecutively to raise their percentage to at least 60%. What will be their winning percentage if they win 10 games in a row?

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : One – to – One Functions		
Learning Competency : Represent real-life situation using one–to–one function (MIIGM-Id-1)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 22

CONCEPT NOTES

The function is **one-to-one** if for any x_1, x_2 in the domain of f, then $f(x_1) \neq f(x_2)$. That is, the same -value is never paired with two different -values.

Example 1. The relation pairing an SSS member to his or her SSS number
Solution. Each SSS member is assigned to a unique SSS number. Thus, the relation is a function. Further, two different members cannot be assigned the same SSS number. Thus, the function is one-to-one.

Example 2. The relation pairing a real number to its square.
Solution. Each real number has a unique perfect square. Thus, the relation is a function. However, two different real numbers such as 2 and –2 may have the same square. Thus, the function is not one-to-one.

Example 3.The relation pairing a person to his or her citizenship.
Solution. The relation is not a function because a person can have dual citizenship (i.e., citizenship is not unique).

EXERCISES

A. Which of the following relations is a one-to-one function?
(a) $\{(0,0), (1,1), (2,8), (3,27), (4,64)\}$
(b) $\{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$
(c) $\{(0,4), (1,5), (2,6), (3,7), \ldots\ (n, n+4), \ldots\}$

B. Which of the following are one-to-one functions?
(a) Books to authors
(b) SIM cards to cell phone numbers
(c) True or False questions to answers

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Inverse of One – to – One Functions		
Learning Competency : Determines the inverse of one-to-one function (M11GM-Id-2)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 23

CONCEPT NOTES

The function f is one-to-one if for any x1, x2 in the domain of f, then f(x1) 6= f(x^2).That is, the same y-value is never paired with two different x-values. **A function has an inverse if and only if it is one-to-one.**

To find the inverse of a one-to-one function:
(a) Write the function in the form y = f(x);
(b) Interchange the x and y variables;
(c) Solve for y in terms of x

Example 1. Find the inverse of f(x) = 2x + 3

Solution: Y = 2x + 3 X = 2y + 3

Solve for y in terms of x:

$\frac{x-3}{2} = \frac{2y}{2}$ $\frac{x-3}{2} = \text{y}$

Therefore the inverse of f(x) = 2x + 3 is $f^{-1}(x) = \frac{x-3}{2}$

Example 2. Find the inverse of the rational function $f(x) = \frac{3x+5}{2x-1}$

Solution: $y = \frac{3x+5}{2x-1}$ $x = \frac{3y+5}{2y-1}$

Solve for y in terms of x:

X(2y – 1) = 3y + 5
2xy – 2x = 3y + 5
2xy – 3y = 2x + 5
Y(2x – 3)=2x + 5

$Y = \frac{2x+5}{2x-3}$

Therefore the inverse of $f(x) = \frac{3x+5}{2x-1}$ is $f^{-1}(x) = \frac{2x+5}{2x-3}$

EXERCISES:

Find the inverse of the following one to one functions:

1. f(x) = 3x – 5
2. f(x) = 9 – 4x

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Table of Values of the Inverse Function		
Learning Competency : Represent the inverse function through table of values (M11GM-Id -3)		
References : General Mathematics by Orlando A. Oronce		LAS No.:24

CONCEPT NOTES

In finding the table of values find first the inverse function, then use it to find the values of the variable y by giving values of x.

Example 1.

Show the table of values of the inverse function f(x) = 2x + 3

Solution: *Solution:* Y = 2x + 3 X = 2y + 3

Solve for y in terms of x:

$\frac{x-3}{2} = \frac{2y}{2}$ $\frac{x-3}{2} = y$

Therefore the inverse of f(x) = 2x + 3 is $f^{-1}(x) = \frac{x-3}{2}$

X	-3	-2	-1	0	1	2	3
$f^{-1}(x)$	-3	-2.5	-2	-1.5	-1	-0.5	0

if x = -3 $f^{-1}(x) = \frac{-3-3}{2} = \frac{-6}{2} = -3$

if x = -2 $f^{-1}(x) = \frac{-2-3}{2} = \frac{-5}{2} = -2.5$

if x = -1 $f^{-1}(x) = \frac{-1-3}{2} = \frac{-4}{2} = -2$

if x = 0 $f^{-1}(x) = \frac{0-3}{2} = \frac{-3}{2} = -1.5$

if x = 1 $f^{-1}(x) = \frac{1-3}{2} = \frac{-2}{2} = -1$

if x = 2 $f^{-1}(x) = \frac{2-3}{2} = \frac{-1}{2} = -0.5$

if x = 3 $f^{-1}(x) = \frac{3-3}{2} = \frac{0}{2} = 0$

EXERCISES: Solve the following problems:

1. Use the inverse of the function f(x) = 5x + 3 then supply the values needed in the table.

X	-3	-2	-1	0	1	2	3
$f^{-1}(x)$							

2. Use the inverse of the function $f(x) = \frac{3x-2}{2x+3}$ then supply the values needed in the table.

X	-3	-2	-1	0	1	2	3
$f^{-1}(x)$							

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Graph of the Inverse Function		
Learning Competency : Represent the inverse function through its graph (M11GM-Id -3) : Graphs inverse functions (M11GM-Ie-1)		
References : General Mathematics by Orlando A. Oronce		LAS No.:25

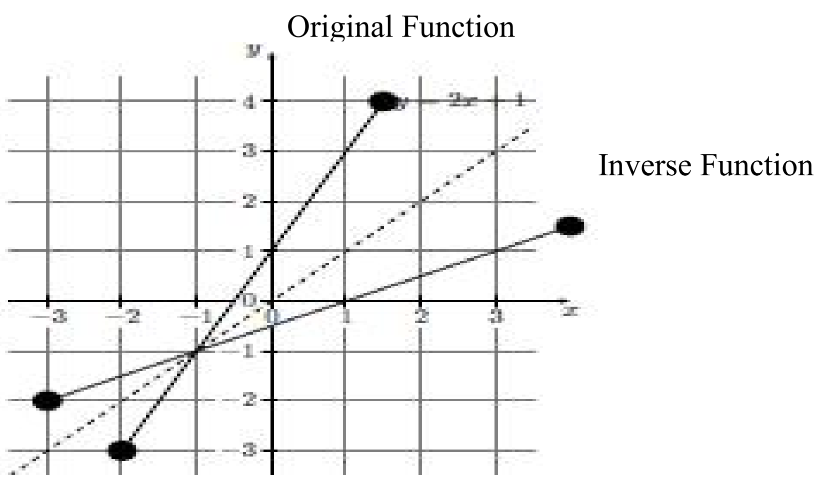
CONCEPT NOTES

First we need to ascertain that the given graph corresponds to a one-to-one function by applying the horizontal line test. If it passes the test, the corresponding function is one-to-one.

