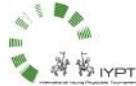




Problem No.16

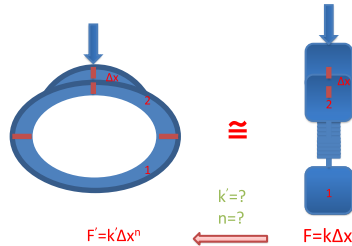
Hoops

Erfan Pirmorad



- An elastic hoop is pressed against a hard surface and then suddenly released. The hoop can jump high in the air. Investigate how the height of the jump depends on the relevant parameters.

| Theoretical Study | Experiments | Conclusion |
|---|---|---|
| <ul style="list-style-type: none"> Initial observation Simple Explanation Spring Break-Off Maximum height Trajectory | <ul style="list-style-type: none"> Stiffness Coefficient Maximum height V_r, Inner Radius V_r, Thickness V_r, Deviation V_r, flexural modulus Trajectory | <ul style="list-style-type: none"> Stiffness Coefficient Maximum height V_r, Inner Radius V_r, Thickness V_r, Deviation V_r, flexural modulus Trajectory |

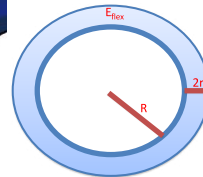
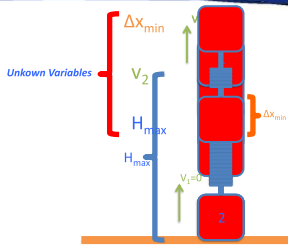


Main Approach

Conclusion

Acknowledgment

$k = f$



$$k = f(R, r, E_{flex})$$

$$k \propto R^n r^b E_{flex}^c$$

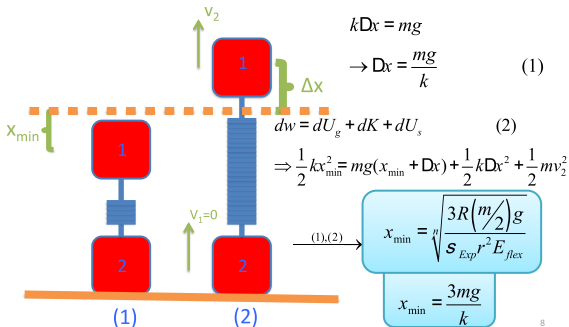
$$[k] = \frac{M.L.T^{-2}}{L}$$

$$\propto k \propto \frac{r^3 E_{flex}}{R^2}$$

$$F = S_{Exp} \frac{r^3 E_{flex}}{R^2} D\Delta x^n$$

Deriving

Δx_{min}



$$dw = dU_g + dK + dU_s$$

$$\Rightarrow \frac{1}{2}kx_{dev}^2 = mg(x_{dev} + Dx) + \frac{1}{2}kDx^2 + \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{k}{m}(x_{dev}^2 - Dx^2) - 2g(x_{dev} + Dx)}$$

$$v_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

$$v_{CM}(0) = \frac{v_2}{2}$$

$$H_{max} = \frac{v_{CM}^2(0)}{2g}$$



$$Dl = r'_a - r'_b - l$$

$$m r'_a = -k(r'_a - r'_b - l) - mg$$

$$m r'_b = +k(r'_a - r'_b - l) - mg$$

$$m(r'_a - r'_b) = -2k(r'_a - r'_b - l)$$

$$v_a = \frac{v_0}{2}(1 + \cos \omega t) - gt$$

$$v_b = \frac{v_0}{2}(1 - \cos \omega t) - gt$$

$$F = \frac{dp}{dt}$$

$$\begin{cases} u = A \sin \omega t + B \cos \omega t \\ t = 0 \rightarrow u = 0 \Rightarrow B = 0, \\ t = 0 \rightarrow \dot{u} = 0 \Rightarrow A = \frac{v_0}{\omega} \end{cases}$$

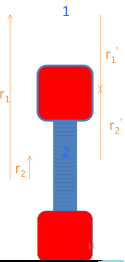
$$u = \frac{v_0}{\omega} \sin \omega t$$

$$\dot{u} = v_0 \cos \omega t$$

$$u = v_0 \phi - v_0 \phi^2 - v_0 \phi^3$$

$$v'_a = -v'_b = \frac{1}{2} v_0 \cos \omega t$$

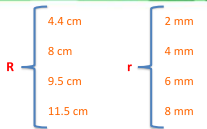
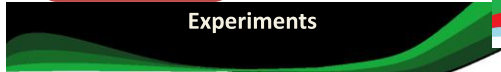
$$v_a = R + v'_a, v_b = R + v'_b$$



$$y_a = \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \sin \omega t + \frac{\partial}{\partial t} \cos \omega t \right) \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \right)$$

$$y_b = \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \sin \omega t - \frac{\partial}{\partial t} \cos \omega t \right) \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} \left(\frac{\partial}{\partial t} \right)$$

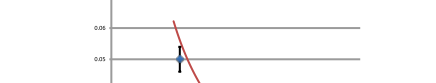
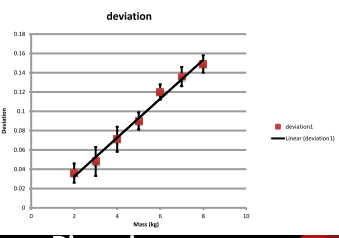
$$u + \frac{2k}{m} u = 0 \rightarrow \omega = \sqrt{\frac{2k}{m}}$$



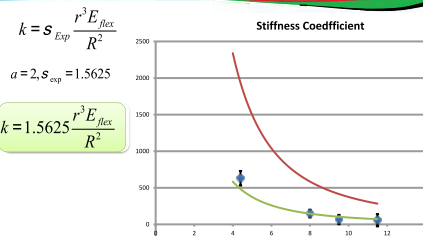
$E_{flex} \rightarrow 2340 \text{ MPa}$



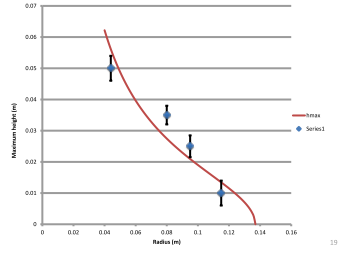
Discussion



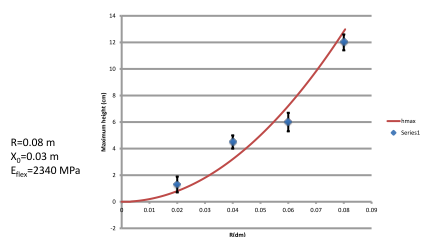
Discussion



$r=0.004 \text{ m}$
 $X_0=0.03 \text{ m}$
 $E_{flex}=2340 \text{ MPa}$



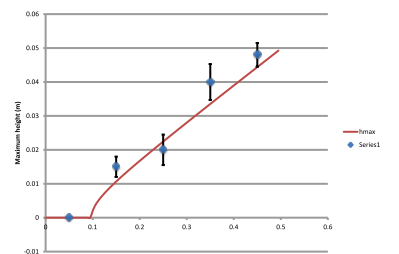
Discussion



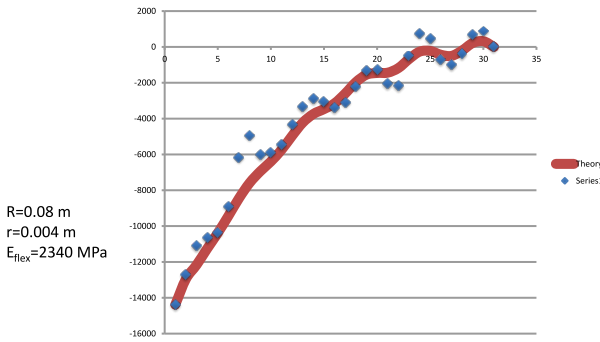
$R=0.08 \text{ m}$
 $r=0.004 \text{ m}$
 $E_{flex}=2340 \text{ MPa}$



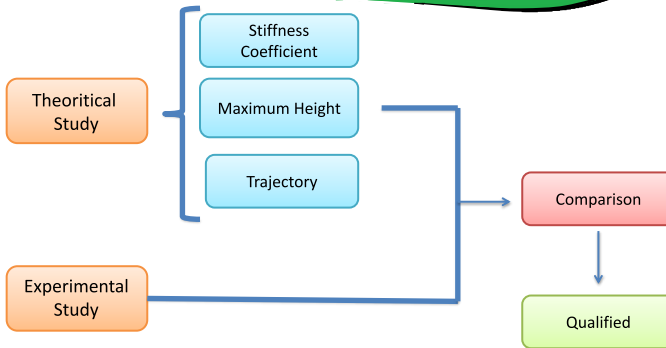
Discussion



Discussion



Conclusion



PROBLEM NO.2 ELASTIC SPACE

problem

The dynamics and apparent interactions of massive balls rolling on a stretched horizontal membrane are often used to illustrate gravitation. Investigate the system further. Is it possible to define and measure the apparent "gravitational constant" in such a "world"?

F.Mokhtari

introduction

- Basic explanation
- Initial observation
- The differences between the real universe and elastic space

theory

- The surface equation in real universe
- Finding the surface equation in elastic space
- Finding the gravitational force in elastic space
- Finding the potential equation
- Finding the orbital motions
- Investigating the Kepler's laws in elastic space

experiments

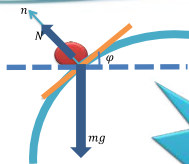
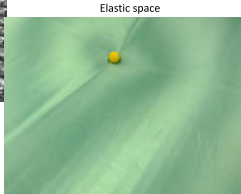
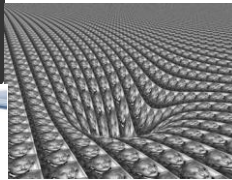
- experimental setup
- The circuit of the ball
- The orbital motion



Main approach

conclusion

- Comparing the experiments with theory
- Summarizing



If the equation of the surface is an analogy of real universe, the tangential force is equal to gravitational force.

$$\Sigma F_t: mg \sin \varphi = ma$$

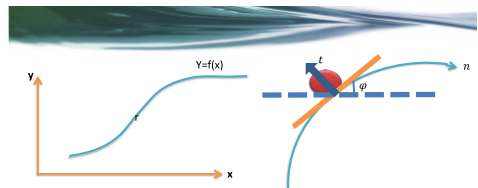
$$mg \sin \varphi = \frac{GMm}{r^2}$$

$$\sin \varphi = \frac{GM}{gr^2}$$

$$r \rightarrow 0 \rightarrow \sin \varphi \rightarrow \infty$$

$$\sin \varphi_{max} = 1 \rightarrow \varphi_{max} = 90 \rightarrow r \geq \sqrt{\frac{GM}{g}}$$

For this limitation our modeling is not responding



$$\tan \varphi = \frac{GM}{\sqrt{r^4 g^2 - G^2 M^2}}$$

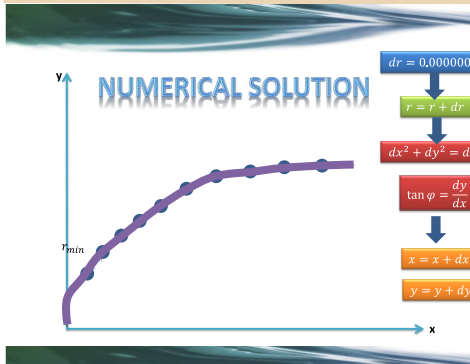
$$\tan \varphi = \frac{dy}{dx}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \frac{GM \frac{d^2y}{dx^2}}{2g \frac{dy}{dx}} \sqrt{\frac{g \frac{dy}{dx}}{GM \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}}}$$

$$\frac{dy}{dx} = \frac{GM}{\sqrt{r^4 g^2 - G^2 M^2}} \rightarrow r = \sqrt{\frac{GM \sqrt{1 + \left(\frac{dy}{dx}\right)^2}}{\frac{dy}{dx} g}}$$

$$r = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

By solving this differential equation we can obtain the surface equation



$$dr = 0,00000001$$

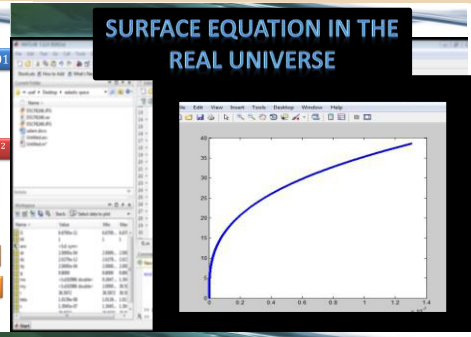
$$r = r + dr$$

$$dx^2 + dy^2 = dr^2$$

$$\tan \varphi = \frac{dy}{dx}$$

$$x = x + dx$$

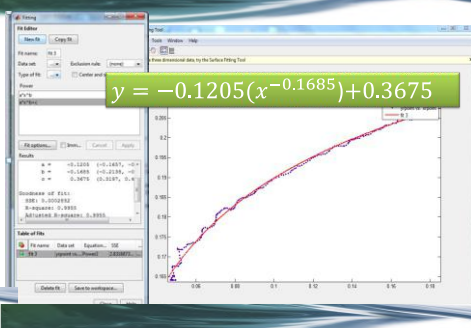
$$y = y + dy$$



What about the surface equation in our world?

It is better to find the surface equation experimentally!!!

Finding the powers of R & M



Power of m

Power of m=?

$$mg \sin \varphi = F_{\text{gravitation}}$$

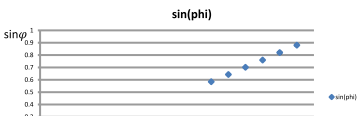
$$F_{\text{gravitation}} \propto ?$$

Power of M=?

$$F_{\text{gravitation}} \propto m$$

Power of r=?

Surface equation is not related to m



$$F_{\text{gravitation}} \propto M$$

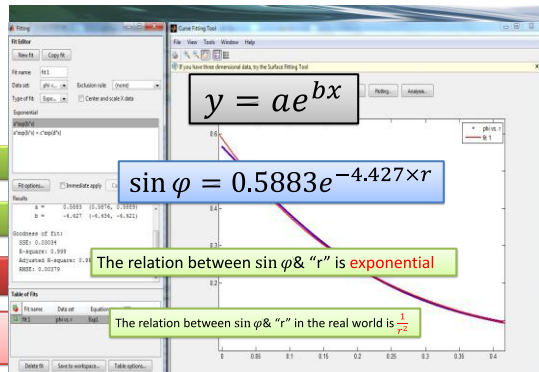
To find the power of r we should draw the $\sin \varphi(r)$. The degree of the diagram is the power of "r".

$$y = -0.1205(x^{-0.1685}) + 0.3675$$

$$y' = 0.02030425x^{-1.1685}$$

$$\sin \varphi = \sqrt{\frac{y'}{1 + y'}}$$

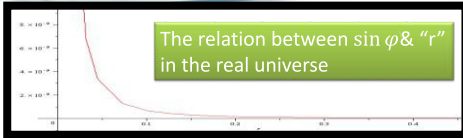
$$r = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$



The relation between $\sin \varphi$ & "r" is exponential

The relation between $\sin \varphi$ & "r" in the real world is $\frac{1}{r^2}$

ART AN AMAZING FACT IN SCIENCE



Gravitational force

REAL WORLD

$$F_g = G \frac{Mm}{r^2}$$

$$= 6.67 \times 10^{-11}$$

OUR WORLD

$$F_g = G' \frac{Mm}{(e^r)^b}$$

$$= g \times a = 5.771223$$

potential function

$$U = - \int_a^b F(r) dr = - \int_r^\infty \frac{MmG'}{e^{br}} dr$$

$$U(r) = - \frac{MmG'}{be^{br}}$$

potential function

REAL WORLD

$$U(r) = -G \frac{Mm}{r}$$

OUR WORLD

$$U(r) = - \frac{MmG'}{be^{br}}$$

Orbital motion

$$E = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2) + U(r)$$

$$\frac{d\theta}{dt} = \frac{l}{mr^2}$$

$$\frac{dr}{dt} = \sqrt{\frac{2}{m} \left[E - \frac{l^2}{2mr^2} - U(r) \right]}$$

$$\frac{d\theta}{dr} = \frac{l}{mr^2 \sqrt{\frac{2}{m} \left(E - \frac{l^2}{2mr^2} - U(r) \right)}}$$

$$\theta = \int_0^r \frac{l}{mr^2 \sqrt{\frac{2}{m} \left(E - \frac{l^2}{2mr^2} - \frac{MmG'}{be^{br}} \right)}} dr$$

solving this differential equation we can obtain the orbital motion of the jet in our world

Orbital period

KEPLER'S LAWS IN OUR WORLD

| REAL WORLD | OUR WORLD |
|---------------|---------------|
| First law) ✓ | First law) ✗ |
| Second law) ✓ | Second law) ✓ |
| Third law) ✓ | Third law) ! |

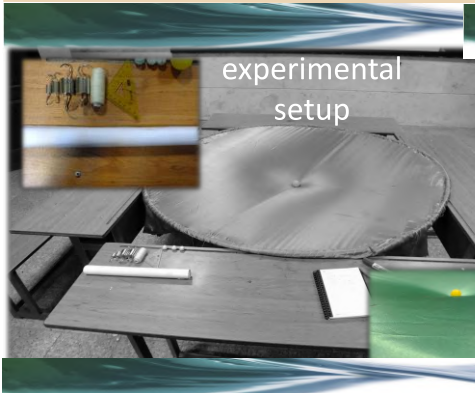
| REAL WORLD | OUR WORLD |
|-------------------|--|
| $T = \sqrt{kA^3}$ | $T = \int_0^R \frac{dr}{\sqrt{\frac{2}{m} \left(E - \frac{l^2}{2mr^2} + \frac{MmG'}{be^{br}} \right)}}$ |

Some Problems of This Modeling

A rolling ball's rotational kinetic energy has no analogue.

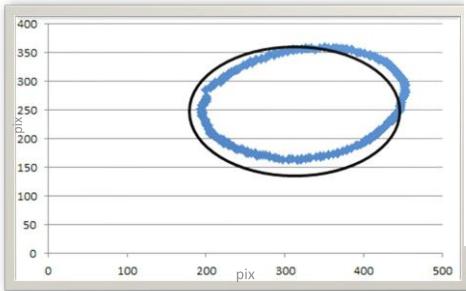
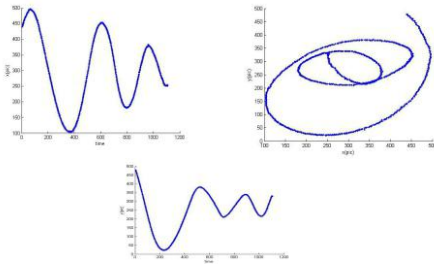
The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

This effect can be reduced by concentrating the ball's mass near its center so that the moment of inertia is small compared to mr^2 .



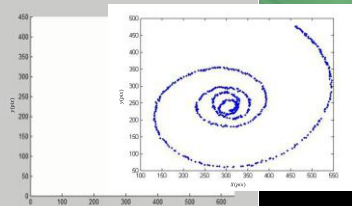
experimental setup

objects weights: 1 kg

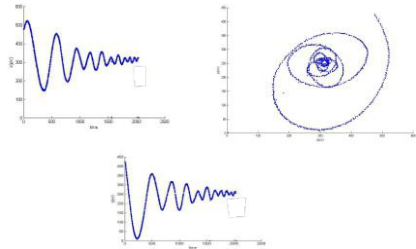


First experiment

objects weights: 0.7 kg



Mass weights: 1.2 kg



review

- 1) We find the surface equation in the real universe
- 2) We find the surface equation in our space (Elastic space) by experimental ways.
- 3) We find the relation of the gravitation force with M , m , r & we find the gravitation force in elastic space.
- 4) We derive the potential function in elastic space
- 5) We investigate the orbital motions and also we explained the Kepler's laws in elastic space
- 6) Finally we investigate the orbital motions in elastic space, experimentally and they were in best agreement with our theory.

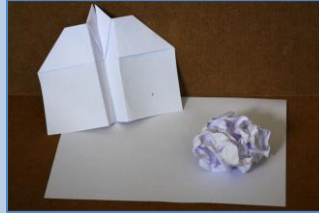


24th International Young Physicists' Tournament



POLAND

15. Slow Descent



Design and make a device, using one sheet of A4 80gram per m² paper that will take the longest possible time to fall to the ground through a vertical distance of 2.5m. A small amount of glue can be used. Investigate the influence of the relevant parameters.

Interpretation of the task

No initial velocity

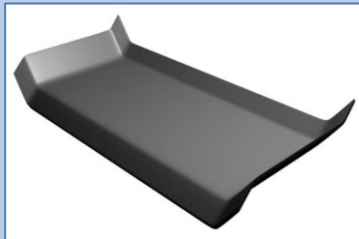
Single device

Whole sheet

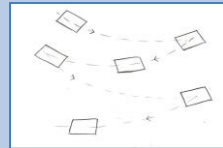
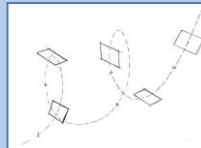


No confetti

The presentation of the final device



Types of card's motion

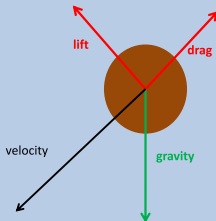


Tumbling

Fluttering

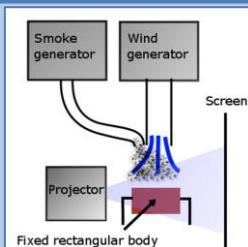
We will study and use autorotation as the source of lift force

Forces on a body moving in fluid medium



Lift and drag will be studied separately

Experimental setup



We studied images appearing on the screen in order to find out how the air flows around the strip

Image on the screen – front edge

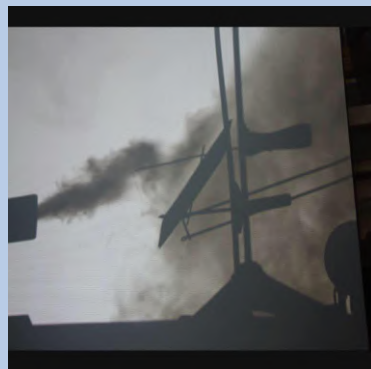
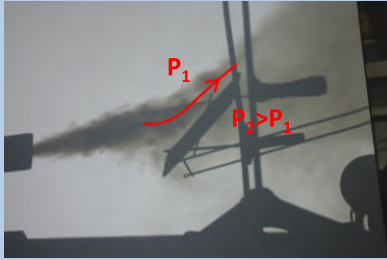


Image on the screen – front edge

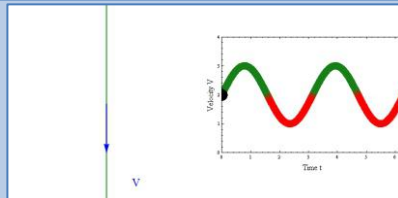


The air would accelerate along the front edge, which indicates pressure reduction along the front edge

Image on the screen – back edge



Sustaining the rotation

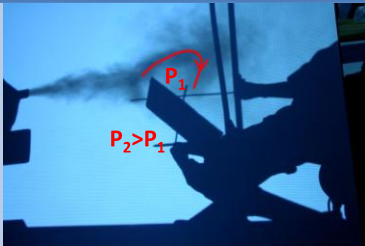


Green – described torques increase angular velocity

Blue – described torques decrease angular velocity

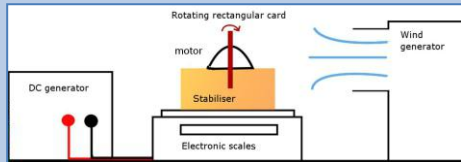
As green corresponds to higher velocity than blue, the net torque tends to increase the angular velocity, counter the drag and sustain stable rotation

Image on the screen – back edge



Whirlpool indicates there is an area of reduced pressure at the top of the trailing edge

Lift force experimental setup



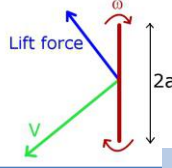
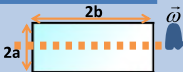
Scales indication changes were measured versus the voltage and wind generator setting

Lift force on a rectangular body

The lift force on a rotating rectangular body moving through a fluid can be found using Kutta-Joukowski theorem

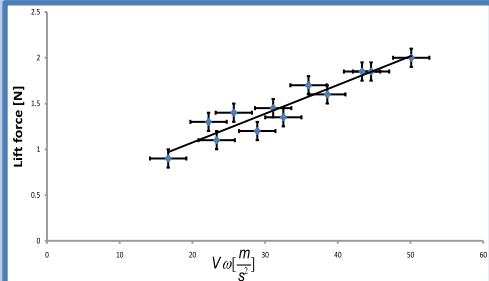
$$F_{lift} = 4\pi a^2 b \omega V \rho$$

Assumptions: very long body, ideal fluid with no viscosity



How does this correspond to a real, viscous situation

Lift force measurements results



Lift force was found to be linearly dependent on $V\omega$

Lift force experimental setup

Angular velocity was found as a function of voltage by means of stroboscopic measurement



Flashing diode

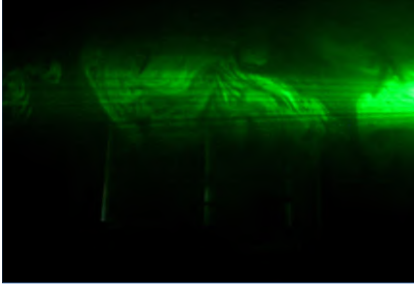
The velocity of wind was found using a Venturi tube with a manometer

Flow characteristic

Reynolds number is a parameter, describing a flow in given setup.

$$Re = \frac{\text{inertia force}}{\text{viscous force}}$$

Reynolds number describes the flow in given setup.

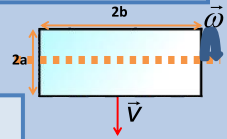


Vortexes can be seen behind the body

Flow characteristic

Reynolds number in our case is given by:

$$Re = \frac{2aV}{\nu}$$



2a – side of the rectangle
V – velocity of the body
 ν – kinematic viscosity of the medium

Theory and experiments described in the presentation will discuss motions with Re of order of 10^3 to 10^4

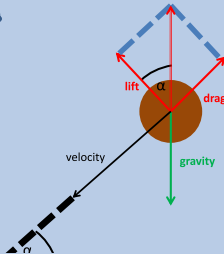
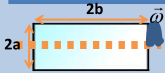
Aerodynamic drag

The value of aerodynamic drag for discussed motion can be approximated as (David L. Finn, "Falling paper and flying business cards"):

$$F_d = \frac{1}{2} C_d \rho AV^2$$

C_d – dimensionless drag coefficient
 ρ – air density
A – area perpendicular to V
V – velocity of the body with respect to air

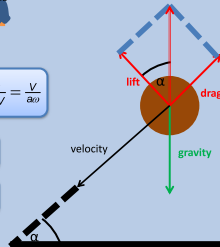
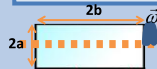
Theoretical parameters of steady motion



$$\text{drag} \propto abV^2$$

$$\text{lift} \propto a^2b\omega V$$

Theoretical parameters of steady motion



$$\tan \alpha = \frac{\text{drag}}{\text{lift}} \propto \frac{abV^2}{a^2b\omega V} = \frac{V}{a\omega}$$

$$\text{drag} \propto abV^2$$

$$\text{lift} \propto a^2b\omega V$$

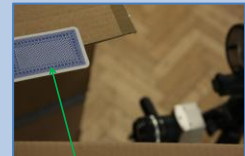
Direct measurements of tumbling bodies

Air motion protection

Distance scale

1000 W lamp

High speed camera



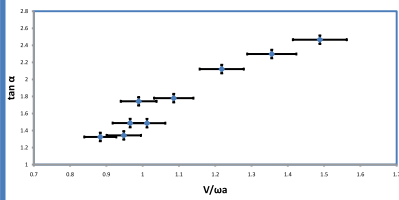
Studied body made of playing card

Measured parameters of motion:

- > Linear velocity
- > Angular Velocity
- > Angle of descent

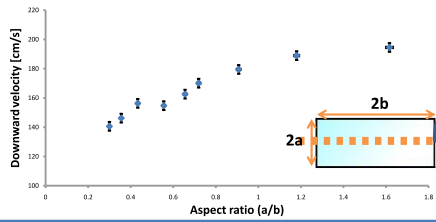
Comparison with theory

Comparison with theory



The plot is not linear but it is increasing, so a moderate agreement with the theory was achieved

Optimisation of aspect ratio



Bending of paper

Unlike playing cards, paper strips to bend during the fall



Typical bending



The core of our device is two glued halves of A4 sheet.

Optimisation – longer edges folded

Typical bending during tumbling of paper with a/b=1/6



With longer edges folded



Optimisation – shorter edges folded

The core of the device are two glued halves of A4 sheet



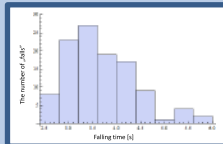
Instabilities (chaotic motion) occur less frequently for paper with shorter edges folded

The experiment determining the influence of winglets on falling time



The falling time of our final device

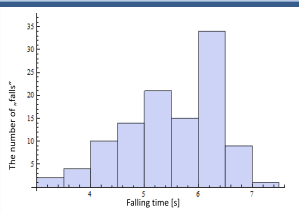
Histogram of falling times of the device without winglets – 110 falls



The mean fall time – 3.66s
Standard deviation – 0.52s
Highest fall time – 5.9s
Lowest fall time – 2.4s

The falling time of our final device

Histogram of falling times of the device with winglets – 110 falls



The mean fall time – 5.49s
Standard deviation – 0.69s
Highest fall time – 7s
Lowest fall time – 3.4s

The final design



Conclusions

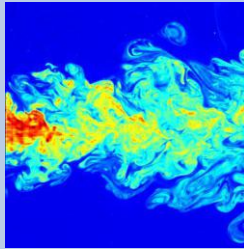
1. During the fall of thin rectangular bodies, autorotation may arise and so lift force may be generated.
2. Simple theoretical treatment allows to qualitatively predict the parameters of motion.
3. Rigid cards with biggest aspect ratio allow slowest fall. However, aspect ratio is limited by bending.

Literature

- P. Dupleich, *Rotation in free fall of rectangular wings of elongated shape*, NACA Technical Memo No. 1201 (1941)
- J. C. Maxwell, *Scientific Papers of J. C. Maxwell*, New York, 1940, Vol. 1, p. 115.
- D.L. Finn, *Falling paper and flying business cards*, SIAM News, 4, 40 (2007)
- A. Belmonte et al., *From flutter to tumble: Inertial Drag and Froude Similarity in Falling Paper*, Phys. Rev. Lett. 2, 81 (1998)
- L. Mehadevan et al., *Tumbling cards*, Phys. Fluids 1, 11 (1999)

Turbulent flow

In turbulent flow is characterized by the lack of stream lines.



15.
Frustrating golf ball
Matej Badin

Problem

- It often happens that a **golf ball escapes** from the hole an instant after it has been **putted** into it. Explain this phenomenon and investigate the conditions under which it can be observed.

Content

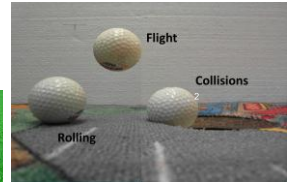
- Definitions
- Analysis of the motion
 - Rolling, flight, collisions
- Simulation
- Experiment
- Conclusion

Our definitions

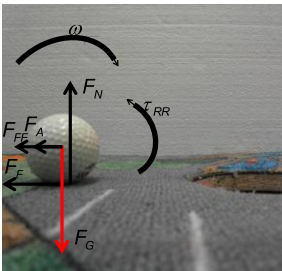
- Golf ball
- USGA** norm – diameter not less than 4.267 cm
 - Our ball – $d=4.27$ cm
- Hole – cylinder
 - Diameter = 10.8 cm
 - Depth = 10.2cm



Analysis of the motion



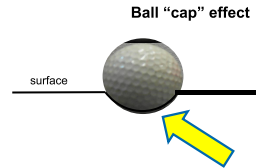
Rolling



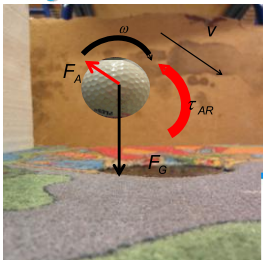
- F_G - Gravity force
- F_N - Normal force
- F_F - Friction force
- F_A - Air drag force
- τ_{RR} - Rolling resistance torque
- F_{FF} - Rolling resistance force

Another resistance in rolling

- Resistance **torque** to rotation around perpendicular axis to horizontal caused by **deformation of the surface or the ball**



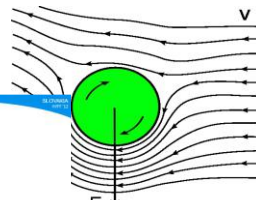
Flight



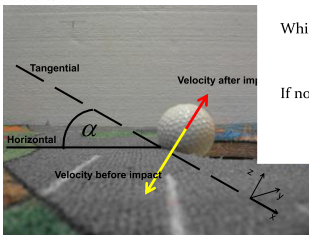
- F_G - Gravity force
- F_A - Drag force
- τ_{AR} - Drag torque

Magnus-Robins effect

- Force caused by **pressure difference** on forward and backward moving side of spinning object.



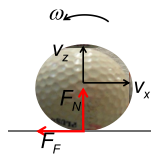
Collisions



- While slipping $F_F = fF_N$
- If not $F_F = 0$

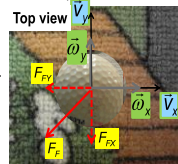
Collision

Translation motion and rotation
Normal force and friction



Slipping during collision

Direction of friction force is opposite to the direction of a contact point



During small time Δt

$$f^2 F_N^2 = F_{Fx}^2 + F_{Fy}^2$$

$$\frac{F_{Fx}}{F_{Fy}} = \frac{v_x - \omega_y R}{v_y + \omega_x R}$$

Used trick - simulation

- Use of Newton's laws
- Simulation over small increments of the momentum Δp
- Calculation of $\Delta p_x, \Delta p_y$ from :

$$f^2 F_N^2 = F_{FX}^2 + F_{FY}^2 \implies f^2 \Delta p^2 = \Delta p_x^2 + \Delta p_y^2$$

$$\frac{F_{FX}}{F_{FY}} = \frac{v_x - \omega_y R}{v_y + \omega_x R} \implies \frac{\Delta p_x}{\Delta p_y} = \frac{v_x - \omega_y R}{v_y + \omega_x R}$$

Simulation

$$v_x \implies v_x - \frac{F_{FX}}{m} \Delta t = v_x - \frac{1}{m} \Delta p_x$$

$$v_y \implies v_y - \frac{F_{FY}}{m} \Delta t = v_y - \frac{1}{m} \Delta p_y$$

$$v_z \implies v_z + \frac{\Delta p}{m}$$

$$\omega_x \implies \omega_x - \frac{F_{FY}}{I} R \Delta t = \omega_x - \frac{R}{I} \Delta p_y$$

$$\omega_y \implies \omega_y - \frac{F_{FX}}{I} R \Delta t = \omega_y - \frac{R}{I} \Delta p_x$$

$$\omega_z \implies \omega_z - \frac{M(F_N)}{I} \Delta t \text{ (Ball cap effect)}$$

Summary of effects

Summary of coefficients

Rolling

- Rolling resistance arm
- Shape coefficient
- Contact area radius
- Frictions coefficients
- Moment of inertia
- Mass
- Radius

Fly

- Shape coefficients

Collisions

- Friction coefficient
- Coefficient of restitution on the edge (in dependence on angle)

Rolling

- Rolling resistance
- Air resistance
- Air resistance torque
- Ball cap effect (deformation of ball or deformation of surface)

Fly

- Magnus effect
- Air resistance
- Air resistance torque

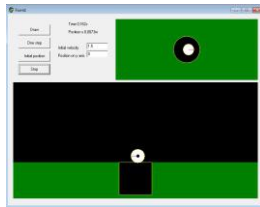
Collisions

- Sliding
- Ball cap effect

Negligible

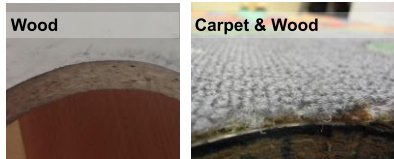
Simulation

- Used described theory
- In collisions we rotated the frame of the reference



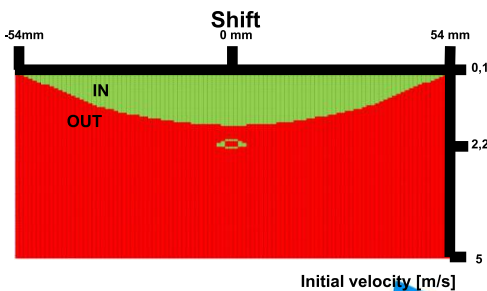
Experiments

- Two surfaces

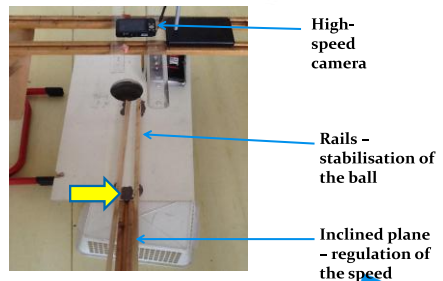


Smooth and smooth edges Similar to grass and rough edges

Wood simulation results



First experimental setup



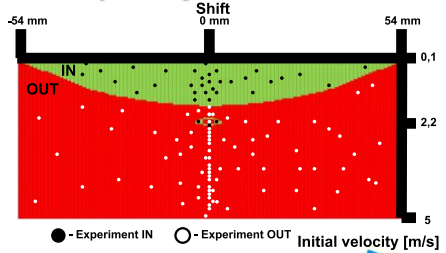
Second experimental setup



Inclined plane
- regulation of
the speed

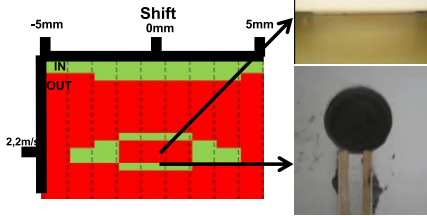
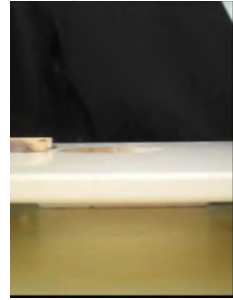
Place of
high-speed
camera

Theory vs. experiment



Comparison - Escape

No shift
Velocity - 2,2 m/s



Comparison - Jump

No shift
Velocity - 2,0 m/s



Comparison - Too much

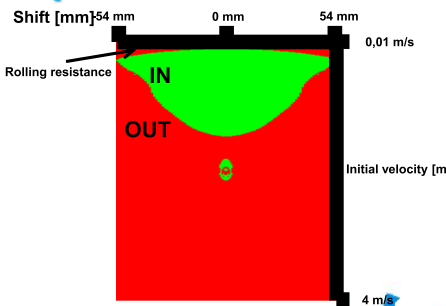
No shift
Velocity - 4,0 m/s



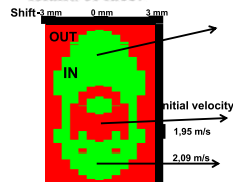
Carpet simulation



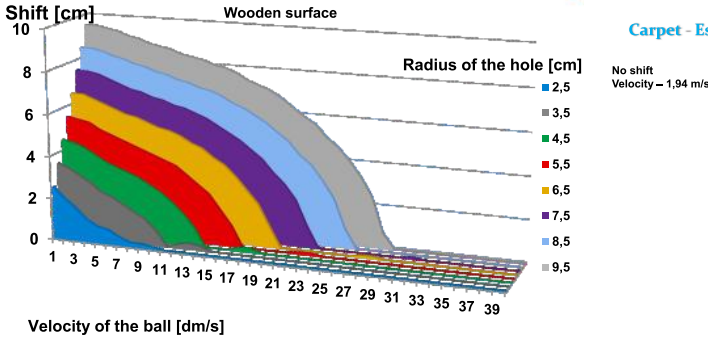
Carpet simulation result



Island or face?



Influence of diameter of the hole



Conclusion

- The most important parameters and effects
 - Rotation
 - Slipping during collision
 - Coefficient of restitution on the edge
 - Coefficient of restitution in the hole

Conclusion

- We developed the model of the motion and collisions
- We theoretically **predicted** the path of the ball
- Prediction **correlates** with experiment
- We explained the most important parameters under which can be phenomenon observable

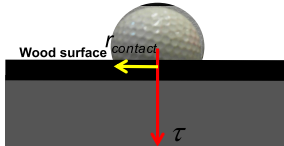
Deformation of the ball

We assume that normal force is evenly distributed to a contact area.
Torque acts on ball in z-axis:

$$\tau = \frac{fF_N}{\pi r_{contact}^2} \int_0^{r_{contact}} 2\pi x^2 dx$$



$$\tau = \frac{2}{3} fF_N r_{contact}$$



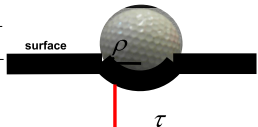
Ball cap effect

We assume that normal force is evenly distributed to a contact area.
Torque acts on ball in z-axis:

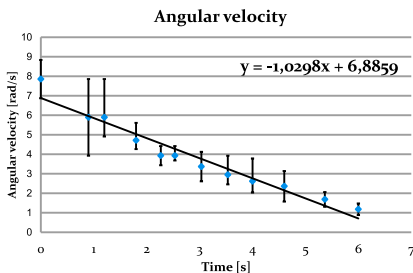
$$\tau = \frac{fF_N}{2\pi R(R - \sqrt{R^2 - \rho^2})} \int_0^{R - \sqrt{R^2 - \rho^2}} 2\pi \sqrt{(R-x)^2 - \rho^2} dx$$



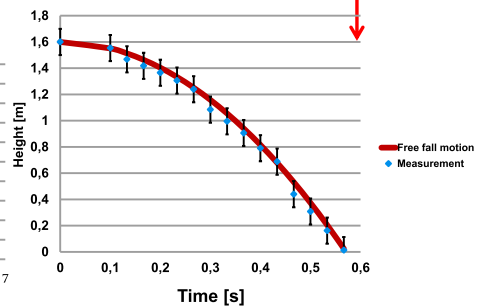
$$= fF_N \frac{\frac{2}{3} R^3 - \rho^2 \sqrt{R^2 - \rho^2}}{R^2 - R\sqrt{R^2 - \rho^2}}$$



Measurement of coefficients



Drag



Summary of coefficients

Carpet

- Rolling resistance $arm - 0.0015 m$
- Shape coefficient (Air drag) – 0.1
- Coefficient of restitution Ball with
 - Carpet 0.4
 - Hole bottom 0.8
 - Hole walls, edges 0.55
- Friction coefficients - ball with
 - Carpet 0.23
 - Hole bottom 0.12
 - Hole walls 0.1
 - Hole edges 0.4
- Ball cap effect – contact radius ball on :
 - Carpet - 0.0004 m
 - Bottom –0.0001 m



4. Fluid bridge

Reporter: Shiva Azizpour



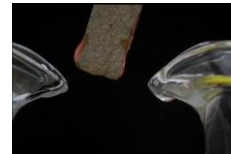
Question

If a **high voltage** is applied to a fluid (e.g. **deionized water**) in two beakers, which are in contact, a fluid bridge may be formed. **Investigate** the phenomenon.



Sparks

The sparks will appear as the bridge breaks down.



Phenomena observation

- Bridge Formation
- Unstable bridge
- Temperature increase
- Flow visualization
- Sparks

Theory

- Formation
 - Sparks
 - Droplets
 - Connection
- Stability
 - Electrical Force
 - Surface tension
 - Gravitation
- Formulation
 - Diameter vs. X
 - critical voltage
 - Reactions in the bridge

Experiments

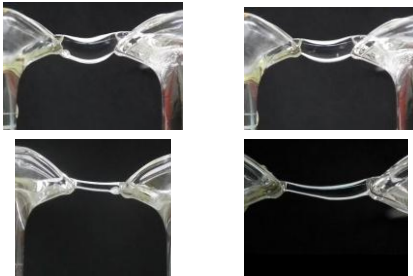
- Setup
- oscillation
- Image Processing
 - Diameter vs. time
 - Deflection vs. time
 - Dip vs. Diameter
 - Diameter vs. X
- Voltage Reduction
 - minimum voltage
 - vs. Length

Main Approach

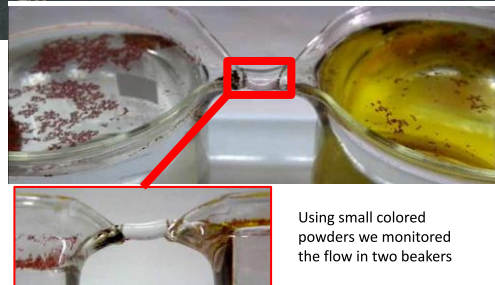
Conclusion

- Theory vs. Experiments
 - minimum Voltage vs. Length
 - Dip vs. Diameter
 - Diameter vs. X
- Reactions in the beakers
- Stratification

Is the Bridge Always Stable?



Detecting the Water Flow



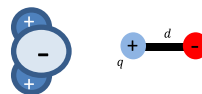
Using small colored powders we monitored the flow in two beakers

Temperature Increase



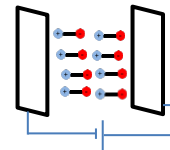
An **increase in the temperature** of the water passing through the bridge would change the refractive index.