Given the graph of a one-to-one function, the **graph of its inverse** can be obtained by reflecting the graph about the line y = x .

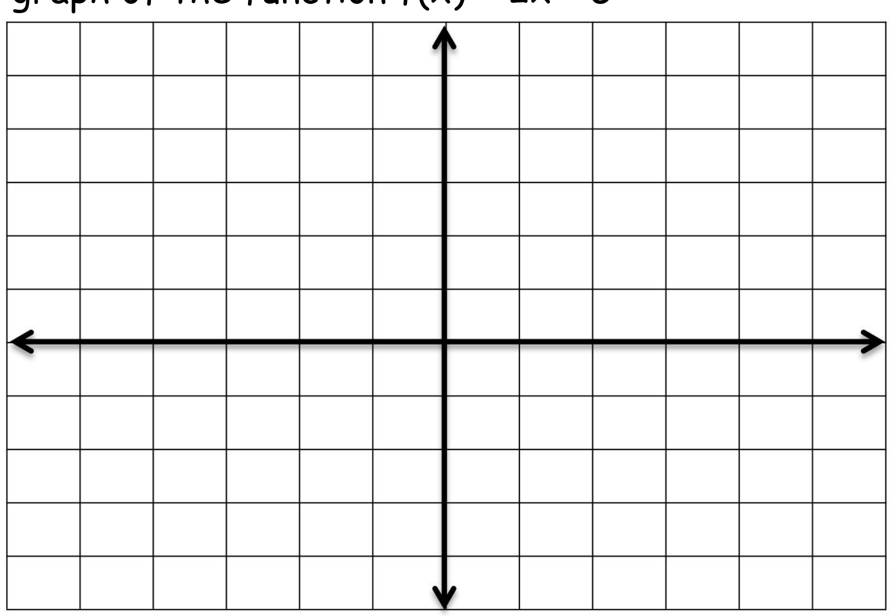
Example 1. Graph $y = f^{-1}(x)$ if the graph of $y=f(x)=2x + 1$ restricted in the domain $\{x / -2 \leq x \leq 1.5\}$ is given below.

Solution. Take the reflection of the restricted graph of y = 2x + 1 across the line y=x.



EXERCISE

Show the graph of the function f(x) = 2x - 5



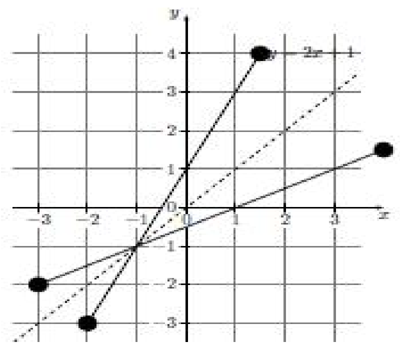
Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Domain & Range of the Inverse Function		
Learning Competency : Find the domain and range of an inverse function (M11GM-Id -4)		
References : General Mathematics by Orlando A. Oronce		LAS No.:26

CONCEPT NOTES

The domain and range of the inverse function can be determined by inspection of the graph.

Example 1:

Y = 2x + 1



original function:

domain is {x ε R/-2 ≤ x ≤1.5 }

range is {y ε R / -3 ≤ y ≤ 4}

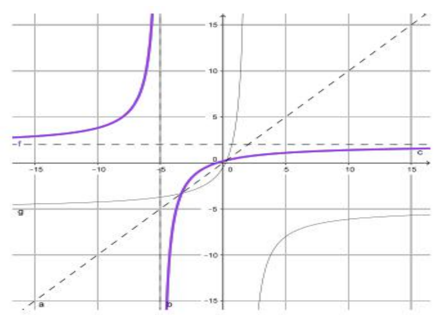
inverse function:

domain is { X ε R/-3 ≤ X ≤ 4}

range is {x ε R/-2 ≤ x ≤1.5 }

Example 2:

f(x) = $\frac{5x-1}{-x+2}$



original function:

domain is {x ε R/ x ≠ ≤ 2 }

range is {y ε R / y ≠ - 5}

Vertical asymptote = 2

Horizontal asymptote = -5

inverse function:

domain is {X ε R/X ≠ - 5}

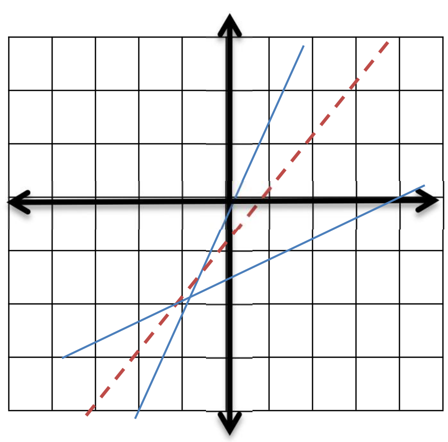
range is {x ε R/y≠ 2 }

Vertical asymptote = -5

Horizontal asymptote = 2

EXERCISE

Give the domain and range of the original and the inverse function



Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Problems Involving Inverse Function		
Learning Competency : Solve problems involving inverse functions (M11GM-Ie -3)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 27

CONCEPT NOTES

Example 1. Think of a nonnegative number, add two to the number, square the number, multiply the result by 3 and divide the result by 2. If the result is 54, what is the original number? Construct an inverse function that will provide the original number if the result is given.

Solution. We first construct the function that will compute the final number based on the original number.

$$f(x) = (x+2)^2 \bullet 3 \div 2 = \frac{3(x+2)^2}{2}$$

The instruction indicated that the original number must be nonnegative. The domain of the function must thus be restricted to $x \geq 0$. The function with restricted domain $x \geq 0$ is then a one-to-one function, and we can find its inverse.

Interchange the x and y variables:

$x = \frac{3(y+2)^2}{2}, y \geq 0$ solve for y in terms of x;

$$x = \frac{3(y+2)^2}{2}$$

$$\frac{2x}{3} = (y + 2)^2 \qquad \sqrt{2x/3} = y + 2$$

$$\sqrt{2x/3} - 2 = y \qquad f\text{-}1(x) = \sqrt{2x/3} - 2$$

Finally evaluate the inverse function at to determine the original number:

$$f^{-1}(54) = \sqrt{2(54)/3} - 2 = \sqrt{108/3} - 2 = \sqrt{36} - 2 = 6 - 2 = 4$$

The original number is 4.

EXERCISE

Think of a positive odd integer, add two to the number, square the number, multiply the result by 3 and divide the result by 3. If the result is 81 what is the original number? Construct an inverse function that will provide the original number if the result is given.

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Represents Real-Life Situations Using Exponential Functions		
Learning Competency :Represents real-life situations using exponential functions(M11GM-le-3)		
References : General Mathematics by Orlando A. Oronce		LAS No.:28

CONCEPT NOTES

Exponential functions occur in various real world situations it can be used to model real-life situations such as population growth, radioactive decay, carbon dating, growth of an epidemic, loan interest rates, and investments.

Definition.
An exponential function with base b is a function of the form $f(x) = b^x$ or $y = b^x$, where b > 0 , b 6= 1.

Example 1. Do the simple activity

a. Get a straw (how many pieces of straw do you have?
b. Step 1 fold into half and cut, how many pieces of straw do you have?
c. Steps 2 fold again those straws that you have and cut, how many pieces of straws do you have?
d. Step 3 fold again those straws that you have and cut, how many pieces of straws do you have?
e. Step 4 fold again those straws that you have and cut, how many pieces of straws do you have?

Steps	0	1	2	3	4
No. of pieces of straw	1	2	4	8	16

Take note that everytime you fold the straws into half and cut, the straws increases the number of pieces. If n is the number of straws and s is the step number, then $n=2^s$

EXERCISES

Complete the table and give the pattern

S	0	1	2	3	4	5	6	7
n	1	3	9					

Pattern: ______________________________

S	0	1	2	3	4	5	6	7
N	1	9	81					

Pattern: ______________________________

S	0	1	2	3	4	5	6	7
N	2	4	8					

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Difference Between Exponential Functions, Exponential Equations & Exponential Inequalities		
Learning Competency : distinguishes between exponential function, exponential equations,and exponential inequality (M11GM-Ie -4)		
References : General Mathematics by Orlando A. Oronce		LAS No.:29

CONCEPT NOTES

	Exponential Equations	Exponential Inequality	Exponential Functions
Definition	An equation involving exponential expressions	An inequality involving exponential expressions	Functions of the form $f(x)=b^x$, where $b>0, b\neq 1$
Example	$7^{2x-x2}=1/343$	$5^{2x} - 5^{x+1} \leq 0$	$F(x)=(1.8)^x$ or $y=(1.8)^x$

EXERCISES

Determine whether the given is exponential function, an exponential equation an exponential inequality or none of these

(a) $f(x) = 2x^3$

(b) $f(x) = 2^x$

(c) $y = e^x$

(d) $2^2(5^{x+1}) = 500$

(e) $625 \geq 5^{x+8}$

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Solve Exponential Equations & Inequalities		
Learning Competency : Solve exponential equations and inequalities (M11GMle-f-1)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 30

CONCEPT NOTES

Example 1. Solve the equation $4^{x-1} = 16$.

Solution. We write both sides with 4 as the base.

$4^{x-1} = 16$

$4^{x-1} = 4^2$

$x - 1 = 2$

$x = 2 + 1$

$x = 3$

Example 2. Solve the equation $125^{x-1} = 25^{x+3}$.

Solution. Both 125 and 25 can be written using 5 as the base.

$125^{x-1} = 25^{x+3}$

$(5^3)^{x-1} = (5^2)^{x+3}$

$5^{3(x-1)} = 5^{2(x+3)}$

$3(x- 1) = 2(x + 3)$

$3x- 3 = 2x + 6$

$x = 9$

EXERCISES

Solve:

(a) $7^{x+4} = 49^{2x-1}$

(b) $4^{x+2} = 8^{2x}$

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Representation of Exponential Functions Through Table of Values		
Learning Competency: Represents an exponential function through table of values(M11GM-le-f-2)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 31

CONCEPT NOTES

Table of values composed of an ordered pairs, where one is the dependent variable and the other is the independent variable

EXAMPLE 1. Construct a table of values of ordered pairs for the given function $f(x)=2^x$

X	-4	-3	-2	-1	0	1	2	3
F(x)	1/16	1/8	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Solution:

If x = -4, $f(-4)=2^{-4}=\frac{1}{2^4}=\frac{1}{16}$

If x = -3, $f(-3)=2^{-3}=\frac{1}{2^3}=\frac{1}{8}$

If x = -2, $f(-2)=2^{-2}=\frac{1}{2^2}=\frac{1}{4}$

If x = -1, $f(-1)=2^{-1}=\frac{1}{2^1}=\frac{1}{2}$

If x = 0, $f(0)=2^0=1$

If x = 1, $f(1)=2^1=2$

If x = 2, $f(2)=2^2=4$

If x = 3, $f(3)=2^3=8$

EXERCISES

1. Construct a table of values of ordered pairs for the given function $f(x)=4^x$

X	-2	-1	0	1	2	3
F(x)						

2. Construct a table of values of ordered pairs for the given function $f(x)=2^{2x}$

X	-2	-1	0	1	2	3
F(x)						

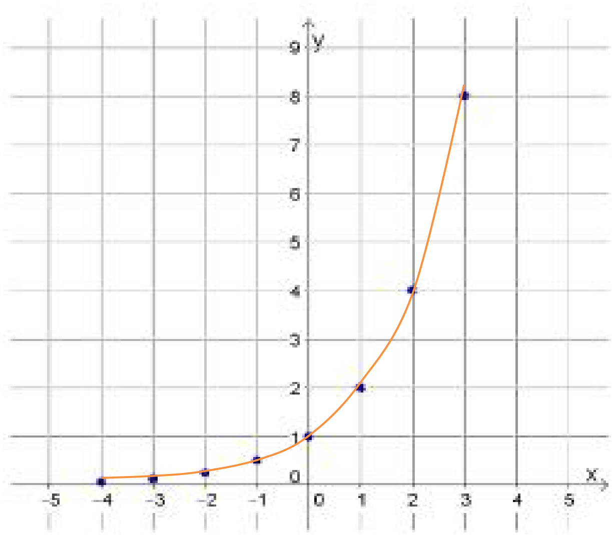
Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Representation of Exponential Functions Through Graph		
Learning Competency: Represents an exponential function through graph(M11GM-le-f-2)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 32

CONCEPT NOTES

The graph of exponential function is also a smooth curve following the ordered pairs made from the exponential functions given.