Dipole in an Electric Field Force Application



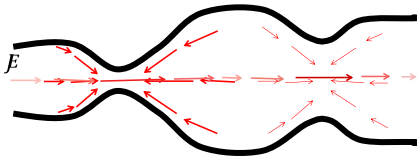
$$P = q \cdot d = \epsilon_0(\epsilon_r - 1)E$$

$$F = P \cdot \nabla E$$



ϵ_0 : vacuum permittivity
 ϵ_r : relative permittivity of the dielectric
 P : dipole moment

Electric Force



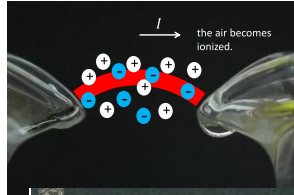
$$E = \rho \cdot J$$

$$J = \frac{I}{A}$$

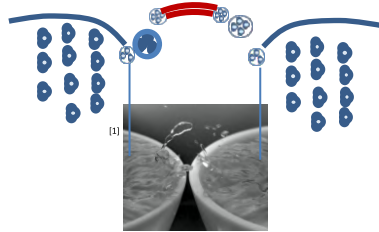
ρ : specific conductance
 A : cross sectional area
 J : current density

Spark

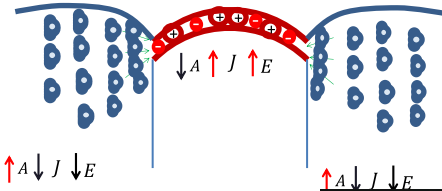
spark is formed when the electric field strength exceeds the dielectric field strength of air.



Droplet Ejection

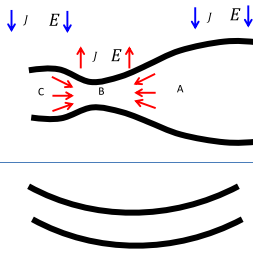


[1] <http://www.ttm.tugraz.at/jw/?seite=water>



The force applied to the dipoles is in the direction of electric field's gradient.

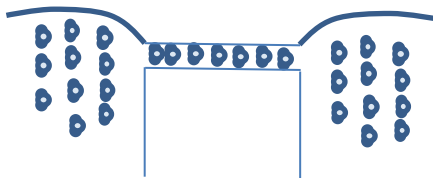
Tending to the Stable Condition



Unstable
 Molecules in A and C will be attracted toward B.

stable

Connection



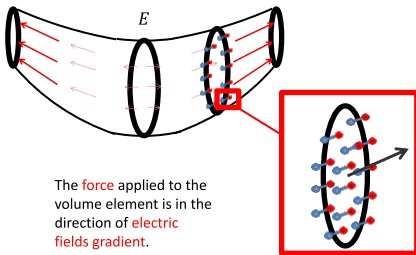
Water is a stronger conductor than air



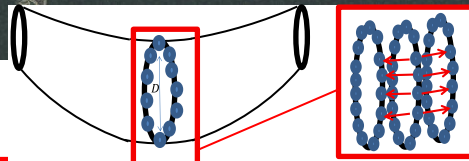
There will be a current through water rather than air.

Surface Tension

Electric Force



The force applied to the volume element is in the direction of electric fields gradient.

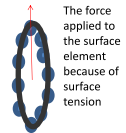


$$\tau = \pi D \gamma$$

$$dl = R d\theta$$

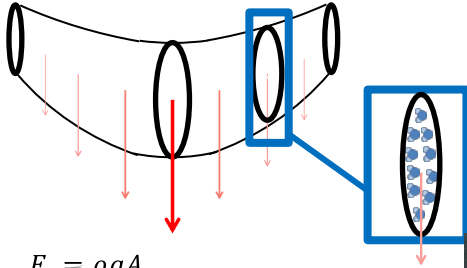
$$F_s = \frac{2\tau d\theta}{dl} = \frac{2\tau}{R}$$

γ : surface tension coefficient
 R : radius of curvature



The force applied to the surface element because of surface tension

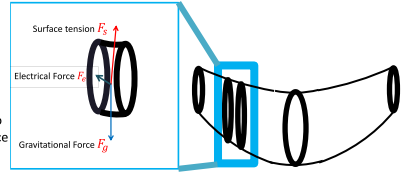
Gravitation



$$F_g = \rho g A$$

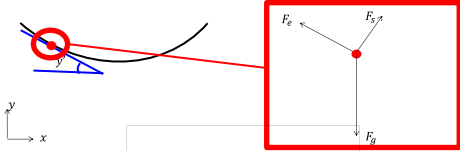
What is Holding the Bridge Against Gravity?

Since there is a slight sag in the bridge the main force cancelling the gravitational force is **Surface tension force**.



Force applied to the surface element due to gravitation

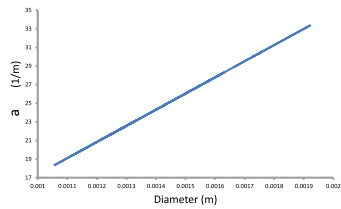
Equilibrium



The bridge is stable So $\sum F = 0$

Sag of the Bridge - D_0

in the center of the bridge: $F_g = F_s \rightarrow a = \frac{\rho g L D_0}{32\gamma}$



Formulation

- Surface tension $F_s = \frac{2\tau}{R}$
- Electrical force $F_e = \frac{K}{D^3} \frac{\partial D}{\partial x}$
- Gravitational Force $F_g = \rho g A$

$$K = \frac{-32}{\pi^2} \epsilon_0 (\epsilon_r - 1) \rho_s^2 l$$

$$\frac{\partial A}{\partial x} = k^2 A^3 x$$

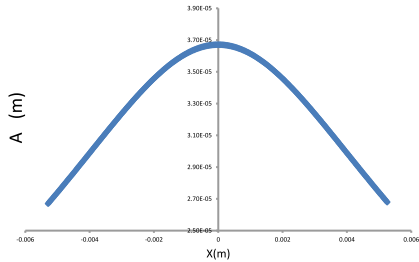
$$A = A(x)$$

Equilibrium of forces:

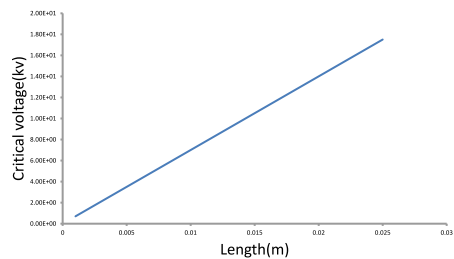
Along the bridge: $F_e + F_s \sin\theta = 0$

Perpendicular to the bridge: $F_s - F_g \cos\theta = 0$

$A(I, x, A_0)$ vs. x



critical Voltage vs. length



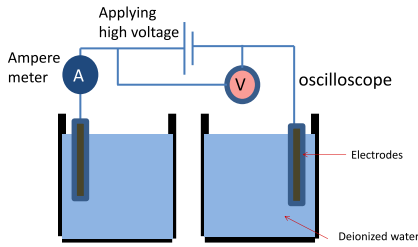
How to Reach a Limitation for Stability

$$R = \rho_s \int_{L/2}^{-L/2} \frac{dx}{A(x, I, D_0)}$$

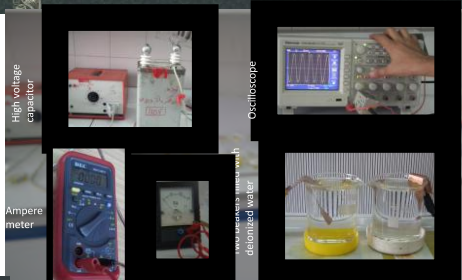
$$V = R I$$

$$\lim_{I \rightarrow 0} V_{(I, L, D_0)} = V_{critical}(L, D_0)$$

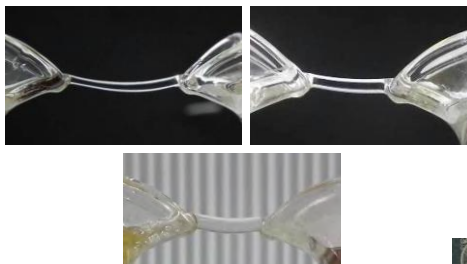
Set Up
Schematic



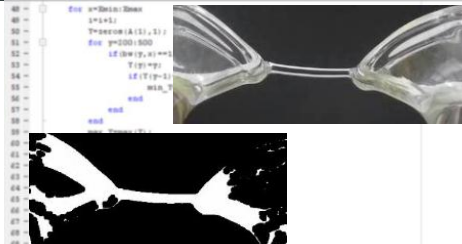
Experimental Set up



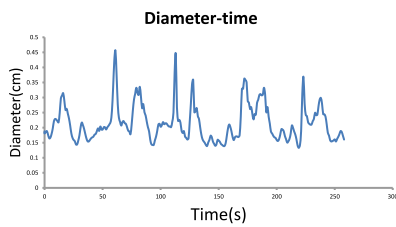
Diameter's oscillation



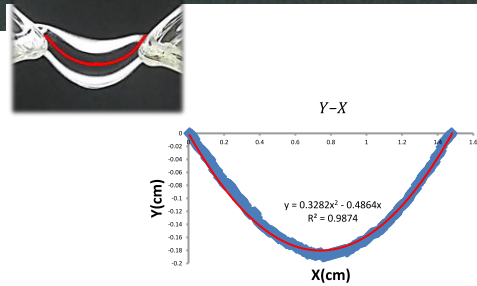
Video Processing:



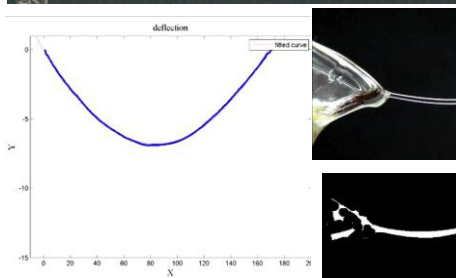
Diameter's oscillations



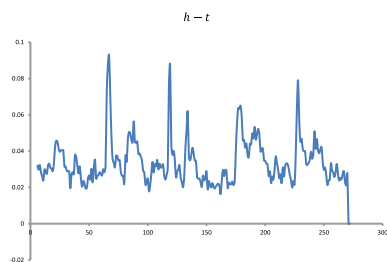
Bridge Coordinate



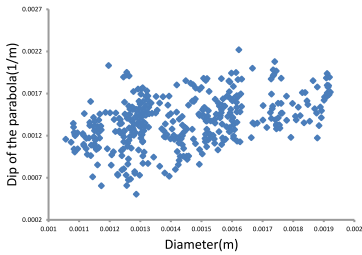
Deflection oscillation



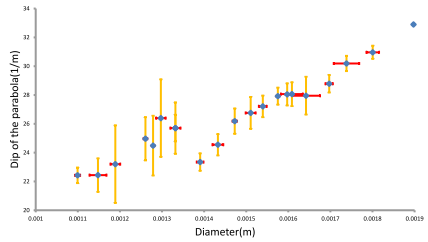
Deflection's Oscillations



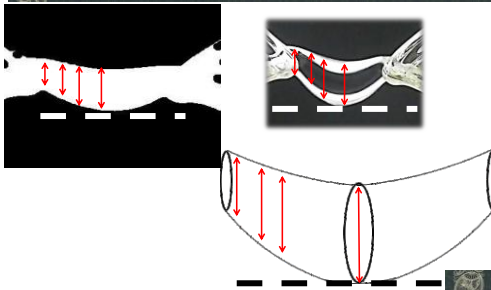
a vs. Maximum Diameter



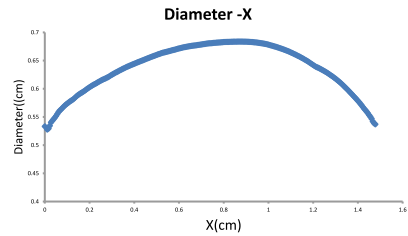
a vs. Maximum Diameter
statistical operation



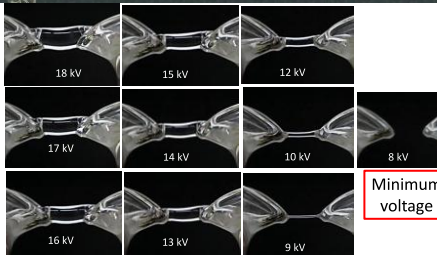
Changes of the Diameter



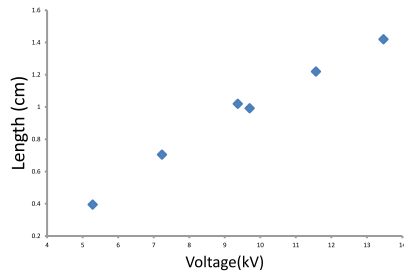
Diameter vs. X



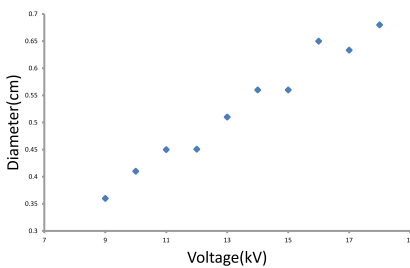
Voltage Reduction



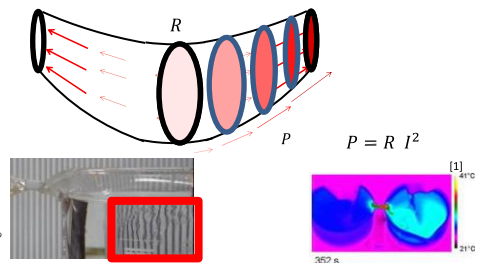
Voltage limit



Maximum Diameter vs. Voltage

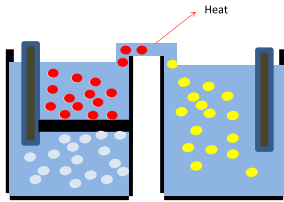


Heating

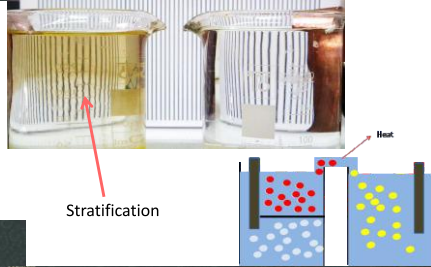


[1]Experiments in a floating water bridge. Jakob Woisetschla¹ger- Karl Gatterer -Elmar C. Fuchs

Stratification

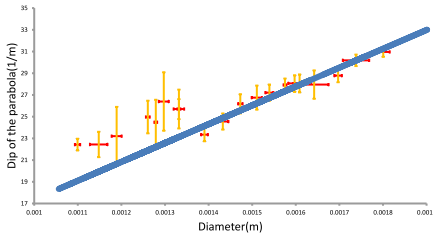


Stratification



a vs. diameter

Theory vs. Experiment



[1] Description of Spherical Aberration and Coma of a Microlens Using Vector Diffraction Theory Glen D. Gillena and Shekhar G.

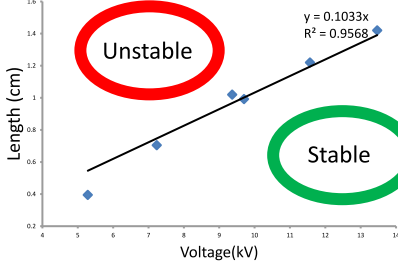
What is holding the bridge?!

The comparison verifies our theory which stated :

The force that is holding the bridge against gravity is the surface tension....

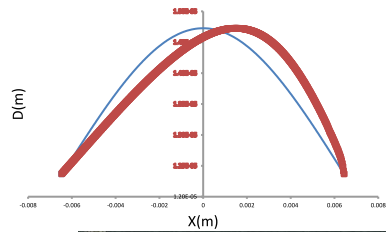
Minimum Voltage vs. Length

Theory vs. Experiment



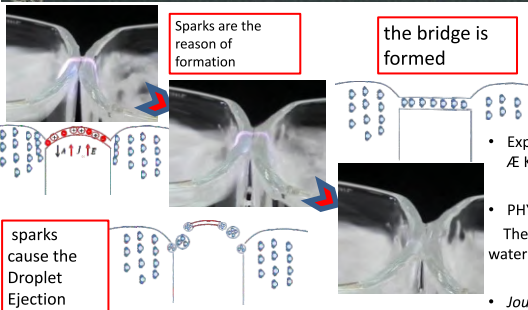
Diameter vs. X

Theory vs. Experiment



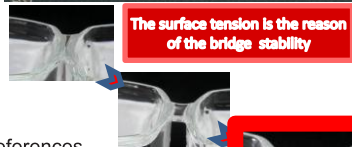
Conclusion

Formation



Conclusion

Stability



References

- Experiments in a floating water bridge „Jakob Woisetschla“ger Æ Karl Gatterer Æ Elmar C. Fuchs(2010)
- PHYSICAL REVIEW E 80, 016301 2009 Theory of the Maxwell pressure tensor and the tension in a water bridge. A. Widom, J. Swain, and J. Silverberg
- Journal of Electrostatics, 26 (1991) 143-156 Elsevier. Journal of Electrostatics, 26 (1991) 143-156 Elsevier. A. Ramos and A. Castellanos

Stearin engine

Solution of the problem "Stearin engine"

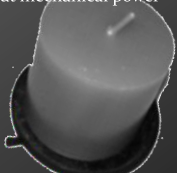
presented by
Belarusian team

Nankai University, 2009

Stearin engine

Formulation of the problem

A candle is balanced on a horizontal needle placed through it near its centre of mass. When the candle is lit at both ends, it may start to oscillate. Investigate the phenomenon. Maximize the output mechanical power of the system.



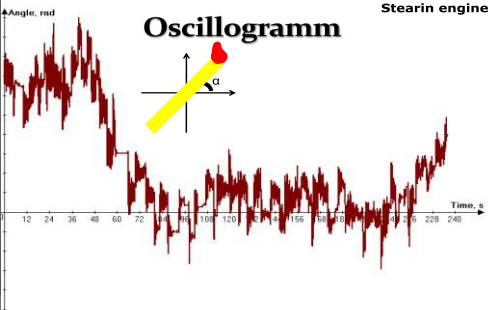
Stearin engine

Demonstration of the effect



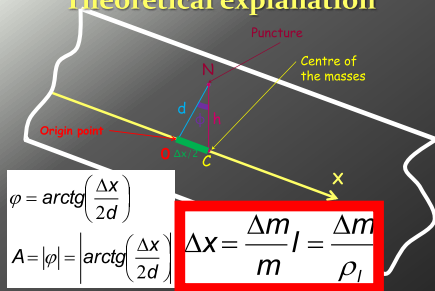
Stearin engine

Oscillogramm



Stearin engine

Theoretical explanation



$$\varphi = \arctg\left(\frac{\Delta x}{2d}\right)$$

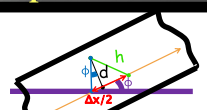
$$A = |\varphi| = \left| \arctg\left(\frac{\Delta x}{2d}\right) \right|$$

$$\Delta x = \frac{\Delta m}{m} l = \frac{\Delta m}{\rho l}$$

Stearin engine

Complicated Explanation

$\Delta x > 0$ when drops from right end




$$\varphi = \arctg\left(\frac{\Delta x / 2 - d * tg \varphi_0}{d}\right) - \varphi_0$$

$$A = |\varphi| = \left| \arctg\left(\frac{\Delta x / 2 - d * tg \varphi_0}{d}\right) - \varphi_0 \right|$$

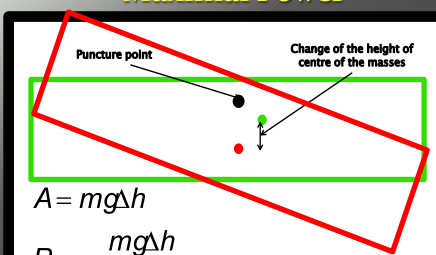
Stearin engine

Puncture near the centre of the masses



Stearin engine

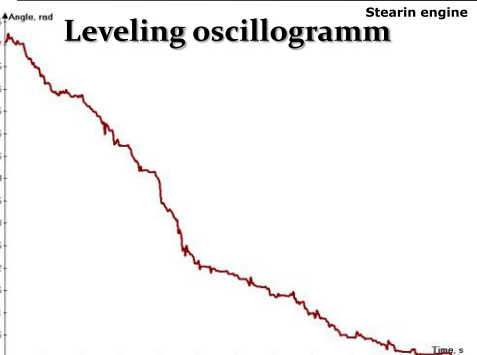
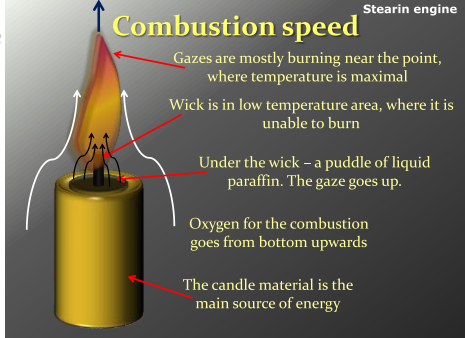
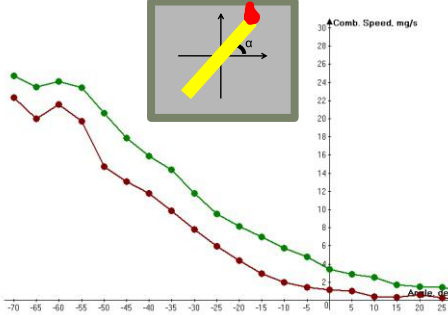
Maximal Power



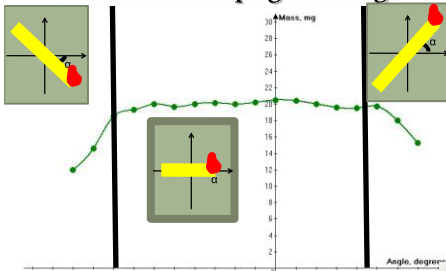
$$A = mg\Delta h$$

$$P_{\max} = \frac{mg\Delta h}{T_{\text{drop}}}$$

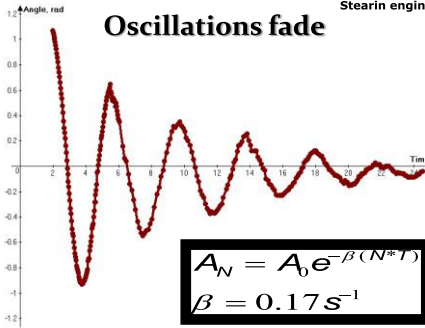
Plot combustion speed (ξ) against angle



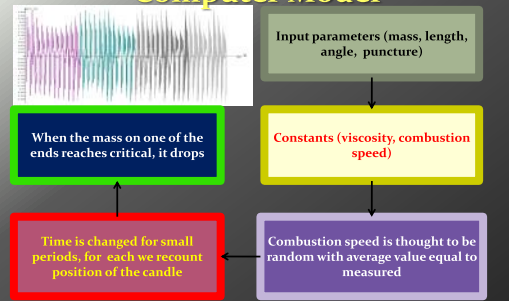
Plot mass of drop against angle

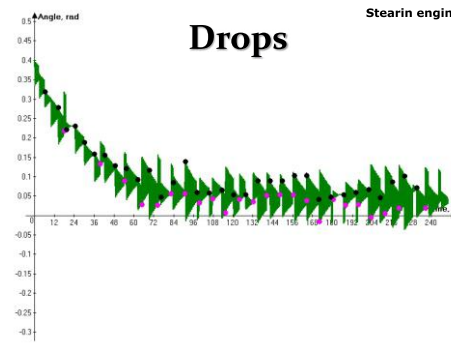
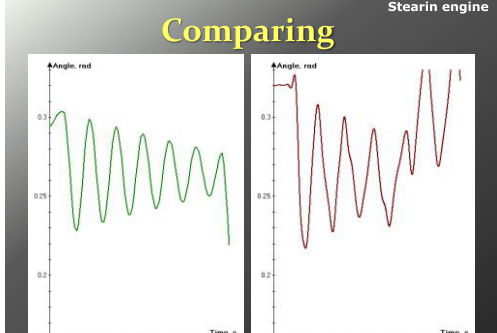
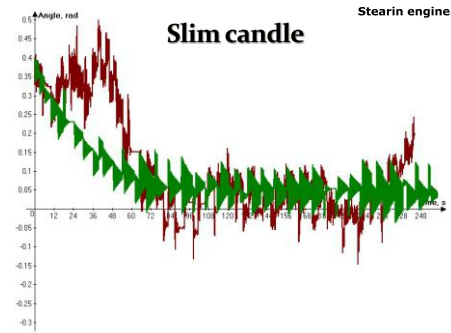