Example: Refer to the table of $f(x)=2^x$

X	-4	-3	-2	-1	0	1	2	3
F(x)	1/16	1/8	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

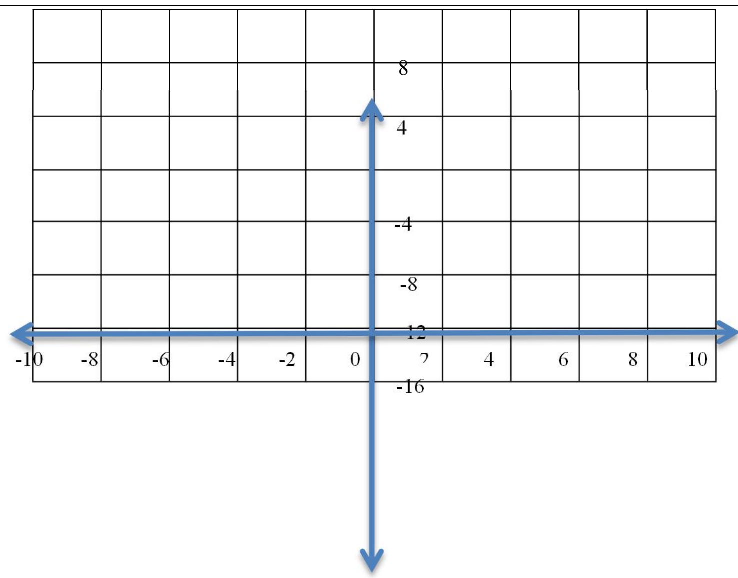


EXERCISE

Graph the function $f(x) = 2^{2x}$

X	-2	-1	0	1	2	3
F(x)						

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Representation of Exponential Functions Through Graph		
Learning Competency: Represents an exponential function through graph(M11GM-le-f-2)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 32



Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title : Representation of Exponential Functions Through Equation		
Learning Competency: Represents an exponential function through equation (M11GM-le-f-2)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 33

CONCEPT NOTES

An exponential equation is one in which a variable occurs in the exponent. It follows the one – to – one property where $b^x = b^y$ if and only if $x = y$.

Example 1. $5^x = 625$

$5^x = 5^4$

$x = 4$

Example 2. $[1/3]^x = 81$

$3^{-x} = 3^4$

$X = -4$

Example 3. $9^{2x-1} = 3^{8x}$

$(3^2)^{2x-1} = 3^{8x}$

$3^{4x-2} = 3^{8x}$

$4x-2 = 8x$

$-2 = 4x$

$-1/2 = x$

EXERCISES

Find the value of x:

1. $4^3 = 8^2$
2. $2^{3x+1} = 16$
3. $9^{3x-1} = 27^{x+3}$

Name:	Date:	Score:
Subject : GENERAL MATHEMATICS		
Lesson Title :Zeroes of an Exponential Function		
Learning Competency:Determine the y-intercepts, asymptotes and zeroes of an exponential function(11GM-If-4)		
References : General Mathematics by Orlando A. Oronce		LAS No.: 34

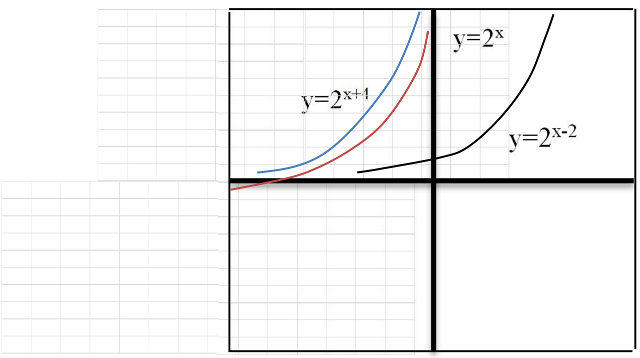
CONCEPT NOTES:

Example:

Use the graph of $y = 2^x$ to graph the functions $y = 2^{x-2}$ and $y = 2^{x+4}$.

Solution.Some y-values are shown in the following table:

x	-3	-2	-1	0	1	2	3
$y = 2^x$	0.125	0.25	0.5	1	2	4	8
$y = 2^{x-2}$	0.031	0.063	0.125	0.25	0.5	1	2
$y = 2^{x+4}$	2	4	8	16	32	64	128



Take note of the following:

a. The y-intercepts changed. To find them, substitute x = 0 in the function. Thus the y-intercept of

$Y=2^{x+4}$ is $2^4 = 16$

$Y=2^{x-2}$ is $2^{-2} = 0.25$

$Y=2^x$ is $2^0 = 1$

b. Translating a graph horizontally does not change the horizontal asymptote. Thus the horizontal asymptote of all three graphs is y = 0

c. No values of x that makes the function zero

EXERCISES

For each of the following functions,

(a) Sketch the graph,
(b) y-intercept, and
(c) Horizontal Asymptote.

1) $y= 3^{x-4}$ 2) $y = 2^{x-5}$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Graphs of Exponential Functions		
Learning Competency : Graphs an exponential functions in the form of $f(x) = b^x \; if \; b > 1$. (M11GM-Ig-1)		
References : General Mathematics Teaching Guide, pp. 105-106		LAS No.: 35

CONCEPT NOTES

In graphing exponential functions, we simply assign values for x and solve for y. After that, plot the points in the Cartesian plane then connect all points. Do not forget to label the graph.

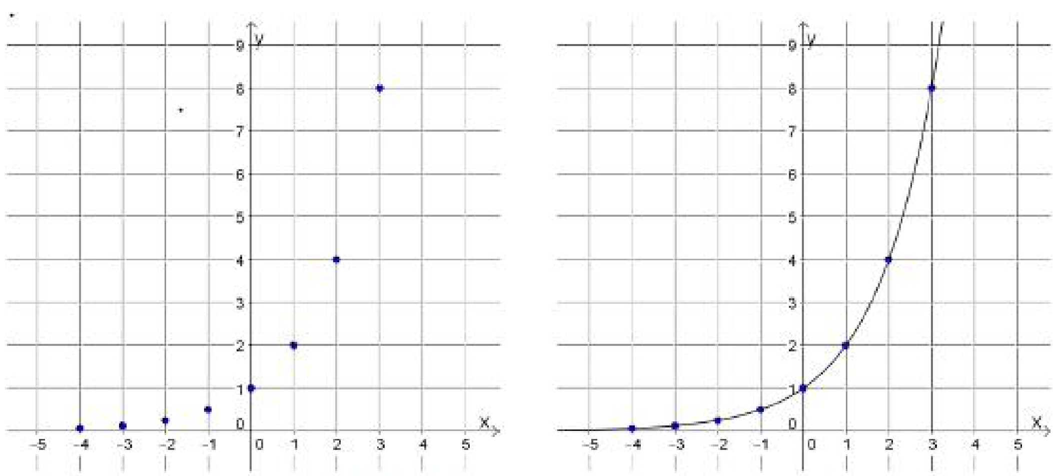
Example: Sketch the graph of $f(x) = 2^x$.

Solution:

Step 1. Construct a table of values of ordered pairs for the given function. The table of values for f(x) is as follows:

x	-4	-3	-2	-1	0	1	2	3
F(x)	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Step 2. Plot the points found in the table and connect them using a smooth curve.



(a) Plotting of points for $f(x) = 2^x$ (b) Graph of $f(x) = 2^x$

EXERCISE

1. Sketch the graph of $f(x) = 3^x$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Graphs of Exponential Functions		
Learning Competency : Graphs an exponential functions in the form of $f(x) = b^x \text{ if } 0 < b < 1$. (M11GM-Ig-1)		
References : General Mathematics Teaching Guide, pp. 106-107		LAS No.: 36

CONCEPT NOTES

In graphing exponential functions, we simply assign values for x and solve for y. After that, plot the points in the Cartesian plane then connect all points. Do not forget to label the graph.

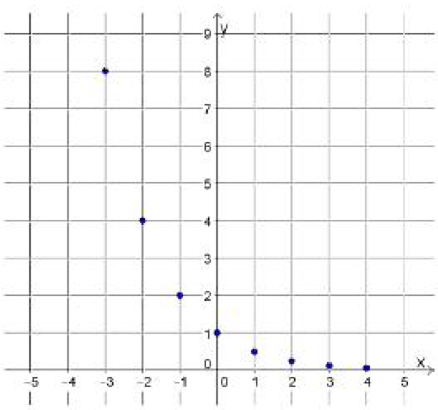
Example: Sketch the graph of $g(x) = (\frac{1}{2})^x$.

Solution:

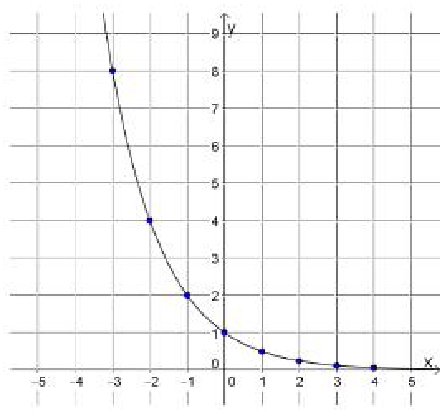
Step 1. The table of values for g(x) is as follows:

x	-3	-2	-1	0	1	2	3	4
F(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

Step 2. Plot the points found in the table and connect them using a smooth curve.



(a) Plotting of points for $g(x) = \left(\frac{1}{2}\right)^x$



(b) Graph of $g(x) = \left(\frac{1}{2}\right)^x$

EXERCISE:

1. Sketch the graph of $f(x) = (\frac{1}{4})^x$.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Population Growth Problem Using Exponential Functions		
Learning Competency : Solves problems involving exponential functions , equations, and inequalities. (M11GM-Ig-2)		
References : General Mathematics Teaching Guide, page 94		LAS No.: 37

CONCEPT NOTES

Exponential functions occur in various real world situations. They are used to model real life situations such as population growth.

Example: Let *t* =time in hours. At *t* =0, there were initially 20 bacteria. Suppose that the bacteria double every 100 hours. How many bacteria will there be after 400 hours? Give an exponential model for the bacteria as a function of *t*.

Solution:

Initially, at t=0	Number of bacteria = 20	= 20
at t=100	Number of bacteria = $20(2)^1$	=40
at t=200	Number of bacteria = $20(2)^2$	=80
at t=300	Number of bacteria = $20\ (2)^3$	= 160
at t= 400	Number of bacteria = $20\ (2)^4$	=320

An exponential model for this situation is $= 20\ (2)^{\frac{t}{100}}$.

EXERCISES: Solve the following problems:

1. Initially, there are 1000 bacteria. It triples every 20 hours. Let t= time in hours.

a. How many bacteria will there be after 20 hours?

b. How many bacteria will there be after 40 hours?

c. Give an exponential model for the bacteria as a function of t.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Radioactive Decay Problem Using Exponential Functions		
Learning Competency: Solves problems involving exponential functions , equations, and inequalities. (M11GM-Ig-2)		
References : General Mathematics Teaching Guide, page 94		LAS No.: 38

CONCEPT NOTES

Exponential functions occur in various real world situations. They are used to model real life situations such as radioactive decay.

The **half-life** of a radioactive substance is the time it takes for half of the substance to decay.

Example: Suppose that the half-life of a certain radioactive substance is 10 days and there are 10g initially; determine the amount of substance remaining after 30 days.

Solution:

Initially, at t =0	Amount of substance = 10g
at t=10 days	Amount of substance = 5g
at t =20 days	Amount of substance = 2.5g
at t= 30 days	Amount of substance = 1.25g

An exponential model for this situation is $y = 10\,(\frac{1}{2})^{\frac{t}{10}}$.

EXERCISE:

1. Initially, there are 100g of bacteria. The half-life of the substance is 15 days.

a. How many bacteria will be left after 15 days?

b. How many bacteria will be left after 30 days?

c. Give an exponential model for the bacteria as a function of t.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Investment Problem Using Exponential Functions		
Learning Competency: Solves problems involving exponential functions , equations, and inequalities. (M11GM-Ig-2)		
References : General Mathematics Teaching Guide, page 95.		LAS No.: 39

CONCEPT NOTES

A starting amount of money (**principal)** can be invested at a certain interest rate that is earned at the end of a given period of time. If the interest rate is **compounded,** the interest earned at the end of the period is added to the principal, and this new amount will earn interest in the next period.

Example:

Mrs. Cruz invested Php100,000 in a company that offers 6% interest compounded annually. How much will this investment be worth at the end of each year for 3 years?

Solution:

Let t be the time in years. Then we have:

Initially, at t=0	Investment = Php100,000
at t=1	Investment = P100,000 (1.06) = P106,000
at t=2	Investment = P106,000 (1.06) = P112, 360
at t=3	Investment = P112,360(1.06) = P119,101.60

An exponential function for this situation is $y = 100{,}000\ (1.06)^t$.

EXERCISE: Solve.

1. If you want to invest P10,000 in a company that offers 7% compounded annually. How much money will you earn at end of each year for 4 years?

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Introduction of Logarithms		
Learning Competency: Rewrites exponential form into logarithmic form and vice-versa		
References : General Mathematics Teaching Guide, page 119.		LAS No.: 40

CONCEPT NOTES

Let a and b positive real numbers such that b ≠ 1. The logarithm of a with base b, denoted by $\log_b a$, is defined as the number such that $b^{\log_b a}$ =a. That is, $\log_b a$ is the exponent that b must be raised to produce a.