Oscillations fade



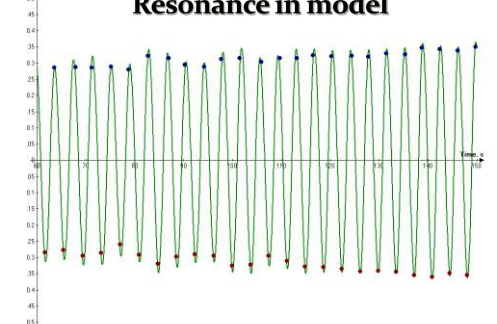
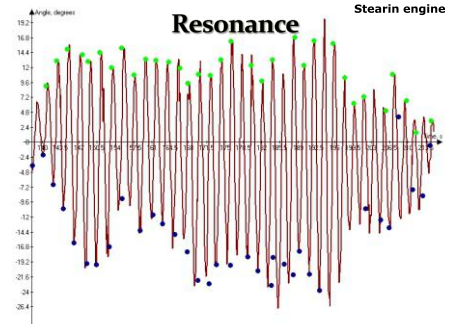
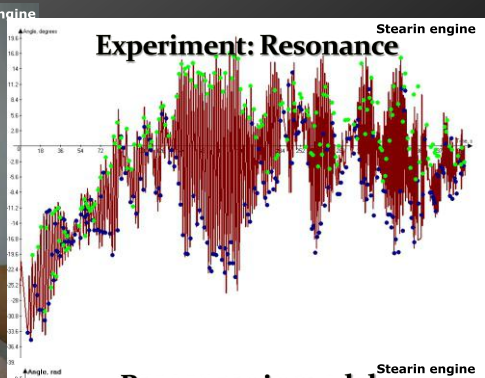
Computer Model

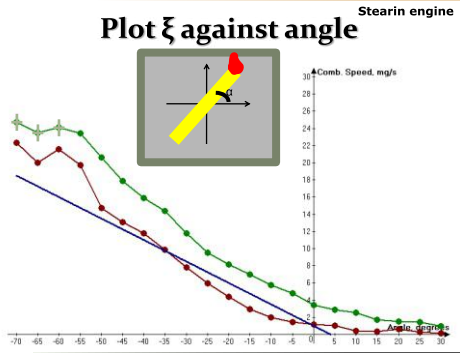
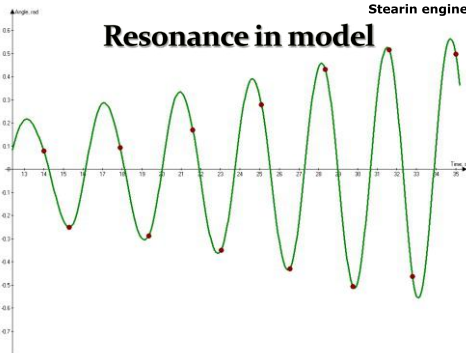




Period of oscillations

Free oscillation period of a candle:

$$T = 2\pi \sqrt{\frac{l}{mgd}} = \pi \frac{l}{\sqrt{3gd}}$$




Resonance conditions

We'll suggest, that resonance occurs, when $T_{free} = T_{drop}$, and will find conditions, needed for the resonance. To do this, we'll find T_{drop} :

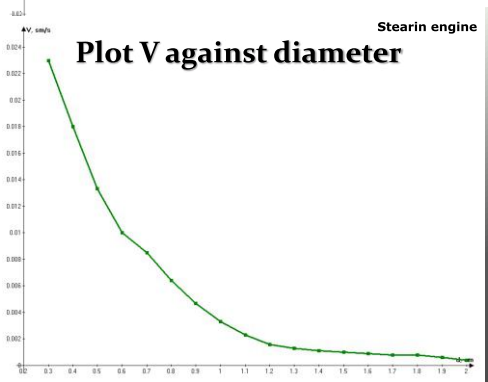
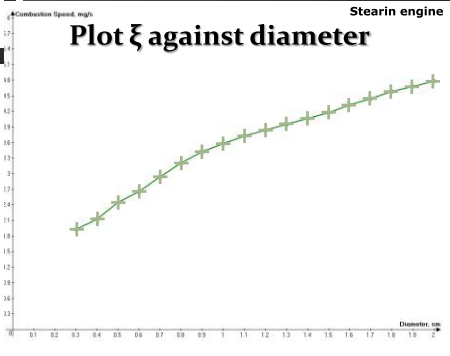
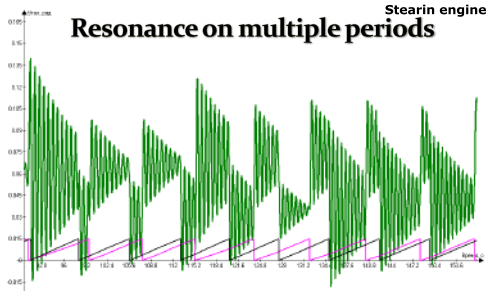
$$m_d = \int_0^{T_{drop}} \xi_i dt = \int_0^{T_{drop}} (2.85 - 0.25 A \sin(\zeta_0 + \omega t)) dt$$

Resonance conditions

$$m_d = 2.85T + \frac{A \cos(\zeta_0 + 2\pi) T}{8\pi}$$

$$m_d = \frac{l}{\sqrt{3gd}} \left(2.85\pi + \frac{\arctg\left(\frac{m_d}{2d\rho_l}\right) \cos \zeta_0}{8} \right)$$

$l \approx 250 \text{ sm } d=0.001 \text{ m } R=0.003 \text{ m}$



Massive candles

We will separate a class of massive candles. Massive candles are such ones, which can't move when one drop falls. It means that the moment of the force is less than moment of frictional forces. Such candle may not oscillate in basic mode, even if punctured at the center of the masses.



Stearin engine

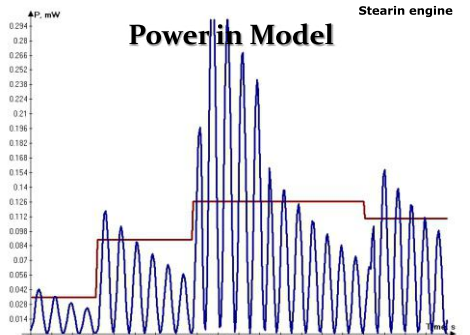
Counting output power

$$A_i = m_c g (h_2 - h_1)$$

$$P_i = \frac{A_i}{T_{drop}} = \frac{A_i}{N * T} =$$

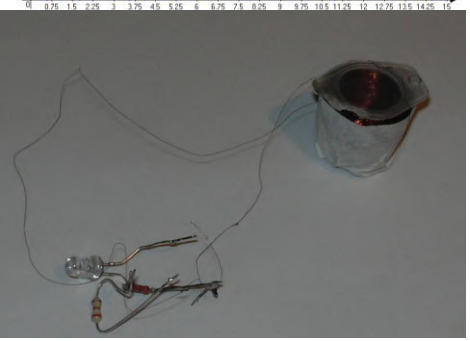
$$= \frac{mg \sqrt{d^2 + (\frac{\Delta x}{2} - d * tg\varphi)^2} - \frac{d}{\cos\varphi} + \frac{\Delta x}{2} \sin\varphi}{N * \pi d / \sqrt{3g \sqrt{d^2 + (\frac{\Delta x}{2} - d * tg\varphi)^2}}}$$

$\Delta x > 0$
right



Stearin engine

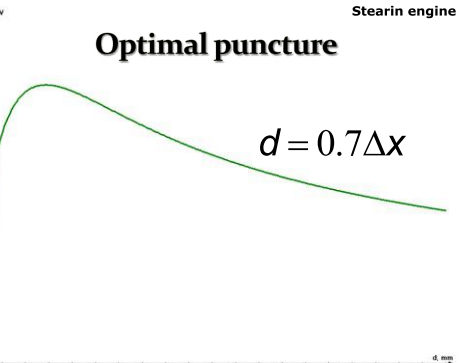
Usage of Power

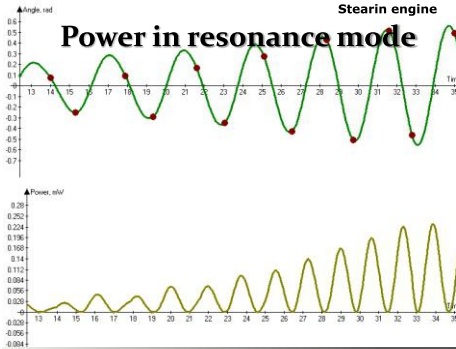
$$P = \frac{\langle \varepsilon \rangle^2}{R}; \langle \varepsilon \rangle \sim \frac{\Phi_0}{T} \sim A\omega; P \sim A^2\omega^2$$


Stearin engine

Output Power

$$P \sim A^2\omega^2 \sim \frac{\arctg(\frac{\Delta x}{2d}) \sqrt{d}}{l}$$

$$\Delta x = \frac{\Delta m}{\rho \pi R^2}$$




Stearin engine

Results:

- The effect of oscillations was explained
- We created a computer model, which lets us imitate effect of the problem
- We found the parameters, which the oscillations depend on.
- We defined optimal parameters for achieving maximal output mechanical power of the system.
- We created a device, transforming the power of oscillation into electromagnetic oscillations.

How fast the candle burns

$$\xi = 50 \cdot R^* e^{-0.04}$$

Stearin engine

Conclusions:

- Stearin engine is not a heat machine
- We need to use candles from paraffin of about 1sm wide.
- The resonance effect is the basic of getting maximal power.
- The optimal puncture is needed for the power to be maximal.
- The best way of using Stearin engine is transforming its mechanical energy into electromagnetic.

Viscosity

$$M_{res} = 2 * \int_0^{l/2} k \sqrt{l} dl = 2 \int_0^{l/2} k \omega l^2 dl = 2k\omega \frac{l^3}{3 * 8}$$

$$\Rightarrow \beta = \frac{k l^3}{12 l} = \frac{k l^2}{m l^2} = \frac{k}{\rho * \pi R^2} = \frac{k}{\rho * \pi R^2}$$

$$M_{fr} = \mu N * r = \mu m g r = \mu \rho l \pi R^2 r$$

Stearin engine

Literature:

- Slobodjanuk A. I. "Advance high school physic"
- Slobodjanuk A. I. "Computer model of physical processes for high school students"
- Y. A. Smorodinsky "Temperature"

Problem No. 1

Invent yourself

Report : F.Mokhtari



Initial information

Initial information

P : axial force

V : shear force

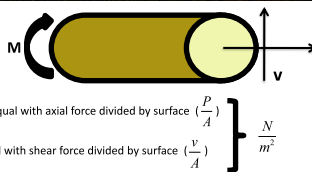
M : momentum

σ : normal stress that is equal with axial force divided by surface $(\frac{P}{A})$
 τ : shear stress that is equal with shear force divided by surface $(\frac{V}{A})$

I : Area moment of inertia

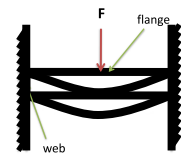
E : young's module

Q : first moment of area



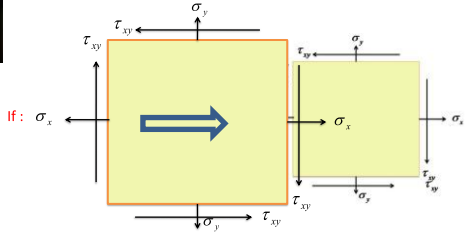
Bending :

Is a deformation that happen due to resistance against transverse force (F) and it happen in all magnitude of transverse force (F), we have maximum bending , in this point section will be



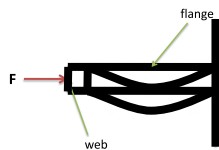
Step 1 : yield or fracture criteria

Von mises is a equation for finding the maximum synthetic stress in an element



Buckling :

Is a deformation that happen after a magnitude of axial force, it does not have any resistance against axial force and it makes a instability in a section, different parts in a structure shall not be buckle.

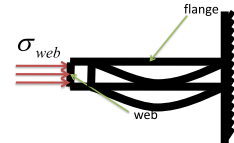


This equation shows the maximum stress that we can have before buckling.

$$\sigma_{buckling} = k \times E \times (\frac{t}{b})^2$$

t: thickness

If : $\sigma_{web} > \sigma_{buckling}$



$$\sigma_{web} \leq \sigma_{buckling}$$

collapse

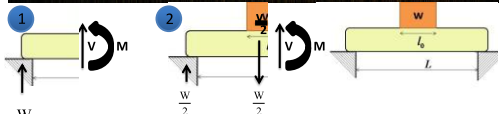
$$\sigma_{von\ mises} \leq \sigma_{ultimate}$$

$\sigma_{ultimate}$: ultimate resistance that consider to type of substance $\sigma_{ultimate} = 50\ Kg/cm^2$

$$\sigma_{von\ mises} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \times \sigma_y + 3 \times \tau_{xy}^2}$$

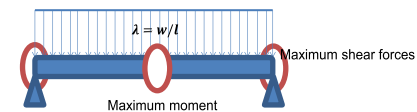
$\sigma_{von\ mises} > \sigma_{ultimate}$

Step 2: internal forces



$$1 \left\{ \begin{aligned} \sum F(y) = 0 &\rightarrow \frac{W}{2} + V = 0 \rightarrow V = -\frac{W}{2} \\ \sum M = 0 &\rightarrow M - \frac{W}{2} \times (\frac{L}{2} - \frac{l_0}{2}) = 0 \rightarrow M = \frac{W}{2} \times (\frac{L}{2} - \frac{l_0}{2}) \end{aligned} \right.$$

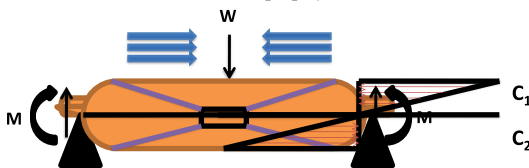
$$2 \left\{ \begin{aligned} \sum F(y) = 0 &\rightarrow \frac{W}{2} - \frac{W}{2} - V = 0 \rightarrow V = 0 \\ \sum M = 0 &\rightarrow M - \frac{W}{2} \times (\frac{L}{2} - \frac{l_0}{4}) = 0 \rightarrow M = \frac{W}{2} \times (\frac{L}{2} - \frac{l_0}{4}) \end{aligned} \right.$$



$$V = \int -\lambda dx \rightarrow V = -\lambda x + \frac{W}{2}$$

$$M = \int V dx \rightarrow M = -\frac{\lambda x^2}{2} + \frac{W}{2}x$$

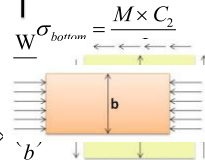
We will construct our bridge considering the maximum moment and shear forces



$$\tau_{xy} = \frac{\sigma_{top}^{op}}{V \times Q} = \frac{M \times C_1}{I} \times \frac{W}{V} \rightarrow \tau_{xy} = \frac{V}{I} \sigma \leq \sigma_{top-buckling}$$

$$\tau_{xy-Buckling} = k \times E \times (\frac{t}{b})^2 \rightarrow k=6 \rightarrow \tau_{xy-Buckling} = 6 \times E \times (\frac{t}{b})^2$$

$$\tau_{xy-Buckling} = k \times E \times (\frac{t}{b})^2 \rightarrow k=8 \rightarrow \tau_{xy-Buckling} = 8 \times E \times (\frac{t}{b})^2$$

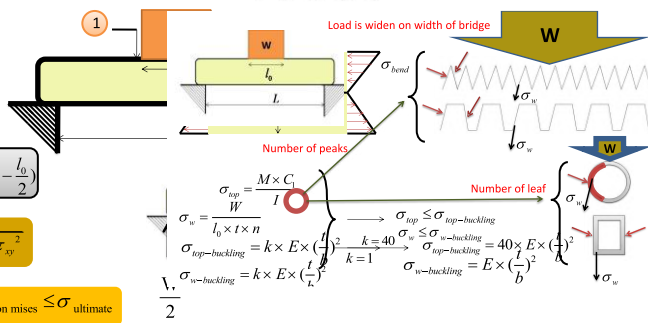


$$\tau_{xy} = \frac{V}{A_w} \rightarrow V = -\frac{W}{2}$$

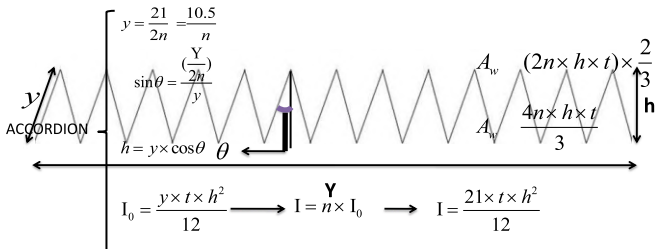
$$\sigma_{bottom} = \frac{M \times C_2}{I} \rightarrow M = \frac{W}{2} \times (\frac{L}{2} - \frac{l_0}{2})$$

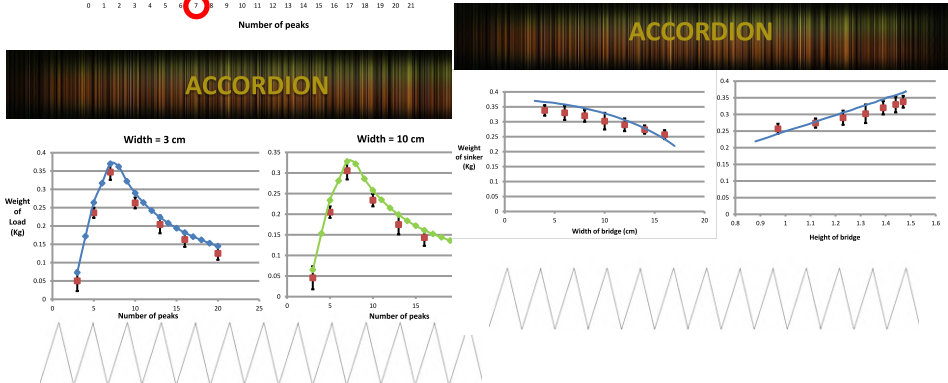
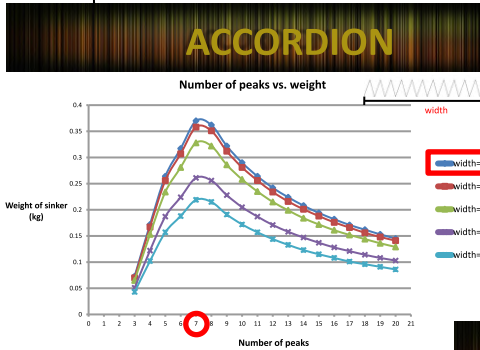
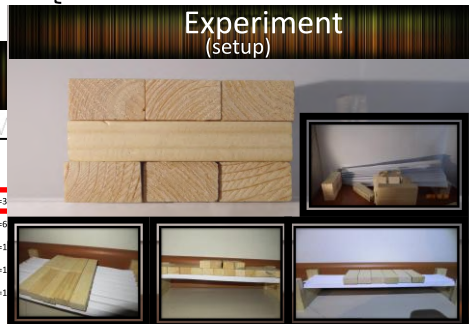
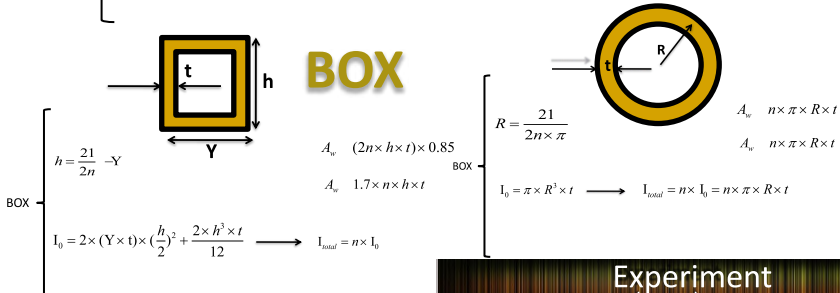
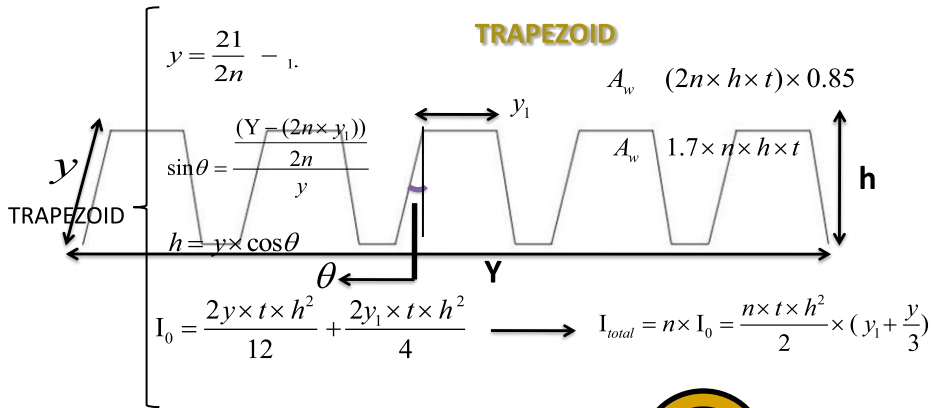
$$\sigma_{von\ mises} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \times \sigma_y + 3 \times \tau_{xy}^2}$$

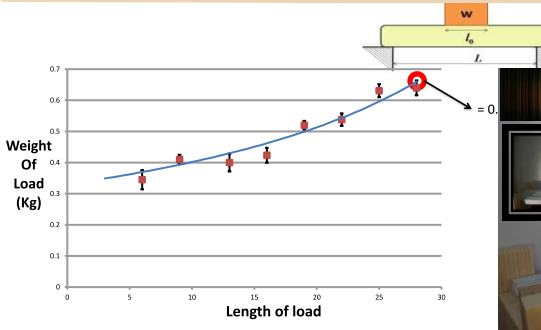
$$\sigma_y = 0 \quad \sigma_x = \sigma_{bottom} \quad \sigma_{von\ mises} \leq \sigma_{ultimate}$$