Example:

A. Rewrite the following exponential equation in logarithmic form

1. $5^3 = 125$ 2. $7^{-2} = \frac{1}{49}$ 3. $4^0 = 1$

Solution:

1. $\log_5 125 = 3$ 2. $\log_7 \frac{1}{49} = -2$ 3. $\log_4 1 = 0$

B. Rewrite the following logarithmic form in exponential form

1. $\log_{10} 100 = 2$ 2. $\log_3 \frac{1}{9} = -2$ 3. $\log_5 1 = 0$

Solution:

1. $10^2 = 100$ 2. $3^{-2} = \frac{1}{9}$ 3. $5^0 = 1$

EXERCISES:

A. Rewrite the following exponential form to logarithmic form

1. $5^2 = 25$ 2. $4^{-3} = \frac{1}{64}$ 3. $7^1 = 7$

B. Rewrite the following logarithmic form to exponential form

1. $\log_6 36 = 2$ 2. $\text{Log}_2 \frac{1}{32} = -5$ 3. $\text{Log}_{11} 1 = 0$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Representation Real Life Situations Using Logarithmic Functions		
Learning Competency: Represents real life situations using logarithmic functions and solves problems involving logarithmic functions, equations and inequalities. (M11GM-Ih-1; M11Gm-Ij-2)		
References : General Mathematics Teaching Guide, page 121-122.		LAS No.: 41

CONCEPT NOTES

Logarithms allow us to discuss very large numbers in more manageable ways. For example, 10^{31} a very large number, may be difficult to work with. But its common logarithm $\log 10^{31} = 31$ is easier to grasp. Because logarithms can facilitate an understanding of very large numbers (or positive numbers very close to zero), it has applications in various situations.

EXERCISE:

In 1935, Charles Richter proposed a logarithmic scale to measure the intensity of an earthquake. He defined the magnitude of an earthquake as a function of its amplitude on a standard seismograph. The following formula produces the same results, but is based on the energy released by an earthquake.

The magnitude R of an earthquake is given by
$R = \frac{2}{4} \log \frac{E}{10^{4.40}}$, where E (in joules) is the energy released by the earthquake (the quantity $10^{4.40}$ is the energy released by a very small reference earthquake).

Suppose the earthquake released approximately 10^{12} joules of energy,
a. What is the magnitude on a Richter scale?
b. How much more energy does this earthquake release than that by the reference earthquake?

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Logarithmic Functions , Equations, and Inequalities		
Learning Competency: Distinguishes logarithmic function, logarithmic equation and inequality (M11GM-Ih-2)		
References : General Mathematics Teaching Guide, page 128-129.		LAS No.: 42

CONCEPT NOTES

	Logarithmic Equation	Logarithmic Inequality	Logarithmic Function
Definition	An equation involving logarithms	An inequality involving logarithms	Function of the form $f(x) = \log_b x$ $(b>0, b\neq 1)$
Example	$\text{Log}_x 2 = 4$	$\log 10 < \log 100$	$F(x) = \log_3 x$

EXERCISE:

Identify whether the given is a logarithmic equation, logarithmic inequality or logarithmic function or none of these.

1. $\log_2 64 = 6$

2. $g(x) = \log_7 x$

3. $\log_3 x > \log_4 4$

4. $f(x)= \log_5 x$

5. $10^2 = x$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Basic Properties of Logarithms		
Learning Competency: Illustrates the basic properties of logarithms (M11GM-Ih-3)		
References : General Mathematics Teaching Guide, page 128-129.		LAS No.: 43

CONCEPT NOTES

Basic Properties of Logarithms:

Let b and x be real numbers such that b > 0 and b≠ 1,

(a) $\log_b 1 = 0$

(b) $\log_b b^x = x$

(c) If x> 0, then $b^{\log_b x} = x$

Example: Use the properties of logarithms to find the value of the following logarithmic expressions.

(a) log 10 (b) $\ln e^3$ (c) $\log_4 64$ (d) log 1 (e) $5^{\log_5 2}$

Solution:

(a) $\log 10 = \log_{10} 10^1 = 1$ (Property 2)

(b) $\ln_e e^3 = \ln_e e^3 = 3$ (Property 2)

(c) $\log_4 4^3 = 3$ (Property 2)

(d) log 1 = 0 (Property 1)

(e) $5^{\log_5 2} = 2$ (Property 3)

EXERCISE:

Use the properties of logarithms to find the value of the following logarithmic expressions.

1. $\log_7 1$

2. $\text{Log}_8 8^2$

3. $\text{Log}_5 125$

4. $9^{\log_9 3}$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Laws of Logarithms		
Learning Competency: Illustrates the laws of logarithms (M11GM-Ih-3)		
References : General Mathematics Teaching Guide, page 134-136.		LAS No.: 44

CONCEPT NOTES

Laws of Logarithms

Let b > 0, b≠ 1 and let n ϵ $\Re$. For u > 0, v > 0, then

1. $\log_b (uv) = \log_b u + \log_b v$
2. $log_b \frac{u}{v} = log_b u - log_b v$
3. $log_b u^n = nlog_b u$

Example: Illustrate the following by applying the laws of logarithms.

1. $log_7(7^3.7^8)$ 2. $log_7(\frac{49}{7})$ 3. $log_7 7^5$

Solution:

1. $log_7(7^3.7^8)$ = $log_7 7^3 + log_7 7^8$
2. $log_7\left(\frac{49}{7}\right) = log_7 49 - log_7 7$
3. $log_7 7^5 = 5log_7 7$

EXERCISE:

Illustrate the following by applying the laws of logarithms.

1. $log_6(6^5.6^4)$
2. $log_5(\frac{125}{5})$
3. $log_{97} 97^5$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Logarithmic Equations		
Learning Competency: Solves logarithmic equations (M11GM-Ih-i-1)		
References : General Mathematics Teaching Guide, page 134-136.		LAS No.: 45

CONCEPT NOTES

Some Strategies for Solving Logarithmic Equations

1. Rewriting to exponential form
2. Using logarithmic properties
3. Applying the one-to-one property of logarithmic functions, as stated below:

One-to One Property of Logarithmic Functions

For any logarithmic function $f(x) = log_b x, if\ log_b u = log_b v\ , then\ u = v.$

Zero Factor Property:

If ab = 0, then a =0 or b=0.

4. Checking if each of the obtained values does not result in undefined expressions in the given equation.

Example: Find the value of x in the equation $log_4(2x) = log_4 10$

Solution:

$log_4(2x) = log_4 10$

2x = 10 (apply directly the one-to-one property)

X= 5 (divide 2 in both sides)

Check: 5 is a solution since $\log_4 (2\cdot 5) = \log_4 (10)$.

EXERCISE:

1. Find the value of x in the following equations.

a. $\log_8 (3x) = \log_8 (15)$

b. $\log_3 (2x-1) =2$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Logarithmic Inequality		
Learning Competency: Solves logarithmic inequalities (M11GM-Ih-i-1)		
References : General Mathematics Teaching Guide, page 144-145.		LAS No.: 46

CONCEPT NOTES

Property of Logarithmic Inequalities

Given the logarithmic expression $\log_b x$,

If $0 < b < 1$, then $x_1 < x_2$ if and only if $\log_b x_1 > \log_b x_2$.

If $b > 1$, then $x_1 < x_2$ if and only if $\log_b x_1 < \log_b x_2$.

Example: Solve the logarithmic inequality $log_3(2x-1) > log_3(x+2)$

Step 1: Ensure that the logarithms are defined.

Then 2x-1 > 0 and x+2 > 0 must be satisfied.

2x-1 > 0 implies $x > \frac{1}{2}$ and x + 2 > 0 implies x > -2

To make both logarithms defined, then x > 1/2 (If $x > \frac{1}{2}$
, then x is surely greater than -2)

Step 2. Ensure that the inequality is satisfied.

The base 3 is greater than 1.

Thus, since $\log_3(2x-1) > \log_3(x+2)$, then:

2x-1 > x +2

x> 3 (Subtract x from both sides, add 1 to both sides)

∴ x > 3

Hence, the solutions is $(3, +\infty)$.

EXERCISE:

Solve the logarithmic inequality $\log_4(3x + 3) > \log_4(x+1)$

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Representation of a logarithmic Function		
Learning Competency: Represents a logarithmic function through its table of values and graph. (M11GM-Ii-2)		
References : General Mathematics Teaching Guide, page 151-153.		LAS No.: 47

CONCEPT NOTES

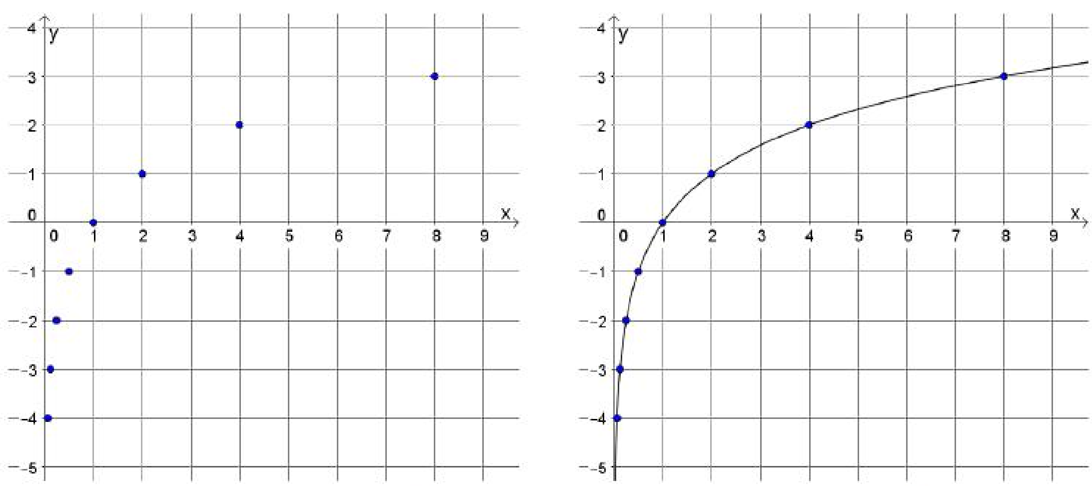
Logarithmic functions can be represented through table of values and graph. Consider the given example:

Sketch the graph of $y = \log_2 x$.

Solution: Step 1. Construct table of values of ordered pairs for the given function. A table of values for $y = \log_2 x$ is as follows:

x	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-4	-3	-2	-1	0	1	2	3

Step 2. Plot the points found in the table, and connect them using a smooth curve.



(a) Plotting of points for $y = \log_2 x$ (b) Graph of $y = \log_2 x$

It can be observed that the function is defined only for x>0. The function is strictly increasing, and attains all real values. As x approaches 0 from the right, the function decreases without bound, i..e., the line x =0 is a vertical asymptote.

EXERCISE:

1. Sketch the graph of $y = \log_3 x$. Complete first the table of values below.

x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	2	3
y						

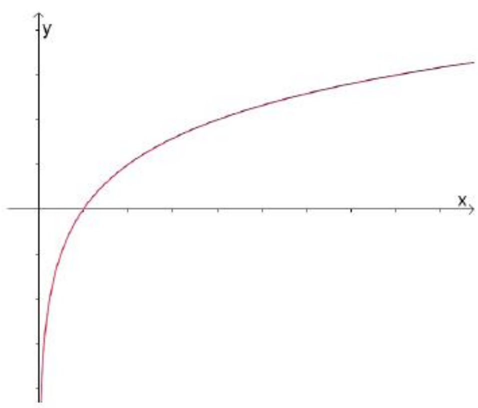
Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Domain and Range of a Logarithmic Functions		
Learning Competency: Finds the domain and range of a logarithmic function . (M11GM-Ii-3)		
References : General Mathematics Teaching Guide, page 153-154.		LAS No.: 48

CONCEPT NOTES

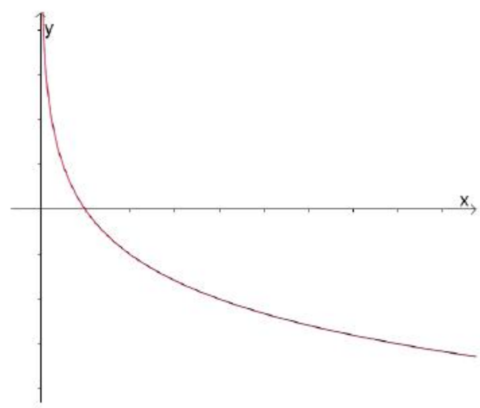
Domain of the function- set of values of x for which a corresponding value of y exists.

Range of the function-set of values of y which correspond to the values of x in the domain.

Consider the following graphs and identify the domain and range.



(a) $y = \log_b x (b > 1)$

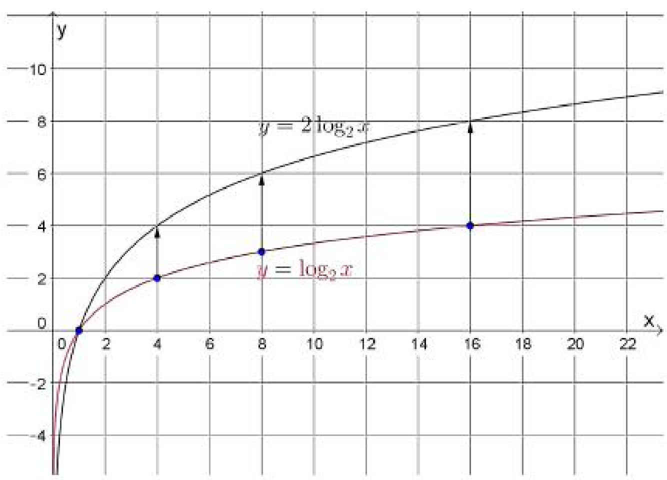


(b) $y = \log_b x (0 < b < 1)$

Using the graph of $y = \log_b x$ ($b > 1$ or $0 < b < 1$) as visual cue, you may elicit the following properties:

1. The domain is the set of all positive numbers, or $\{x \in \Re \ / \ x > 0\}$.
2. The range is the set of all real numbers.