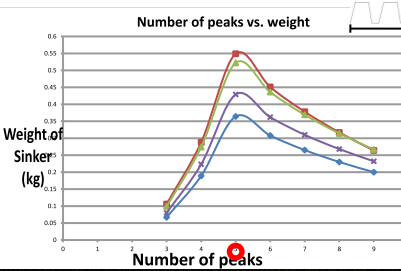
ACCORDION



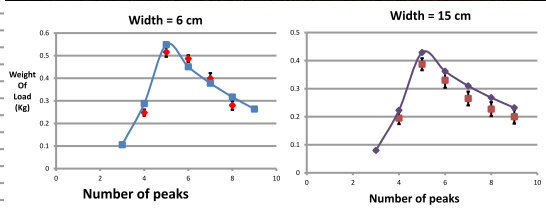




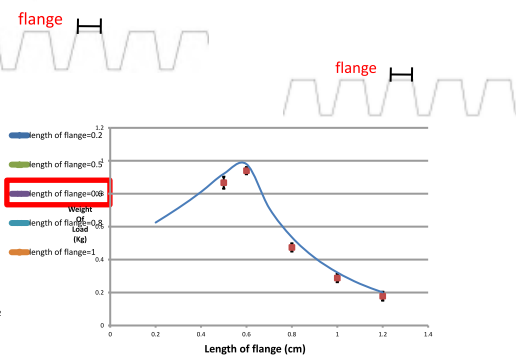
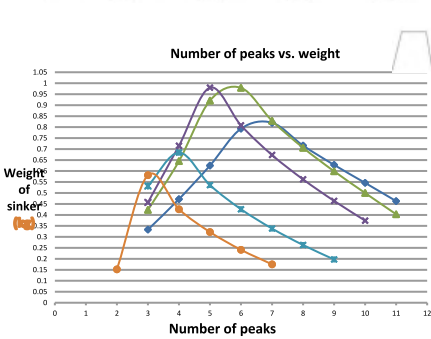
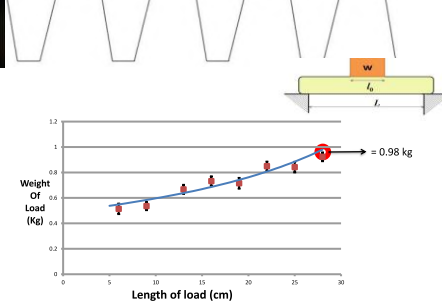
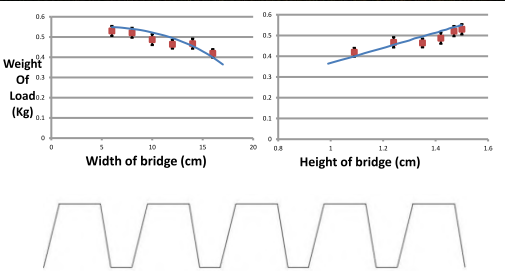
TRAPEZOID (THEORY RESULTS)

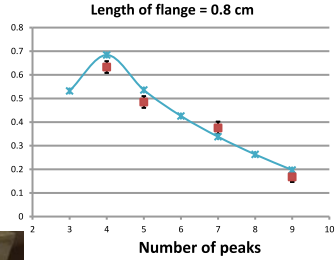
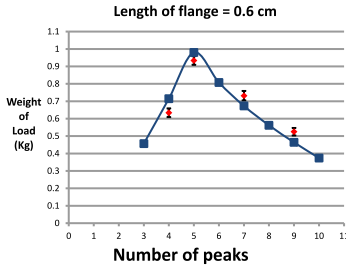


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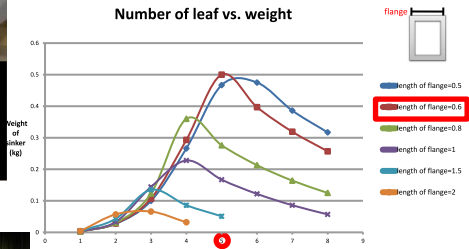


TRAPEZOID

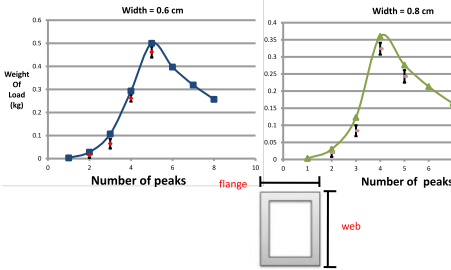




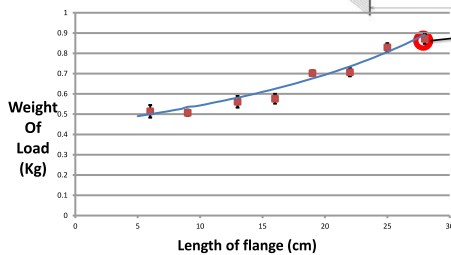
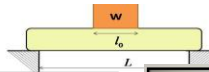
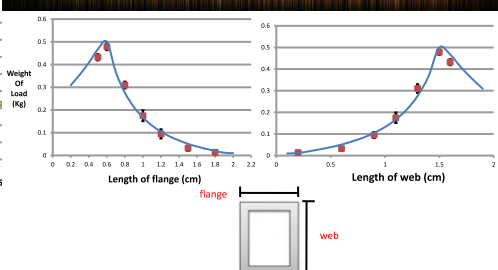
BOX (THEORY RESULTS)



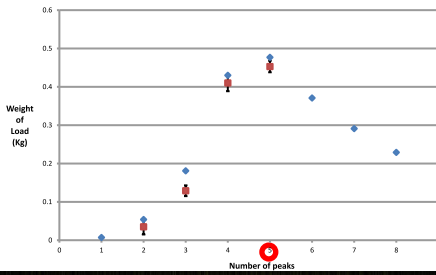
BOX



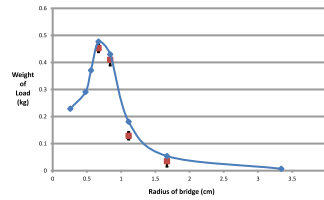
BOX



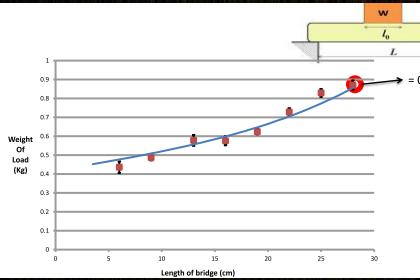
TUBE



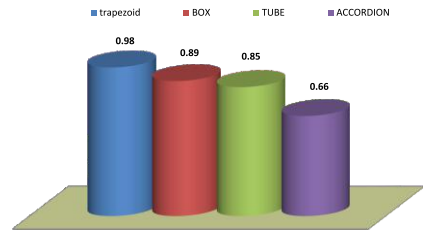
TUBE



TUBE



Maximum resistance

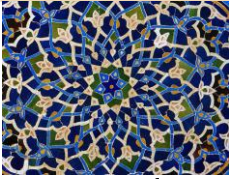


Review

In this research we find the best shape that have maximum resistance against weight and we calculate all the stresses and forces and we compares this parameters.

Reference

- Plastic analysis of plates and shapes _ professor Tomasz Wierzbicki
- Mechanics of materials _ Ferdinand P.Beer, E.Russel Johnston
- Mechanics of materials _ Egor P. Popov
- The stability of beams with buckled compression flanges _ Cherrys
- Handbook of structure stability _ Gerard G , Becker H
- Theory of plates and shells _ Stephan p. Timoshenko



Hovercraft

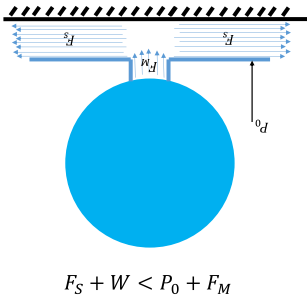
Amirhossein Ebadi

Problem
 A simple model hovercraft can be built using a CD and a **balloon** filled with air attached via a tube. Exiting air can **lift** the device making it float over a surface with **low friction**. Investigate how the relevant parameters influence the **time** of the 'low friction' state.

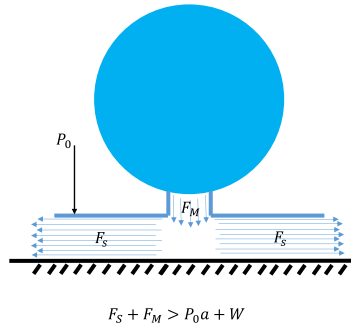
Observation



Second idea



First idea



Inconsistency

$$F_S + F_M > P_0 + W \quad \times$$

$$F_S + W < P_0 + F_M$$

$$F_S + W < P_0 + F_M$$

The pressure under the CD is less than the pressure of air

Adiabatic

$$p \cdot v^\gamma = \text{constant}$$

$$p_1 \cdot v_1^\gamma = p_2 \cdot v_2^\gamma \quad \rightarrow \quad p = \frac{\text{cons}}{v^\gamma}$$

(γ is $\frac{c_p}{c_v}$)

P: pressure

V: volume

Criticizing the Navier-Stocks

Navier-Stokes momentum equation (compressible fluid)

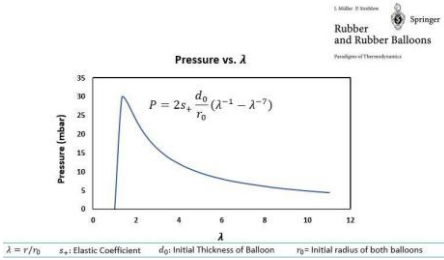
$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u} + \frac{1}{3} \mu \nabla (\nabla \cdot \mathbf{u}) + \rho \mathbf{g}, \quad [1]$$

$$\frac{\partial u}{\partial t} = 0$$



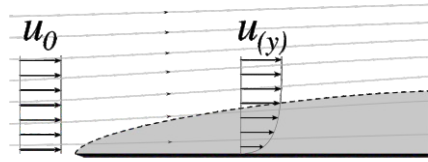
Unsteady

P-V Chart



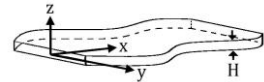
Flow under the hovercraft

Boundary layer:



Hele-Shaw

$$u = \frac{v_p}{2\mu} (z^2 - H^2)$$



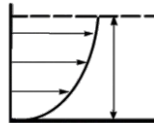
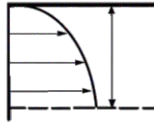
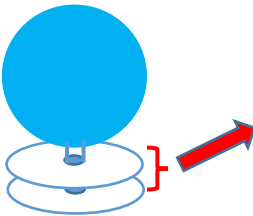
u-velocity of flow

P(x,y,t)-Local pressure

$z = \pm H$

μ - Flow's viscosity

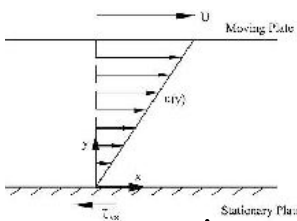
Flow under the CD



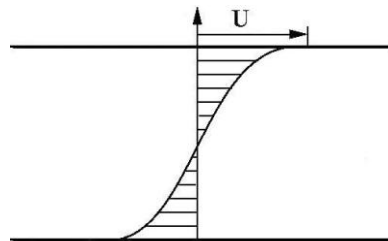
Flow under the CD (Couette flow)

$$v_{CD} < v_{flow}$$

$$v_{CD} = v_{flow}$$



$$v_{CD} > v_{flow}$$



Simulation

$$\dot{m} = \lim_{\Delta t \rightarrow \infty} \frac{\Delta m}{\Delta t} = \frac{dm}{dt}$$

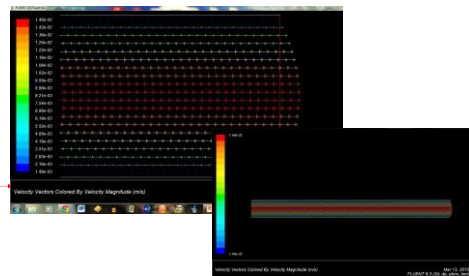
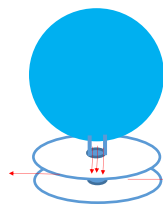
$$\dot{m} = \rho \cdot v \cdot A$$

ρ = pressure

V = volume

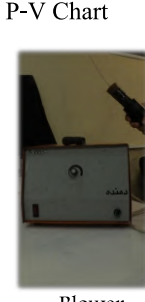
A = Area

$$\dot{m} = \lim_{\Delta t \rightarrow \infty} \frac{\Delta m}{\Delta t} = 0.14 \frac{g}{s}$$

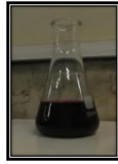




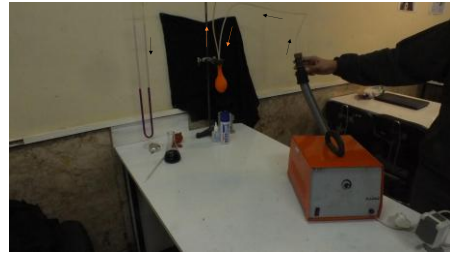
Barometer



Blower

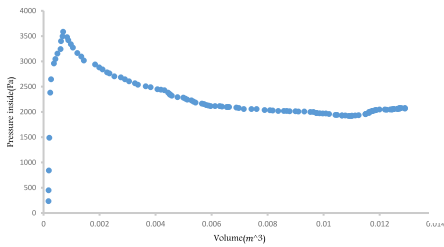


Potassium permanganate ($KMnO_4$)



Processing

Outcome



The pressure inside the balloon *isn't* stable

Reynolds number

$$Re = \frac{INERTIAL\ FORCES}{VISCIOUS\ FORCES} = \frac{\rho \cdot V \cdot L}{\mu}$$

V=VELOCITY OF FLOW (m/s)

ρ =DENSITY (kg/m³)

μ =DYNAMIC VISCOSITY (N·s/m²)

L-is a characteristic linear dimension(m)

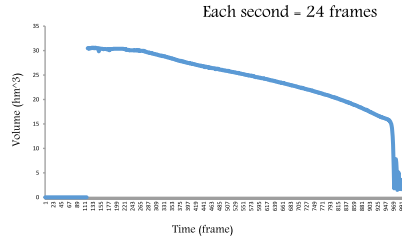
$$Re = \frac{1.225 \times 0.647 \times 0.0012}{0.00018} = 79$$

Effective parameters

- Air pressure inside the balloon
- Diameter of the CD
- Area of the nozzle
- Hovercraft's weight
- Surface of the ground
- Volume of air in the balloon
- Height of the CD from the ground
- Density of air inside the balloon



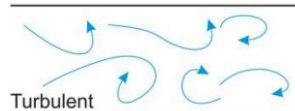
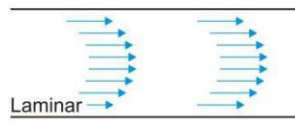
V-t chart



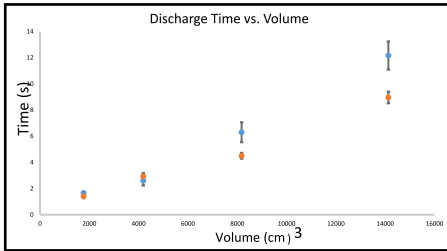
Laminar/Turbulent

$Re < 2000$ laminar

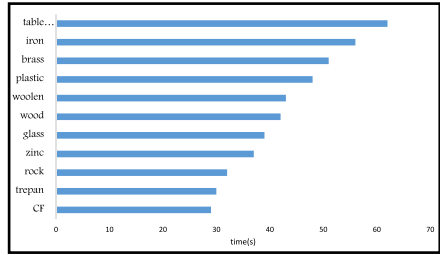
$Re > 4000$ turbulent



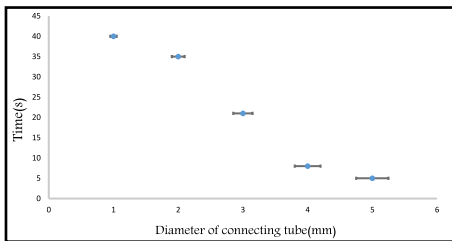
Volume



The surface effect

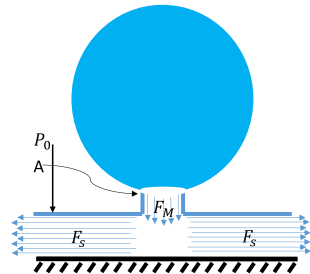
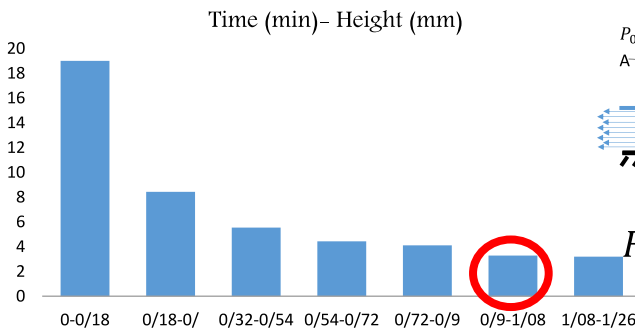


Diameter of connecting tube



Measuring the FM

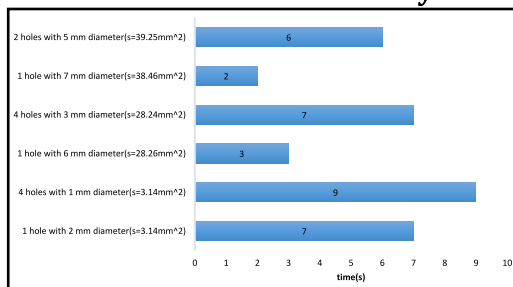
Initial height of the hovercraft



$$F_M = \frac{m\dot{V}}{A} + m\ddot{V}$$

$$m = \int \dot{m} dt$$

Diameter of holes on valve



General theory

14. Fountain

Construct a fountain with a 1m "head of water". Optimise the other parameters of the fountain to gain the maximum jet height by varying the parameters of the tube and by using different water solutions.

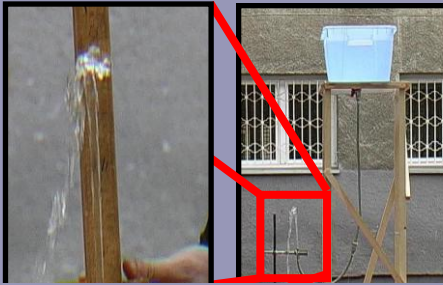
To calculate what height will the ideal liquid's stream reach with given variables, we may use Bernoulli's equation:

Knowing that, we have constructed a fountain, thinking that the jet height would reach a little less than one meter.

$h = 1m$

Which is an answer to our problem for ideal conditions.

Our first fountain



Pressure losses

Why is that theory wrong?

If the liquid in the fountain was ideal, and the resistances equal zero, the speed on the jet would be equal to:

$$V_{\max} \approx 4,4 \frac{m}{s} \approx 15,9 \frac{km}{h}$$

Those factors have influence on this velocity:

- Linear resistance of flow
- Local resistances of flow
- Air resistance
- Liquids viscosity
- Air's density and pressure
- Jet type
- Type of the flow of water
- Material, that the pipes are made of

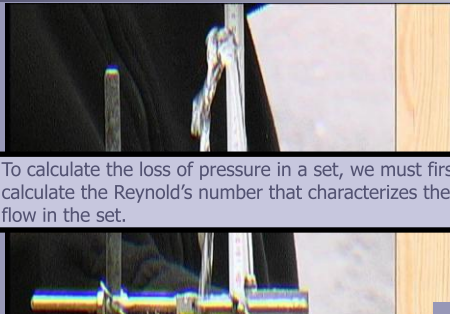
Reynold's number

Reynold's number specifies whether the flow is laminar or turbulent.

Reynold's number depends on viscosity coefficient ν , diameter of the flow and liquid's average velocity in the conduit;

$$Re = \frac{V \cdot l}{\nu} = \frac{\rho \cdot V \cdot l}{\mu}$$

For each fountain the Reynold's number must be calculated separately. It's magnitude should be the smallest possible, because the energy lost by turbulences in the flow is the lowest.



To calculate the loss of pressure in a set, we must first calculate the Reynold's number that characterizes the flow in the set.

Absolute roughness

Here is a list of the absolute roughness coefficient value for some common materials:

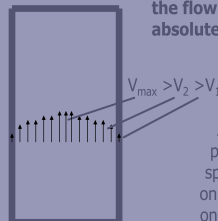
| | |
|----------------|-------------|
| glass | 0,0015-0,01 |
| aluminium | 0,015-0,06 |
| Steel | 0,02-0,10 |
| Corroded steel | 0,40 |
| Cast iron | 0,25-1,0 |

$$\epsilon = \frac{k}{D}$$

Where ϵ is the relative roughness, k is absolute roughness and D is the flow's diameter

Relative roughness

The width of the zone in which the flow is slowed depends on the absolute roughness k .



At some distance from the pipe's surface the flow has a speed which does not depend on the absolute roughness (but on viscosity only). The width of this zone is inversely proportional to relative roughness.

On this picture, the influence of viscosity is omitted.

Linear flow resistance

It is the proportion between loss of pressure in the pipe and the specific gravity weight. It is calculated with equation:

$$h_L = \frac{p_1 - p_2}{\gamma} = \lambda \frac{L}{D} \frac{v^2}{2g}$$

where λ is the linear resistance coefficient

Local resistances

Local resistances are calculated from formula:

$$h_{lost} = \zeta \frac{V^2}{2g}$$

Where ζ is the local resistances coefficient, and V is an average velocity in the cross-section after the obstacle

Even smallest increase or decrease of flow's diameter, each valve, bend, etc cause some loss of pressure.

Local resistances

The influence of local resistances is effectively visualized by the height that the stream from one of the fountains we have constructed reached:



Each narrowing in this pipe has decreased the speed of the water. The stream's height in this case was near zero.



What pipe will be the best?

As said before, linear flow resistance are described by equation:

$$h_f = \lambda \frac{L}{D} \frac{V^2}{2g}$$

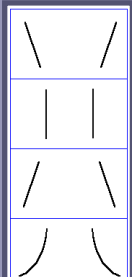
To minimize velocity losses in the flow, we must design such fountain, that length L of the flow would be the smallest, and the diameter D the biggest possible.

Choosing pipes with lowest roughness, having sizes as above and for low Reynolds numbers, we may calculate:

$$\lambda \approx 0.0006 \text{ so } h \approx 0.0006 \approx 6 \cdot 10^{-4}$$

Water solutions

Resistances at waterspout



Conic widening mouthpiece: turned out to be less effective. Water splashed heavily.

Cylindric mouthpiece: It gave various effects, depending on the fountain.

Conic narrowing mouthpiece: gave very good results.

Curved mouthpiece (Weisbach's jet) results nearly identical to those from conic narrowing mouthpiece.

Liquid properties:

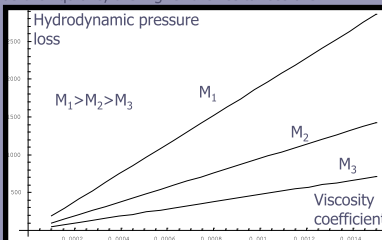
- density
- viscosity
- specific heat
- compressibility
- temperature
- surface tension

We must now find out which of these properties have influence on the height of the liquid's stream.

Viscosity and, indirectly, temperature, seem to have the biggest influence on the height of the stream.

Liquid's viscosity

From the value of the kinematic viscosity coefficient depends the velocity of the flow lost due to local resistances. The more viscous the liquid is, the higher the resistances are:



The best water solution

To choose the best water solution, we must consider how will it change the water viscosity. Basing on the information from literature, we were able to divide substances into those that:

Lower the viscosity

- soap
- ethyl alcohol
- fenol

Increase the viscosity

- sugar
- salt

To maximize the stream's height, we must minimize the linear resistances, so the solution's viscosity must be lowest possible.

Our experiments

We have tried out three different solutions: water with soap, salt and sugar. Then, we've compared the gathered data with the results for clean water (experiment was carried out on a 0,75 cm diameter conic narrowing mouthpiece)

| | |
|---------------------|-------|
| Clean water | 87 cm |
| Salt solution [1%] | 85 cm |
| Sugar solution [3%] | 84 cm |
| Soap solution | 88 cm |

The results vary, however, very slightly - they are on the verge of measurement error. However, they seem to confirm our presumptions.

$h = 0,82m$

Sugar solution [15%]



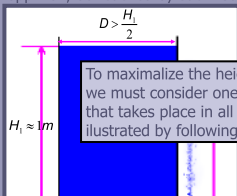
Our experiments

$h = 0,79m$

Sugar solution [20%]

Optimal fountain

The highest stream height will, in our opinion, be reached by such fountain:



To maximize the height of the stream, we must consider one more phenomenon that takes place in all fountains. It is illustrated by following photos.

- Reynolds number for the flow in the tank is very high
- The flow is linear
- Local resistances exist only at the mouthpiece – everywhere else they are omittable.
- Highest possible speed water

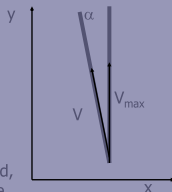
„Falling water“

While designing the optimal fountain it is worthy to consider deviating the stream from perpendicularity with gravity field to minimize this effect.

However, it is connected with decreasing the maximal reached height, because the speed of water on vertical axis will be:

$$V = V_{\max} \cos \alpha$$

It needs to be empirically checked, whether it is better to deviate the mouthpiece or not.

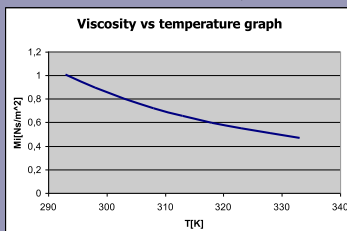


Our experiments

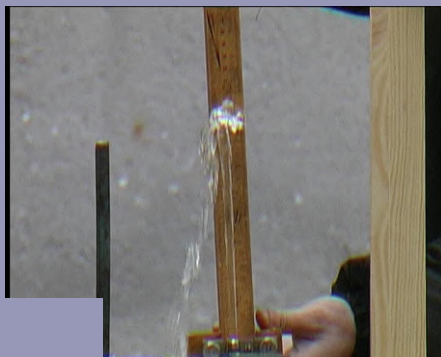
Liquid's temperature influence

As the temperature increases, liquid's viscosity decreases:

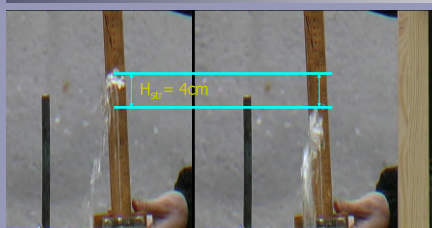
$$\mu = Ae^{\frac{E}{RT}}$$



Our experiments confirmed this theory: hot (ok 60°C) water



„Falling water“



The height lost in this way may reach even 25% maximal height. What can be done to bypass this effect?

Our fountain

Having analyzed the theory, we began constructing our own „optimal” fountain.

We have built a fountain for which the biggest pressure losses are those on mouthpiece, and for which linear and local losses are much smaller, nearly ommitable.

We have used a conic narrowing mouthpiece (Weisbach's jet was too hard to make in school conditions).

We have also decided to deviate the stream from perpendicular of about 4 degrees.

Our best result

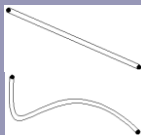


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- *Mechanika płynów w inżynierii środowiska* – Z. Orzechowski, J. Prywer, R. Zarzycki
- *Hydraulika i hydrologia* – B. Jaworowska, A. Szuster, B. Utrysko
- *Tablice i wykresy do obliczeń z mechaniki płynów* - W. Stefański, K. Wyszowski

Pipe's shape

Any pipe may connest two holes. The be connection, from the hydrodynamic point is such, that the pipe is ideally straight, the pressure losses are minimal.



In such pipe only resistance influer flow.

In such pipe, each narrowing, bend, a loss of pressure

Ideal fountain's general assumptions

We aim at a set, in which the influence of all earlier mentioned factors will be optimised for depreciating the flow's resistance and to increase the discharge coefficient:

- smallest possible pipe's length, large diameter and possibly smallest roughness: linear flow resistance $\rightarrow 0$;
- lack of narrowings, bends and any other obstacles on water's way: local flow resistances $\rightarrow 0$;
- Weisbach's jet instead of usual round hole \rightarrow discharge coefficient $\rightarrow 1$;
- Low viscosity liquid: flow resistances $\rightarrow 0$

Pipe types

Each pipe is characterized by few parameters:

- length – L
- material of which it is made and, connected with absolute roughness –
- internal diameter – D

Crimping, plastic pipe – extremely large roughness



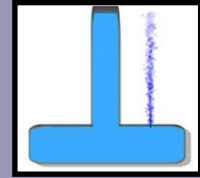
This pipe is, inside, made of rubber

Calculations for our fountain



$Re < 10$ (for the flow in the tank)

$$V \approx 4,15 \frac{m}{s} \approx \frac{95}{100} V_{max}$$



Conclusion

Our fountain's efficiency reached more than 90% (92 cm exactly).

There is one more thing to mention: with the problem formulated this way, it was possible to obtain a height of a stream much greater than one meter by, for example, creating a pressure difference between „head of water” and mouthpiece or by constructing so-called Hero's fountain.

However, we decided not to consider those matters, I focus on strictly hydrodynamic problems.

Local resistances

Examplyary local resistance coefficients:

| Nazwa przeszkody | Współczynnik oporów miejscowych ζ |
|---|---|
| Wlot z przewodu do zbiornika | $\zeta = 1$ |
| Nagle zwiększenie przekroju z D_1 do D_2 (tylko dla $Re > 3500$) | $\zeta = \left[\left(\frac{D_1}{D_2} \right)^2 - 1 \right]^2$ |
| Nagle zmniejszenie przekroju z D_2 do D_1 | $\zeta = \frac{1}{2} \left[1 - \left(\frac{D_2}{D_1} \right)^2 \right]$ |
| Wodomierz tłoczkowy | $\zeta = 12$ |
| Wodomierz skrętowy | $\zeta = 6$ |
| ompensator dławikowy | $\zeta \approx 0,2$ |

Resistances at waterspout

| Type of hole and mouthpiece | Coefficients | | | |
|-------------------------------|-------------------|-------------------|-----------------|--------------------------|
| | throttling ϕ | Velocity α | discharge μ | Local resistance ζ |
| Round hole | 0,64 | 0,97 | 0,62 | 0,6 |
| Conic widening mouthpiece | 1,0 | 0,45-0,50 | 0,45-0,50 | 3-4 |
| Cylindric internal mouthpiece | 1,0 | 0,707 | 0,707 | 1,0 |
| Cylindric external mouthpiece | 1,0 | 0,82 | 0,82 | 0,5 |
| Conic narrowing mouthpiece | 0,98 | 0,96 | 0,94 | 0,09 |
| Curved mouthpiece | 1,0 | 0,98 | 0,98 | 0,04 |

Taken from: *Tablice i wykresy do obliczeń z mechaniki płynów* - W. Stefański, K. Wyszowski

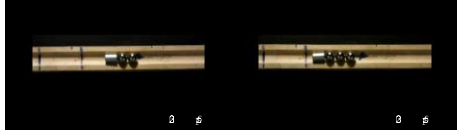


1. Gaussian Cannon

Reporter : S. M. Ali Modarressi



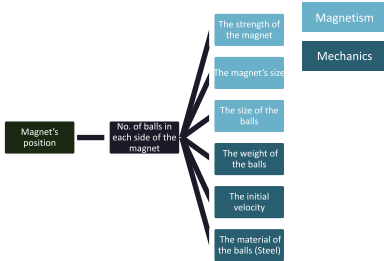
Initial Observations




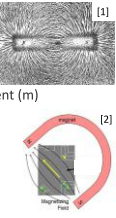
- A high velocity ejection

$$E_1 \neq E_2$$

Effective parameters

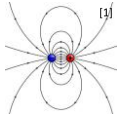


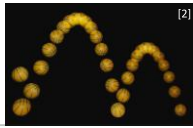
Theory Background

- Magnetism
 - Magnetic field 
 - Magnetization (M) and magnetic moment (m) 

[1] : en.wikipedia.org/wiki/Magnetic_field
 [2] : en.wikipedia.org/wiki/Magnetic_domains

Theory Background

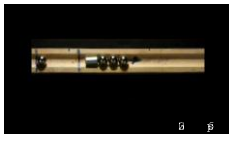
- Magnetic dipoles 
- Mechanics
 - Coefficient of restitution
 - $C_R = \frac{\Delta v_2[S]}{\Delta v_1}$



[1] : en.wikipedia.org/wiki/Magnetic_field
 [2] : en.wikipedia.org/wiki/Coefficient_of_restitution


Theory Base The Phenomenon

1 The main reason



$$K_1 + E_{mag1} = E_{mag2} + K_2$$

$$E_{mag1} > E_{mag2}$$

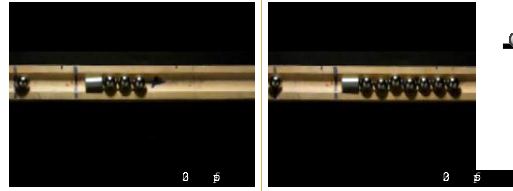
$$K_2 > K_1$$


$$E_1 = E_2$$

Theory Base The Phenomenon

Theory Base The Phenomenon

2 The energy loss

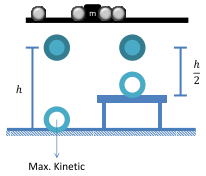
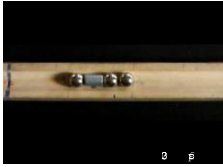


$$E_1 \times \left(\frac{1 + C_r}{2} \right)^2 = E_2$$

$$E_1 \times \left(\frac{1 + C_r}{2} \right)^N = E_2$$

Theory Base The Phenomenon

3 The left side of the magnet



⇒ 0 Balls on the Other side of the magn

Theory Base The Phenomenon

4 Multi ball ejection

Very strong magnet



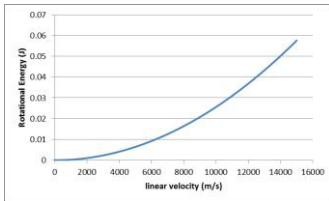
Normal magnet



Note: The Initial velocity are the same

Theory Base The Phenomenon

5 The rotational energy



⇒ The rotational energy is negligible

Theory Governing equation

1 The theory formulation

We mesh the whole model

In each point we have:

$$B = \frac{\mu_0}{4\pi} \frac{3(n \cdot m) - m^2}{|x|^3} \quad [1]$$

$$\sum B = B_0$$

$$M = \left(\frac{3}{\mu_0} \right) \left(\frac{\mu - \mu_0}{\mu + 2\mu_0} \right) B_0^2 [2]$$

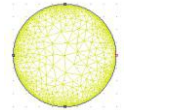
And the magnetic moment is determined by:

$$MV = m^2 [3]$$

And the magnetic energy is calculated by

$$U = -m \cdot B [4]$$

Note: the magnet has a multi-dipole modeling



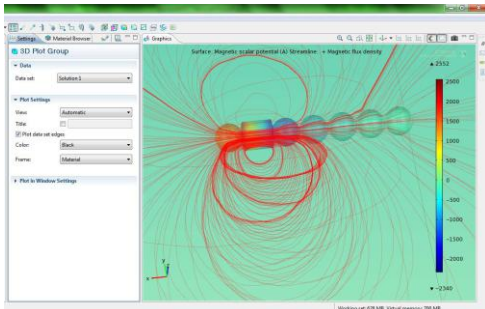
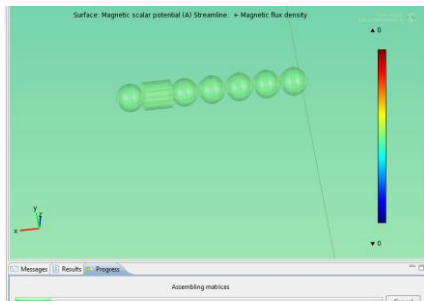
Theory Governing equation

2 Simulation



- Making the geometry
- Meshing the model
- Materializing the model
- Inserting initial values
- Simulating
- Plotting graphs
- Measuring the parameters (m, B, H, ...)

2 Simulation



Theory Governing equation

3 Modeling the cannon

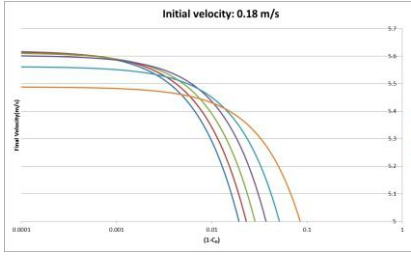
| Time | Position | Velocity | Acceleration | Force | Magnetic Moment | Rotational Energy | Translational Energy | Total Energy |
|-------|----------|----------|--------------|-------|-----------------|-------------------|----------------------|--------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 0.002 | 0.004 | 0.008 | 0.016 | 0.032 | 0.004 | 0.004 | 0.004 | 0.004 |
| 0.003 | 0.009 | 0.027 | 0.054 | 0.108 | 0.009 | 0.009 | 0.009 | 0.009 |
| 0.004 | 0.016 | 0.064 | 0.128 | 0.256 | 0.016 | 0.016 | 0.016 | 0.016 |
| 0.005 | 0.025 | 0.125 | 0.250 | 0.500 | 0.025 | 0.025 | 0.025 | 0.025 |
| 0.006 | 0.036 | 0.216 | 0.432 | 0.864 | 0.036 | 0.036 | 0.036 | 0.036 |
| 0.007 | 0.049 | 0.343 | 0.686 | 1.372 | 0.049 | 0.049 | 0.049 | 0.049 |
| 0.008 | 0.064 | 0.512 | 1.024 | 2.048 | 0.064 | 0.064 | 0.064 | 0.064 |
| 0.009 | 0.081 | 0.729 | 1.458 | 2.916 | 0.081 | 0.081 | 0.081 | 0.081 |
| 0.010 | 0.100 | 1.000 | 2.000 | 4.000 | 0.100 | 0.100 | 0.100 | 0.100 |

- Simulation
- Magnetic field of each ball is obtained
- Magnetization of each ball is calculated
- Magnetic moment of each ball is calculated
- The potential energy of each ball is calculated
- $$(1 + K_1) \times \left(\frac{1 + \Omega}{2} \right)^n - 1 \cdot E$$

$$\Rightarrow E_2$$
- Total velocity is obtained

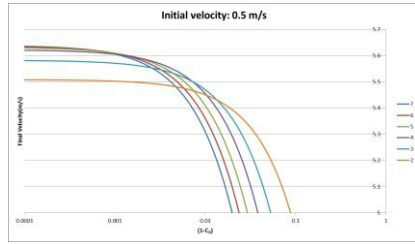
Theory Results

1 The effects of the coefficient of restitution



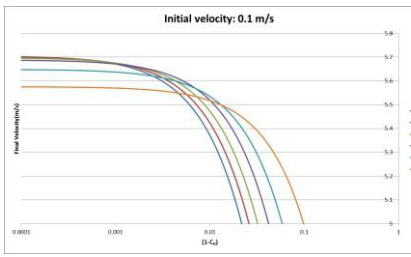
Theory Results

1 The effects of the coefficient of restitution



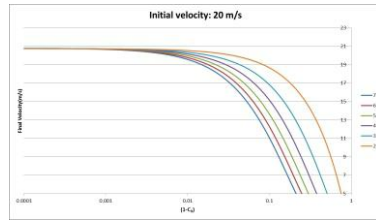
Theory Results

1 The effects of the coefficient of restitution



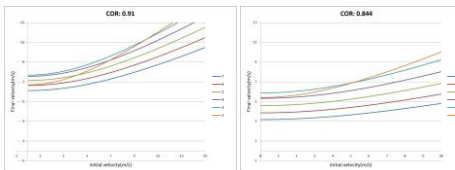
Theory Results

1 The effects of the coefficient of restitution



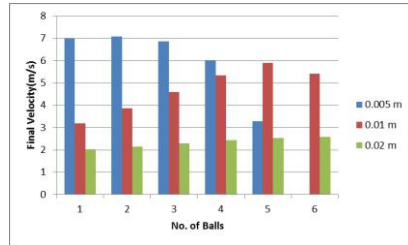
Theory Results

2 The effects of the initial velocity



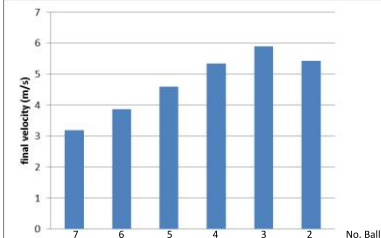
Theory Results

3 The effects of the size of the ball



Theory Results

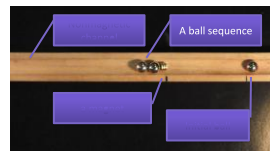
4 The theory results by the reality's properties



Experiment Setup

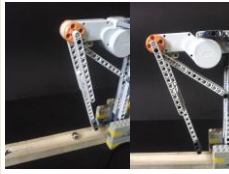
1 The main structure

- Our Variables:
 - The No. Balls
- Our Setup



Experiment Setup

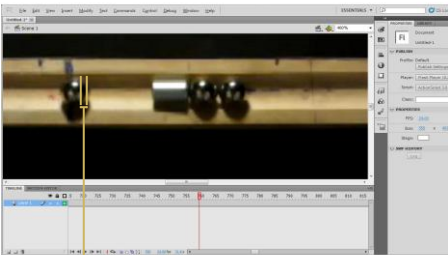
1 The Shooter



- Our Variables:
 - The No. Balls



Experiment Speed Measurement



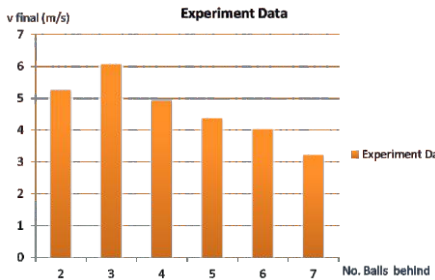
Displacement 4 pixels

$$\left. \begin{aligned} \Delta x &= 4 \text{ pixels} \rightarrow x \frac{0.01}{20} \rightarrow \Delta x = 0.002 \text{ m} \\ \Delta t &= 4 \text{ frames} \rightarrow x \frac{1}{320} \rightarrow \Delta t = 0.0125 \text{ s} \end{aligned} \right\} \frac{\Delta x}{\Delta t} = \text{velocity} = \frac{0.002}{0.0125} = 0.16 \frac{\text{m}}{\text{s}}$$

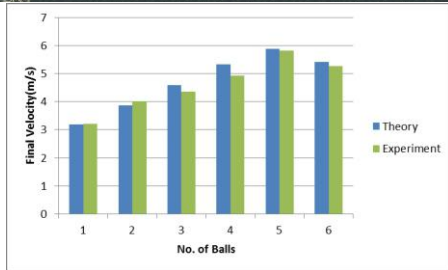
Global Function

$$v = \frac{\text{displacement (pixels)}}{\text{frame}_2 - \text{frame}_1} \times 0.16$$

Experiment Results



Theory vs. Experiment



Conclusion



The phenomenon occurs due to the difference of the attractions of the first and

| Situation | Optimum position |
|---------------------------------|--------------------------------------|
| Our experiments | 3 balls |
| Increasing the initial velocity | Decreases the number of balls |
| Increasing the COR | Increases the number of balls |
| Increasing the size of the ball | Decreases the number of balls |

- [1] : Jackson, John David (1999). Classical electrodynamics (3 rd ed.). New York, [NY]: Wiley, ISBN 0-471-30932-X p. 186
- [2] : Jackson, John David (1999). Classical electrodynamics (3 rd ed.). New York, [NY]: Wiley, ISBN 0-471-30932-X p. 200
- [3] : Kazimierzuk, Marian K. (2009). High-Frequency Magnetic Components, 1st edition Wiley . ISBN 0-470-71453-0 p. 55
- [4] : Jackson, John David (1999). Classical electrodynamics (3 rd ed.). New York, [NY]: Wiley, ISBN 0-471-30932-X p. 190
- [5] : Cross, Rod (2006). The bounce of a ball. Physics Department, University of Sydney, Australia. Retrieved 2008-01-16 p. 1, l. 24



Problem No. 12

Lanterns

Kamran.K.Hedayat



IYPT 2012
BAD SAULGAU - GERMANY

Background

- What is Lantern?
- Why does it rise?
- Designs
- Parameters
- Theoretical Approach

Theory

- Governing equations
- Stages of rising
- Stage 1
- Stage 2
- Simulations

Experiments

- Objectives
- Setup
- Measurements
- Cases & Results

Main Approach

Conclusion & Discussion

- Tea Light or Not?
- Theory Vs. Experiments
- Optimization

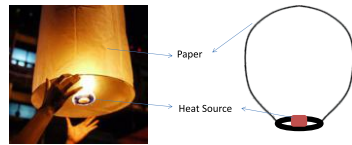
Problem

Paper lanterns float using a candle. **Design and make** a lantern powered by a **single tea-light** that takes the **shortest time** (from lighting the candle) to float up a vertical height of **2.5 m**. Investigate the influence of the relevant **parameters**.



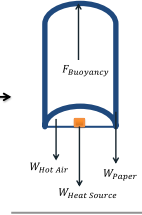
Background - Lanterns

- Lanterns are constructed using paper and a flammable wax as a heat source.

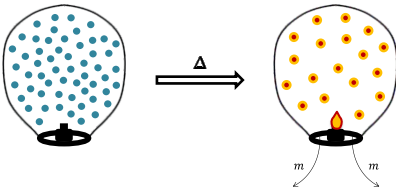


Background - Lantern's Free Body Diagram

Due to pressure difference we have the buoyancy force.



Background - Flying Mechanism



Background - Finding the best Design

- So the best "shape" for the lantern is spherical.