EXERCISE: Identify the domain and range of the graphs below.

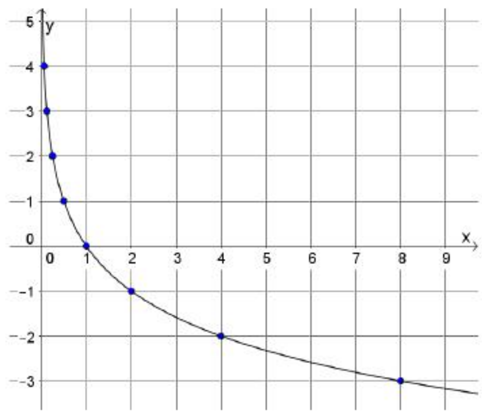


Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Intercepts, Zeroes and Asymptotes of Logarithmic Functions		
Learning Competency: Determines the intercepts, zeroes, and asymptotes of logarithmic function . (M11GM-Ii-4)		
References : General Mathematics Teaching Guide, page 153-155.		LAS No.: 49

CONCEPT NOTES

Intercepts- the value of x or y that the graph passes through the axes
Zeroes of the function- the value of x where the f(x) = 0
Asymptotes- the graph approaches to but never touches the line.

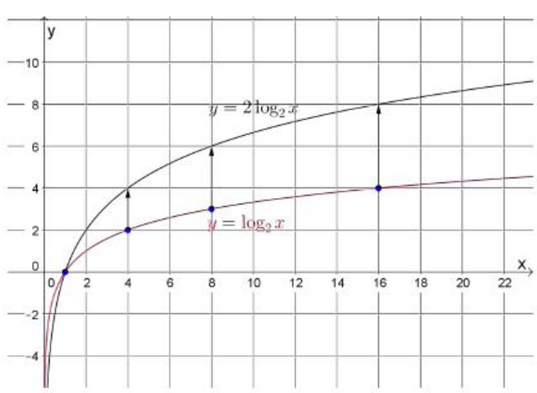
Consider the graph below and identify the intercepts, zeroes and asymptotes of the logarithmic function.



(b) Graph of $y = \log_{\frac{1}{2}} x$

Using the graph of y = $\log_b x$ (b >1 or 0<b<1) as visual cue, you may elicit the following properties:
1. The x-intercept is 1. The graph does not pass through y-axis, therefore there is no y-intercept.
2. The zero of the function is x=1, since the value of y = 0 at x=1.
3. The vertical asymptote is the line x=0 (or the y-axis). There is no horizontal asymptote.

EXERCISE: Identify the intercepts, zeroes, and asymptotes of the logarithmic functions.



Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Graph of Logarithmic Functions		
Learning Competency: Graphs rational functions.(M11GM-Ij-1)		
References : General Mathematics Teaching Guide, page 154-157.		LAS No.: 50

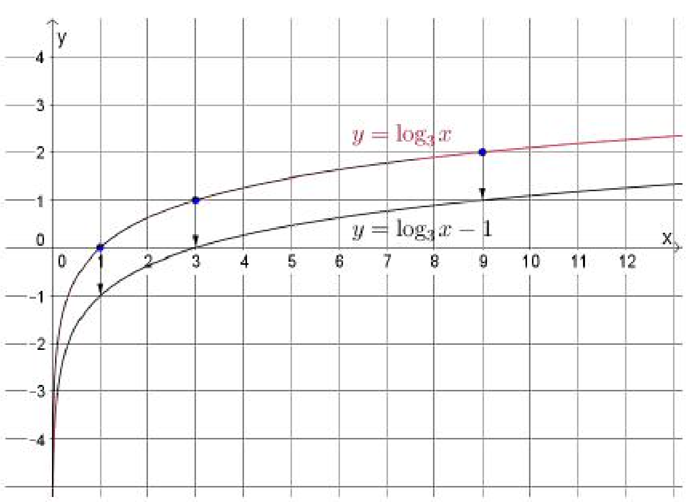
CONCEPT NOTES

Graph of $f(x) = a \cdot \log_b (x-c) + d$

- The value of b (either $b>1$ or $0<b<1$) determines whether the graph is increasing or decreasing.
- The value of a determines the stretch or shrinking of the graph. Further, if a is negative, there is a reflection of the graph about the x-axis.
- Based on $f(x)= a \cdot \log_b x$, the vertical shift is d units up (if $d>0$) or d units down (if $d<0$), and the horizontal shift is c units (if $c>0$) or c units to the left(if $c<0$).

Example: Sketch the graph of $y = \log_3 x - 1$.
Solution: Sketch first the graph of $y = \log_3 x$. Note that the base 3 is greater than 1.
The "-1" means vertical shift downwards by 1 unit.
Some points on the graph of $y=\log_3 x$ are (1,0), (3,1), and (9, 2)
Shift these points 1 unit down to obtain (1, -1), (3,0), and (9,1).

Plot these points.
The graph is shown below.



Analysis:
1. Domain: $\{x/x \in \Re, x > 0\}$
2. Range: $\{ y/y \in \Re\}$
3. Vertical Asymptote: $x = 0$
4. x-intercept is 3
5. Zero of the function is 3.

EXERCISE:
1. Sketch the graph of $y = \log_2 x + 2$. Determine the domain, range, vertical asymptote, x-intercept and zero of the function.

Name:	Date:	Score:
Subject : General Mathematics		
Lesson Title : Solving Problems Involving Logarithmic Functions		
Learning Competency: Solves problems involving logarithmic functions.(M11GM-Ij-2)		
References : General Mathematics Teaching Guide, page 147-148.		LAS No.: 51

CONCEPT NOTES

Logarithmic Functions can be applied in real world problems specifically in the interest compounded annually.

Consider the problem below:

Using the formula $A = P(1 + r)^n$ where A is the future value of the investment, P is the principal, r is the fixed annual interest rate, and n is the number of years, how many years will it take an investment to double if the interest rate per annum is 2.5%?

Solution: Doubling the principal P, we get A = 2P, r = 2.5%=0.025

$A = P(1+r)^2$

$2P = P(1+0.025)^n$ (Substitute the given information)

$2 = (1.025)^n$ (Divide P to both sides)

$\text{Log } 2 = \log (1.025)^n$ (Get the log of both sides)

$\text{Log } 2 = n \log (1.025)$ (Apply the laws of logarithm)

$n = \frac{\log 2}{\log 1.025} \approx 28.07 \; years$ (Solve for n)

Answer: It will take approximately 28 years for the investment to double.

EXERCISE:

1. Using the formula $A = P(1 + r)^n$ where A is the future value of the investment, P is the principal, r is the fixed annual interest rate, and n is the number of years, how many years will it take an investment to triple if the interest rate per annum is 5%?