Background - Theoretical Approach

$$F_B \text{ \& \; } W_{hs} \text{ \& \; } W_p \text{ cte.}$$

$$W_{ha} \text{ Variable} \rightarrow W_{ha} = m_{ha}g = V_{ha} \cdot \rho_{ha} \cdot g$$

$$\rho \propto T \rightarrow \text{Min}\{\text{Energy loss}\} \rightarrow E_l \text{ c}$$

Therefore the ideal shape is the one where the ratio between Volume & Surface is large.

$$\frac{\text{Volume}}{\text{Surface}} = \text{MAX}$$

$$\text{Dissipation} = \text{MIN}$$



Theory – Governing Equations

Theory – Stages of Rising

$$\text{Force : } \sum F = ma$$

II. Rising the distance of 2.5(m).

$$\text{Energy : } q_{in} - q_{out} = q_s$$

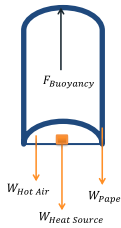
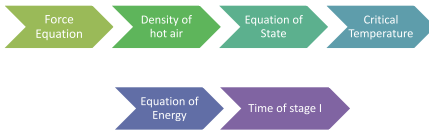
$$\text{State : } PV = nRT$$



I. Reaching the moment of flying.

Theory – First stage theoretical Approach

Theory – Stage I (Finding the critical Density)



At the moment of flying : $\sum F = 0$

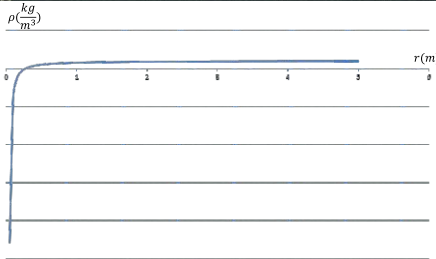
$$-m_{ha}g - m_{hs}g - m_p g + \rho_a V g = 0$$

The hot air density should reach the amount below :

$$\rho_{ha} = \frac{\rho_{hs} V_{hs} + \rho_p A - \rho_a V}{V_{ha}}$$

Theory – Stage I (Finding the critical Density)

Theory – Stage I (Finding the critical Temperature)



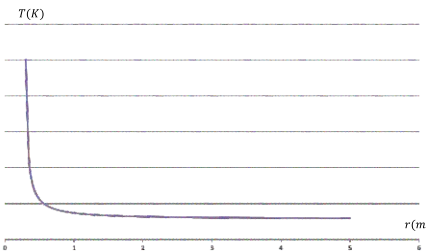
Using the equation of state : $PV = nRT$

$$T = \frac{M \cdot P}{\rho_{cr} \cdot R}$$

ρ_{cr} : Hot air Density measured using Force equation
 P : Atmospheric Pressure
 M : Molecular Mass
 R : Gas Constant

Theory – Stage I (Finding the critical Temperature)

Theory – Stage I (Finding the time)



$$\text{Energy : } q_{in} - q_{out} = q_{stg}$$

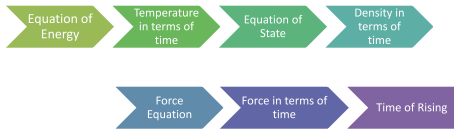
$$q_{in} - q_{out} = mc \frac{dT}{dt}$$

$$q_{in} - \frac{T_{in} - T_{out}}{R} = mc \frac{dT}{dt}$$

By solving the differential equation the Temperature in terms of time can be measured.

Theory – Second stage theoretical Approach

Theory – Stage II (Finding the temperature in terms of time)



$$\text{Energy : } q_{in} - q_{out} = q_{stg}$$

$$q_{in} - q_{out} = mc \frac{dT}{dt}$$

$$q_{in} - \frac{T_{in} - T_{out}}{R} = mc \frac{dT}{dt}$$

By solving the differential equation the Temperature in terms of time can be measured.

Theory – Stage II (Finding the density in terms of)

Theory – Stage II (Finding the displacement in terms of time)

Using the equation of state : $PV = nRT$

$$\rho = \frac{MP}{RT}$$

ρ_{air} : Hot air Density measured using Force equation
 P : Atmospheric Pressure
 M : Molecular Mass
 R : Gas Constant

$$\sum F = ma \longrightarrow \rho_a g V - \rho_p A g - \rho_{ha} V g - m_{hs} g = m_T a$$

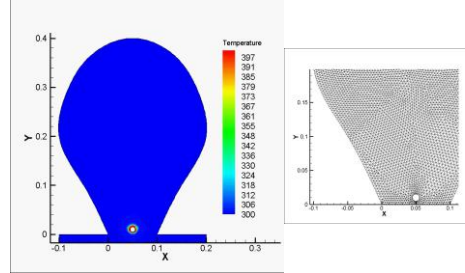
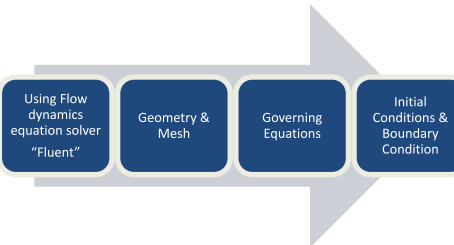
Acceleration is a function of the parameters above where Volume , Surface & density are variable.

$a(t)$ could be found.

$$x(t) = \iint a . dt$$

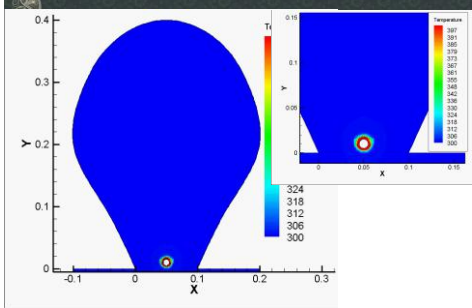
Theory - Simulation

Theory - Simulation



Theory - Simulation

Experiments – Objectives & Set up

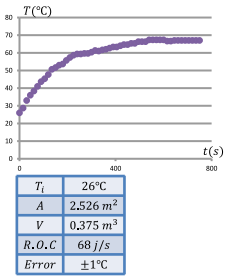
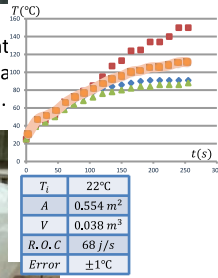


- Finding the temperature in terms of time.
- Finding the force in terms of time.
- Finding the displacement in terms of time.

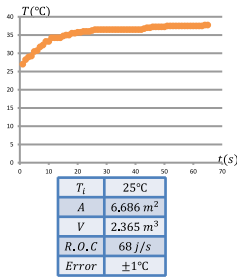


Experiments – Temperature - Time

- To calculate the average temperature of the lantern several thermometers were used in different places of the lantern using tea light as the heat source.

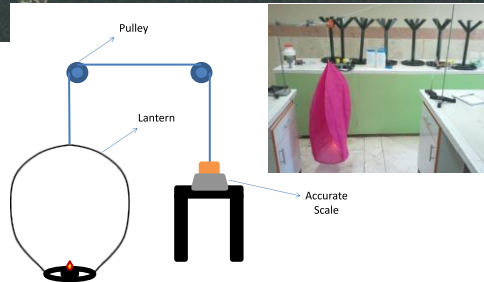


Experiments – Temperature - Time



Using Tea light as the heat source.

Experiments – Force (Setup)

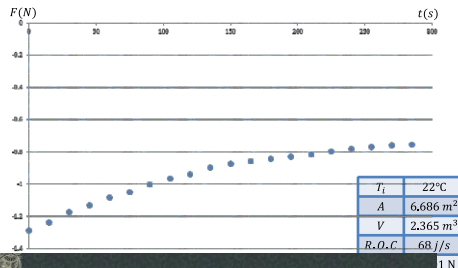


Experiments – A problem called tea light!!

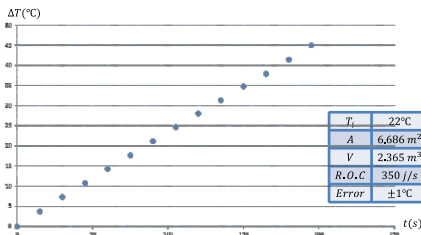
- It was seen in the experiments that a tea light can not produce the energy needed for a lantern to go up. So another heat source was used.



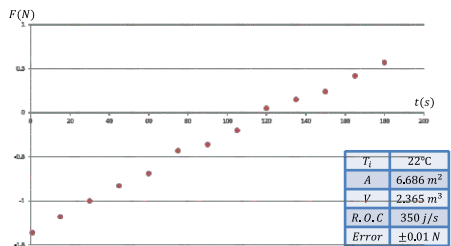
Experiments – Force - Time



Experiments – Temperature - Time



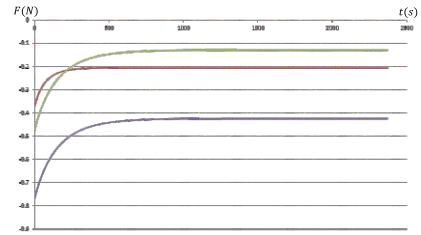
Experiments – Force - Time



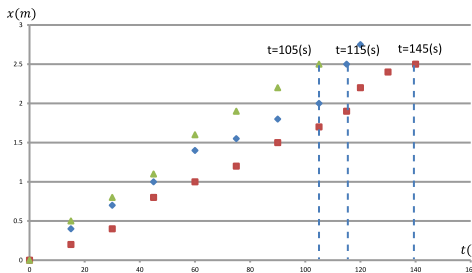
Experiments - Explanation

- It was seen that the lantern rise using this heat source so we continued the experiments to find best lantern.
- The lantern which rises the 2.5 meter height in the least time is the optimized model.

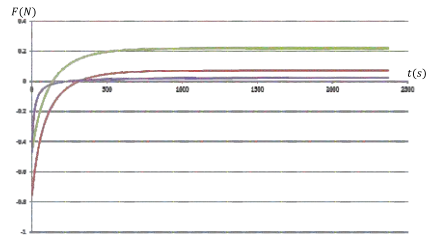
Discussion – Tea light or not??



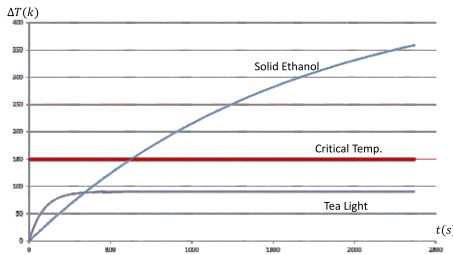
Experiments – Displacement - Time



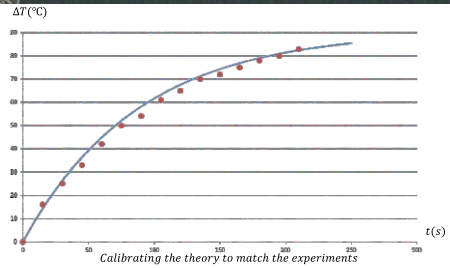
Discussion – Tea light or not??



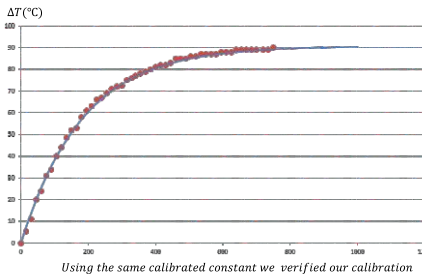
Discussion – Tea light or not??



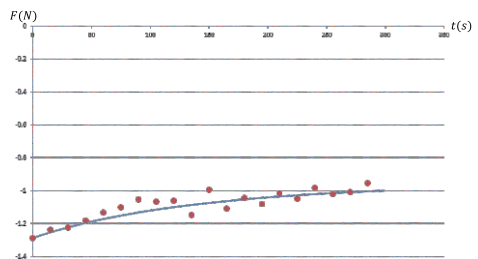
Discussion – Theory Vs. Experiments



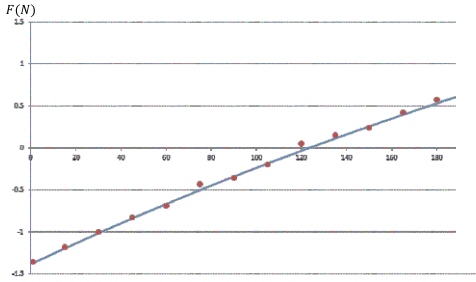
Discussion – Theory Vs. Experiments



Discussion – Theory Vs. Experiments

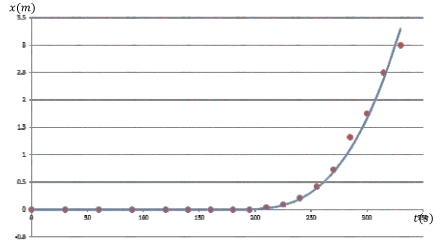


Discussion – Theory Vs. Experiments

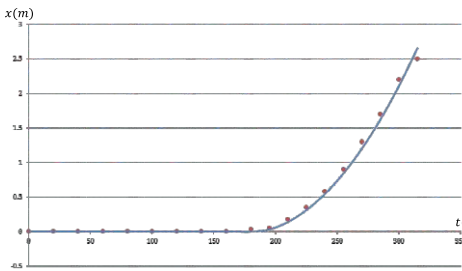


Using the same calibrated constant we verified our calibration

Discussion – Theory Vs. Experiments



Discussion – Theory Vs. Experiments

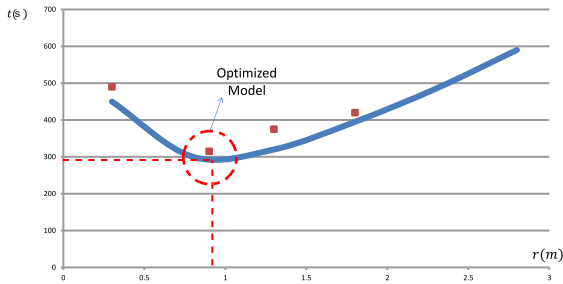


Conclusion – Heat source

- Tea light can not produce the heat needed for a lantern to rise.
- Using another heat source the time of rising has been optimized.



Optimization





Formulation of the problem

6. Roundabout

Put a plastic cup on a thin layer of liquid on a flat solid surface.
Make the cup rotate.

On what parameters does the rotational deceleration of the cup depend?

Contents:

- o 1) The main forces of deceleration
- o 2) Determination of the depth of liquid under cup
 - A. Research of flows under cup
 - B. Mathematical model of laminar mode
 - C. Turbulence mode
- o 3) Parameters of racing
 - A. Initial angular velocity
 - B. Duration of racing
- o 4) Properties of cup and liquid

Conclusions

I. The forces of deceleration



$$F_v \sim A \cdot \mu$$

Viscosity of water and air:

$$\mu_{\text{water}} = 1.00 \times 10^{-3} \text{ Pa} \cdot \text{s}$$

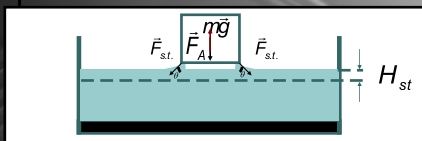
$$\mu_{\text{air}} = 17.4 \times 10^{-6} \text{ Pa} \cdot \text{s}$$

~~$F_{\text{air}} \approx 0.7 \times 10^{-8}$~~ $F_{\text{water}} \approx 4.5 \times 10^{-6}$

The main forces of deceleration



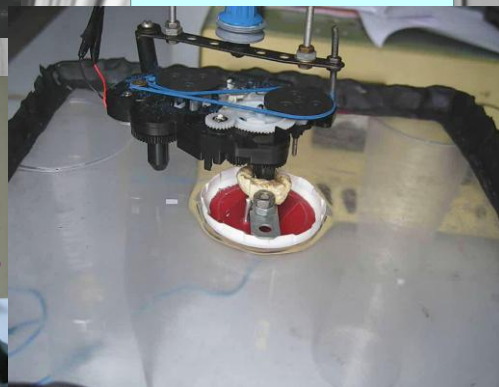
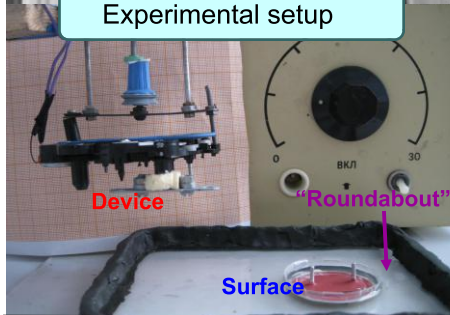
Subsidence of static cup



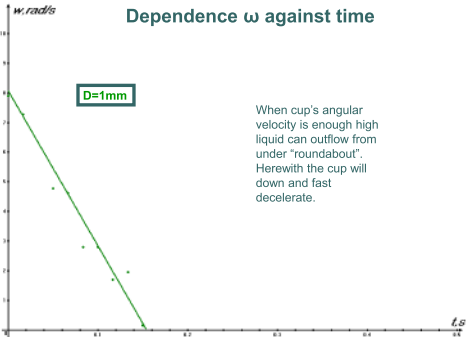
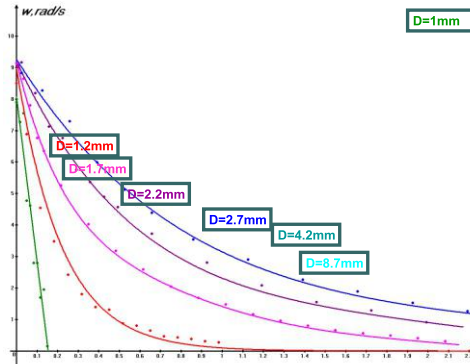
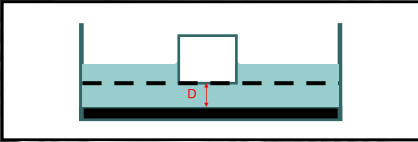
$$H_{st} = \frac{m}{\pi \cdot \rho \cdot R^2} + \frac{2 \cdot \sigma}{\rho \cdot g \cdot R} \cdot \cos \theta$$

m – mass of the cup R – radius of cup
 ρ – density of liquid θ – angle of contact

Experimental setup



II. Dependence of deceleration against depth of layer of liquid



Mathematic model (laminar flow)

The linearization(prediction)

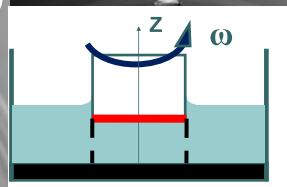
If flow under cup is laminar moment of viscous forces is proportional to the angular velocity.

The basic equation of rotational motion's dynamics:

$$J \frac{d\omega}{dt} = -\alpha \cdot \omega$$

$$\omega = \omega_0 \exp\left(-\frac{t}{\tau}\right)$$

ω – angular velocity of cup t – time of rotation
 ω_0 – initial angular velocity τ – characteristic time



Elementary force (Newton's theory):

$$dF = \mu \cdot dS \frac{dv}{dz}$$

Gradient of liquid's velocity on z direction

$$\frac{dv}{dz} \text{ ?}$$

Mathematic model (prediction)

The Couette flow

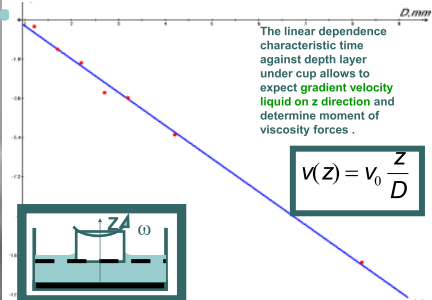
Simple solution of Navier

Liquid's velocity on z direction $v(z) = v_0 \frac{z}{D}$

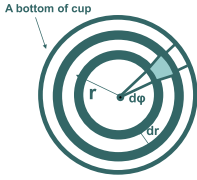
Moment of viscosity friction inversely layer's depth therefore characteristic time must be proportional D

$$M \sim \frac{1}{D} \Rightarrow \tau \sim D$$

Dependence characteristic time against depth layer for the "laminar" part of deceleration



Moment of viscosity forces



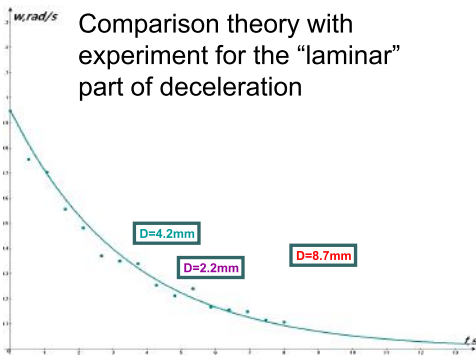
Velocity of liquid under cup on z direction:

$$v(z) = v_0 \frac{z}{D}$$

Elementary force:

$$dF = \mu \frac{dv}{dz} \cdot dS = \mu \frac{\omega \cdot r}{D} \cdot d\phi \cdot r \cdot dr$$

$$M = \int_0^{2\pi} \int_0^R dF \cdot r = \frac{\pi \mu R^4 \omega}{2D}$$

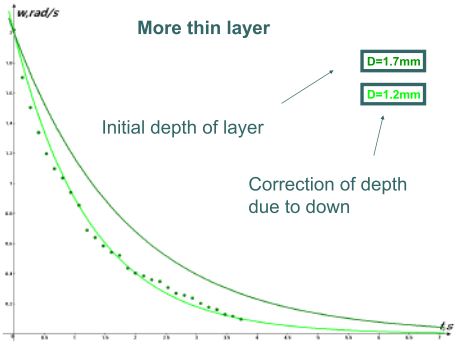


Comparison theory with experiment for the "laminar" part of deceleration

The law of cup's motion

$$\omega = \omega_0 \exp\left(-\frac{\pi \mu R^4}{2DJ} \cdot t\right)$$

- ω – angular velocity of cup
- ω_0 – initial angular velocity
- J – moment of inertia
- D – layer's depth of liquid
- t – time of rotation
- τ – characteristic time
- R – cup's radius
- μ – viscosity

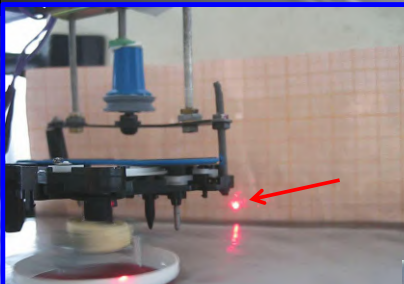


More thin layer

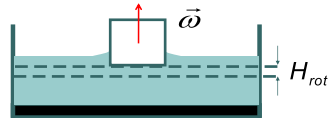
Initial depth of layer

Correction of depth due to down

Movie with down



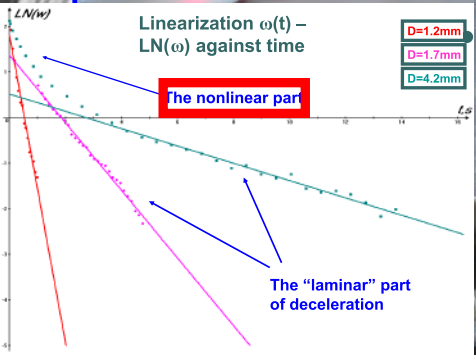
Subsidence due to the rotation



Condition for H_{rot} :

Forces of inertia = Hydrostatic forces

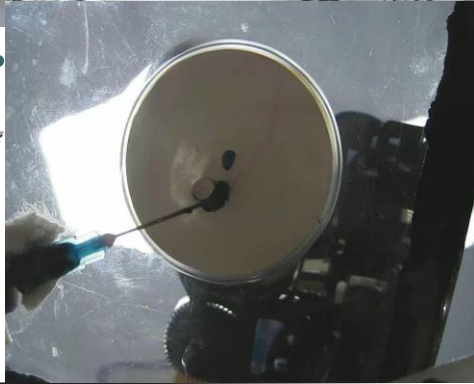
$$H_{rot} = \frac{\omega_0^2 \cdot R^2}{9 \cdot g} = 0,4mm$$



Linearization $\omega(t)$ – $LN(\omega)$ against time

The nonlinear part

The "laminar" part of deceleration



The linearization(turbulence)

When flow under cup is turbulence moment of viscous forces is proportional a square angular velocity.



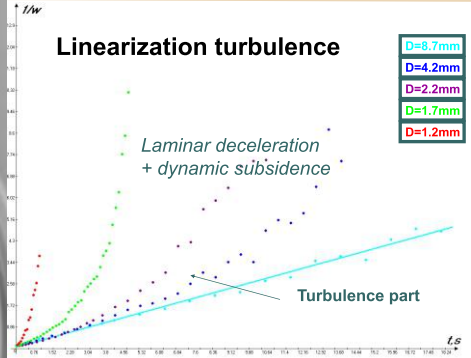
$$M \sim \omega^2$$

The law of cup's motion :

$$\omega = \frac{1}{\frac{1}{\omega_0} + \frac{t}{\tau}}$$

ω – angular velocity of cup t – time of rotation
 ω_0 – initial angular velocity τ – characteristic time

Linearization turbulence



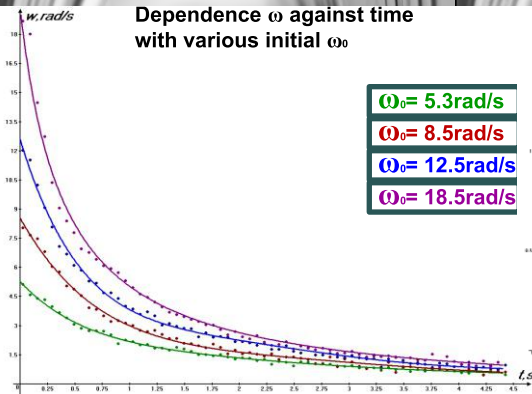
The kinds of deceleration:

- o 1) Gluing
- o 2) $M \sim \omega$, laminar flow
- o 3) $M \sim \omega^2$, turbulence

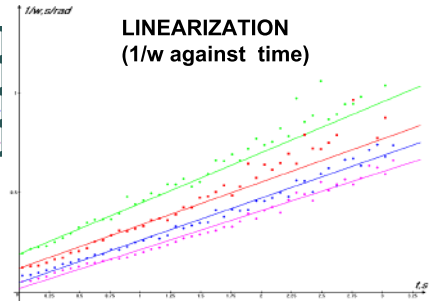
III. Parameters of racing

- o 1) Initial angular velocity
- o 2) Time of racing

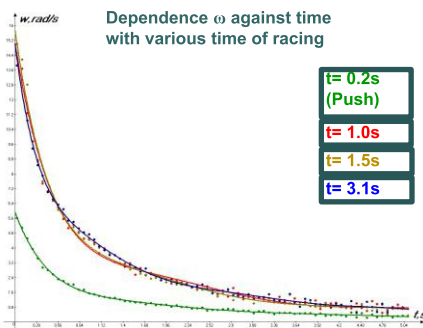
Dependence ω against time with various initial ω_0



LINEARIZATION (1/w against time)



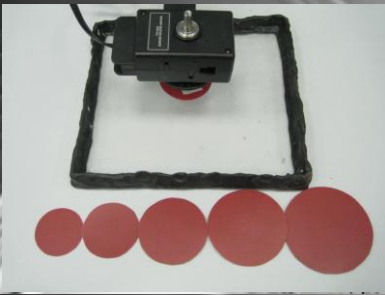
Dependence ω against time with various time of racing



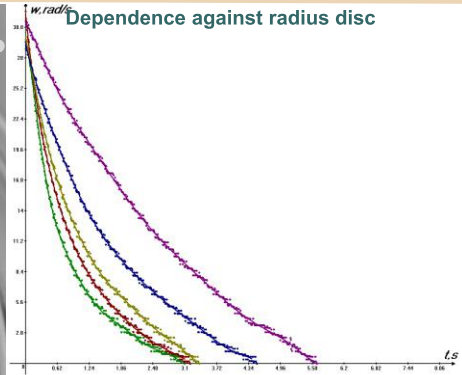
IV. Parameters of cup

- 1) Mass
- 2) Radius(square of bottom)
- 3) Moment of inertia
- 4) Angle of contact

Experiment with radius disc



Dependence against radius disc



V. Parameters of liquid

- o 1) Viscosity
- o 2) Density
- o 3) Surface tension

Parameters of the roundabout

- 1) Depth of liquid under cup
- 2) Initial angular velocity
- 3) Duration of racing
- 4) A cup:
 - a) Mass of cup
 - b) Radius of cup
 - c) Moment of inertia
 - d) Contact's angle
- 5) A liquid:
 - a) Viscosity
 - b) Density
 - c) Surface tension

Conclusions :

- o 1) Depth of liquid layer determines character of deceleration
- o 2) There are 3 different modes of cup deceleration:
 - a) gluing
 - b) laminar flows
 - c) turbulence
- o 3) Turbulent and laminar deceleration may be obtained during a single experiment.
- o 4) Laminar mode is explained by model of Couette flow
- o 5) The characteristic time doesn't depend on depth of liquid layer for turbulence mode.

Summary:

- o The experimental base of deceleration was built for explanation phenomenon.
- o The most important parameters were marked.
- o The experimental investigation shows some

References:

Wikipedia: Couette flow.
 A. D. Glinkin and V. A. Rukavishnikov.
 Laminar flow of a viscous incompressible liquid over the surface of solids of revolution. J. Eng. Phys. Thermophys., 39, 1 (1980)
 Slezkin. Hydrodynamics.

The conditions for laminar flow

$$Re = \frac{\rho \cdot v \cdot l}{\mu}$$

ρ – density of liquid μ – viscosity
 v – characteristic velocity (R)
 l – characteristic size (D)

Boundary value in our experiments:
 $Re \approx 100$

Team Brazil

Team of Brazil
Problem 13: Honey coils

Problem 13 Honey coils

Liana Guinsberg



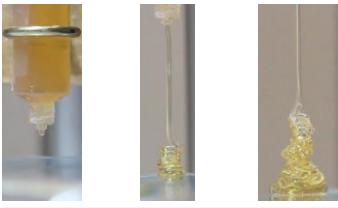
Problem 13

A thin, downward flow of viscous liquid, such as honey, often turns itself into circular coils. Study and explain this phenomenon.

Team of Brazil
Problem 13: Honey coils

Team of Brazil
Problem 13: Honey coils

Video

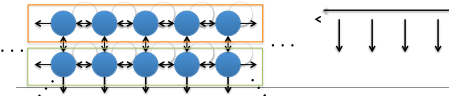


Increasing height

Team of Brazil
Problem 13: Honey coils

Surface tension

- The interior molecules have as many neighbors as they can.
- For the liquid to minimize its energy state, the number of higher energy boundary molecules must be minimized.
- The minimized quantity of boundary molecules: minimized surface area.



Introduction

Theoretical formulation

- Surface tension
 - Plateau-Rayleigh instability
- Viscosity
- Newtonian fluids
- Fluid conditions
 - Initial conditions
- Coiling conditions
- Torsions and Tensions
- Regimes

Experiments

- Honey
- Cane molass
- Oils
- Shampoo

Comparison between the theory and the experiments

- Graph comparison
- Regimes and conditions

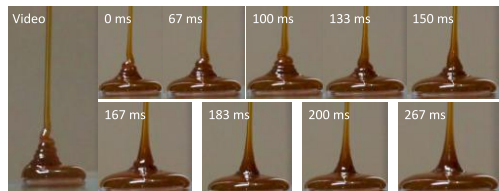
Team of Brazil
Problem 13: Honey coils

Plateau-Rayleigh instability

- The surface tension causes some oscillations in the jet, sometimes breaking it into droplets, to minimize surface area.

Instabilities

- Surface tension:

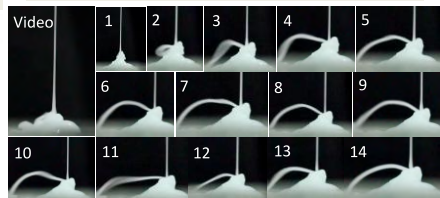


Shear thinning

- Happens on non Newtonian fluids
- The viscosity decreases with the increase of the shear stress.
- Causes phenomena like Kaye effect
- We can study it briefly with shampoo, glycerin and many others.



Kaye effect



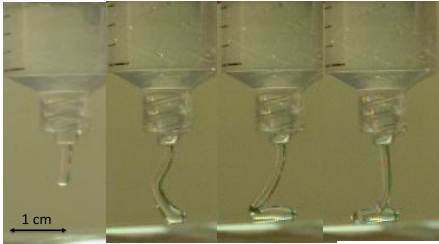
Initial conditions

- First, we suppose the fluid stream is completely vertical before touching the solid surface:

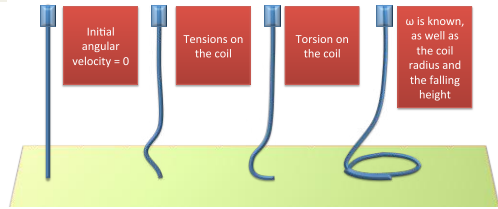


That implies:

- No coiling formation before the fluid touches the surface



Boundary conditions



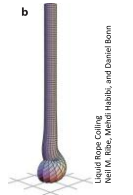
Torsions and tensions along the fluid rope

- Why does the jet changes its format after its collision with a rigid surface?
 - The jet has a velocity when touching the rigid surface
 - It has to slow down to zero, so there's a force directed upwards, that goes along the fluid stream and changes its form
 - There's a torsion caused by this tension
 - Thus, for the smaller energy, we can have the coiling phenomena.

Coiling conditions

- Minimal viscosity for the coil to happen
- Maximum height, because of the Plateau-Rayleigh instability

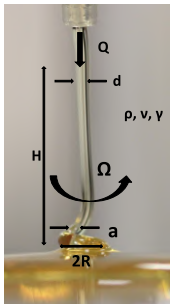
Surface tension relevance, for visible coils minimum height.



Regimes

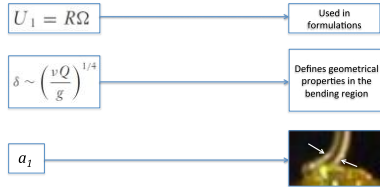
- We can see a relation of the coiling frequency with the height of fall
- As we increase the height, some buckling instabilities appear, and we define 4 regimes.

Theoretical analysis



- H- Height
- R- Coil radius
- Ω - Rotational frequency
- ρ - Density
- ν - Viscosity
- γ - Surface tension
- a- Fillet radius at the contact point
- d- Injection diameter
- Q- Volumetric rate of fluid insertion

Definitions

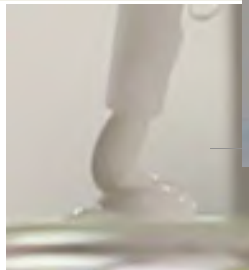


Theoretical analysis

- Viscous regime:
 - Smaller heights
 - Gravitational effect negligible
 - Inertial effect negligible

$$R \sim H$$

$$\Omega \sim \frac{Q}{Ha_1^2}$$



Theoretical analysis

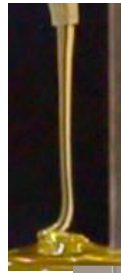
- Gravitational regime:
 - Gravitational effects are the most relevant ones

$$\delta \sim \left(\frac{\nu Q}{g}\right)^{1/4}$$

$$R \sim \delta \left(\ln \frac{H}{\delta}\right)^{1/2}$$

$$\Omega \sim \frac{U_1}{\delta} \left(\ln \frac{H}{\delta}\right)^{-1/2}$$

Gravitational regime movies



Honey, 2000 cst
60 fps
Fall height: 5 cm
Flow rate:
25% of original velocity

Silicone oil, 5000 cst
2000 fps
Fall height: 5 cm
Flow rate:
1.5% of original velocity

Theoretical analysis

- Inertial-Gravitational regime
 - Both gravitational and inertial forces are considerable
 - There're resonant frequencies



Inertial regime movies



Theoretical analysis

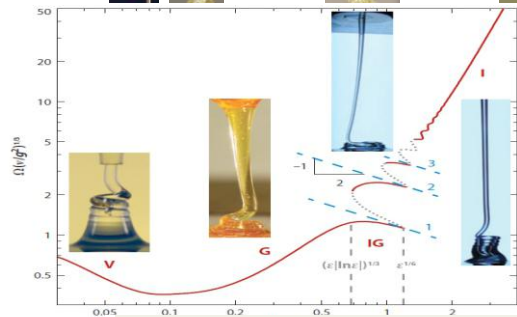
- Inertial regime:
 - Inertial effects are more important than the gravitational and viscous

$$R \sim \left(\frac{\nu a_1^4}{Q}\right)^{1/3}$$

$$a_1 \sim (\nu Q / g H^2)^{1/2}$$

$$a_1 \sim (Q^2 / g H)^{1/4}$$

$$\Omega \sim \left(\frac{Q^4}{\nu a_1^{10}}\right)^{1/3}$$



Liquid Rope Coiling, Neil M. Ribe, Mehdi Habibi, Z and Daniel Bonn, Annu. Rev. Fluid Mech. 2012.

Instabilities

- From the equations: $R \sim \left(\frac{\nu a_1^4}{Q}\right)^{1/3}$ $a_1 \sim (\nu Q / g H^2)^{1/2}$
- We see, from the substitution of R, we have:

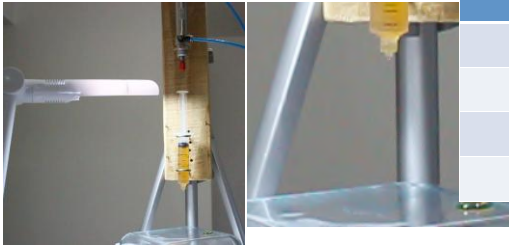
$$R \sim \left(\frac{\nu^3 Q}{g H^2}\right)^{1/3} \propto \nu \quad a_1 \sim \left(\frac{\nu Q}{g H^2}\right)^{1/2} \propto \sqrt{\nu}$$

- There's a moment when the radius R is smaller than the filament radius a. The coil formation is ceased.

Experimental setup



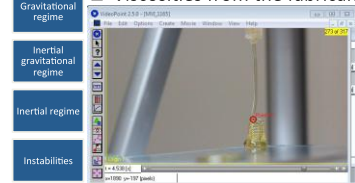
Experimental procedure



| Material used | Viscosity |
|---------------|-----------|
| Corn syrup | 1800 cst |
| Honey | 2000 cst |
| Silicone oil | 5000 cst |
| Silicone oil | 500 cst |

Obtaining data

- We used a high speed camera (2000 fps)
- Height of the fluid measured by using a fixed ruler and Video Point®
- Viscosities from the fabricant of each fluid

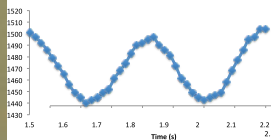


Viscous regime – Honey, 2000

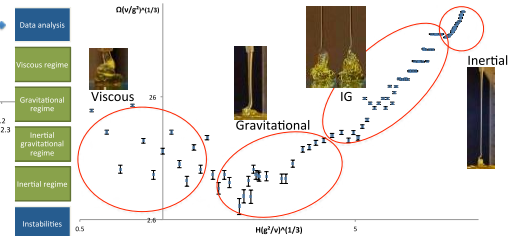
| Data analysis | Fall height | Angular frequency | Theoretical prediction | Relative error |
|-------------------------------|-------------|-------------------|------------------------|----------------|
| Viscous regime | 1.0 cm | 25 Hz | 27 Hz | 7.4% |
| Gravitational regime | 4.0 cm | 45 Hz | 39.0 Hz | 15.3% |
| Inertial gravitational regime | 8.0 cm | 140 Hz | 148 Hz | 5.1% |

Viscous regime
Gravitational regime
Inertial regime

Obtaining data



Honey - Graph

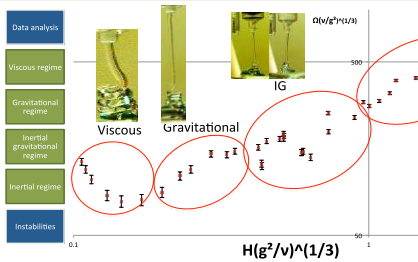


Viscous regime – Silicone oil, 5000 cst

| Data analysis | Fall height | Angular frequency | Theoretical prediction | Relative error |
|-------------------------------|-------------|-------------------|------------------------|----------------|
| Viscous regime | 0.5 cm | 121.2 Hz | 111.2 Hz | 8.2% |
| Gravitational regime | 3.2 cm | 154.8 Hz | 156.4 Hz | 0.9% |
| Inertial gravitational regime | 13.6 cm | 689.2 Hz | 662.8 Hz | 3.8% |

Viscous regime
Gravitational regime
Inertial regime

Silicone oil graph – 5000 cst

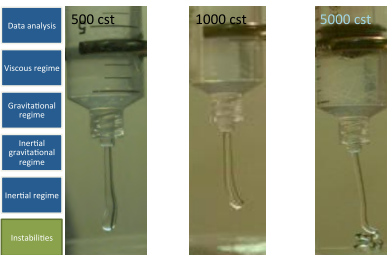


Viscosity variation

For the viscosity variation, we used silicone oils with the same surface tension, density and fall height.

| Viscosity | Surface tension | Density |
|-----------|-----------------|-------------------------|
| 500 cst | 21.2 dynes/cm | 0.970 g/cm ³ |
| 1000 cst | 21.2 dynes/cm | 0.970 g/cm ³ |
| 5000 cst | 21.4 dynes/cm | 0.975 g/cm ³ |
| 60000 cst | 21.5 dynes/cm | 0.976 g/cm ³ |
| 63775 cP | -- | 1.03 g/cm ³ |

Comparative videos



Comparing theoretical and experimental results

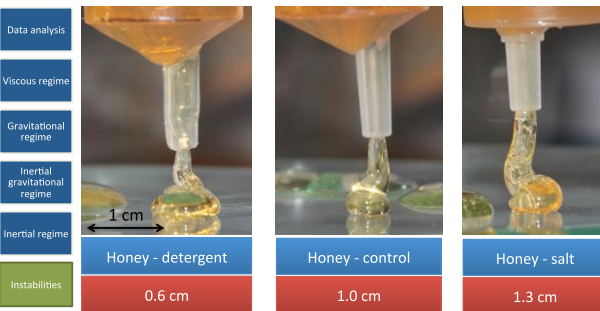
- From our instability analysis:
 - Surface tension reduces the visibility of the phenomena
- Experiment to see its effects:
 - Observe the smaller heights for the phenomena still be seen:
 - Honey (control)
 - Honey with salt (higher surface tension)
 - Honey with a bit of detergent (smaller surface tension)

Experimental setup – Honey and water

- Cup with water
- Honey
- Syringe
- Tripod
- Pneumatic piston

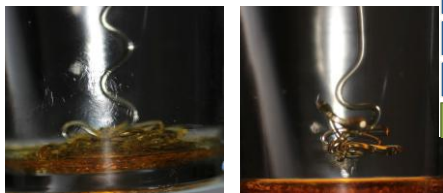


Comparing theoretical and experimental results



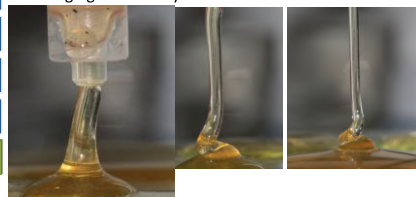
Fluid variation

- We can use another surrounding medium, such as water.
- We get some interesting phenomena:



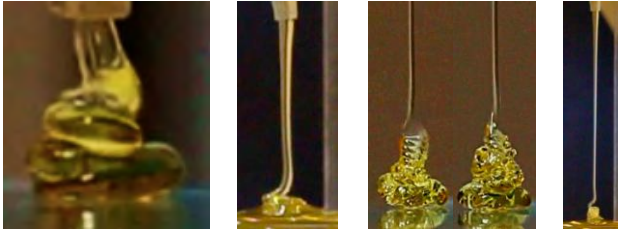
Comparing theoretical and experimental results

- Viscosity
- It's possible to notice changes in the coil formation by changing its viscosity

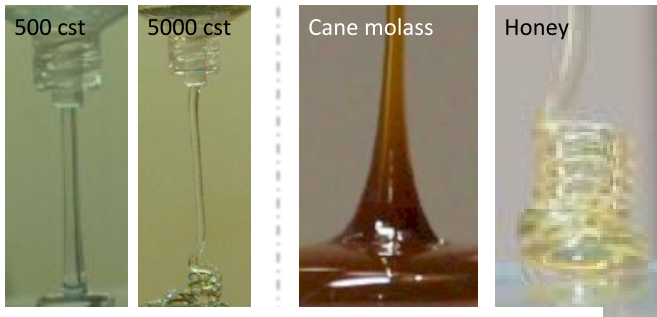


Conclusion

- The phenomena can be divided in 4 phases, depending on the fall height:



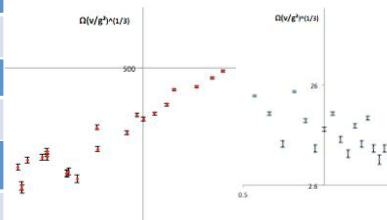
- The coiling can appear in many fluids, but the visualization depends on the surface tension and viscosity.



Conclusion

- We can analyze the problem in a quantitative way, depending on height, viscosity and flow rate.

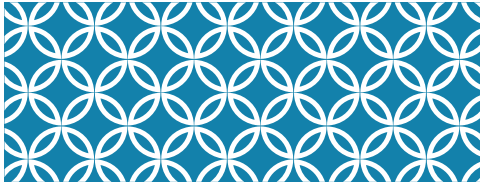
| height | Angular frequency | Flow rate | Theoretical Angular frequency | Error |
|---------|-------------------|-------------------------|-------------------------------|-------|
| 0.5 cm | 30.3 Hz | 0.49 cm ³ /s | 27.8 Hz | 8.2% |
| 3.20 cm | 38.7 Hz | 0.49 cm ³ /s | 39.1 Hz | 0.9% |
| 13.6 cm | 172.3 Hz | 0.49 cm ³ /s | 165.7 Hz | 3.8% |



| Fall height | Angular frequency | Flow velocity | Theoretical Angular frequency | Error | |
|-------------------|-------------------|--------------------|-------------------------------|-------------------------------|--------|
| 1.0 cm | 25 Hz | 25 m/s | 27 Hz | 7.4 % | |
| Fall height | Flow velocity | Angular frequency | Ω parameter | Theoretical angular frequency | Error |
| 4 cm | 30 cm/s | 45 Hz | 0.93 cm | 39 Hz | 15.3 % |
| Angular frequency | Fall height | Ω parameter | Theoretical angular frequency | Error | |
| 140 Hz | 8.0 cm | 0.2 | 148 Hz | 5.1 % | |

References

- *Multiple coexisting states of liquid rope coiling* By N. M. RIBE1, H. E. HUPPERT2, M. A. HALLWORTH2, M. HABIB3,4 AND DANIEL BONN
- *Coiling of viscous jets* By Neil M. Ribe
- *Liquid Rope Coiling*, by Neil M. Ribe, Mehdi Habibi and Daniel Bonn, *Annu. Rev. Fluid Mech.* 2012
- Mahadevan L, Ryu WS, Samuel ADT. 1998. Fluid 'rope trick' investigated. *Nature*
- *The Bouncing Jet: A Newtonian Liquid Rebounding off a Free Surface*, Matthew Thrasher, Bonghwan Jung, Yee Kwong Pang, Chih-Piao Chuu, and Harry L. Swinney
- *The meandering instability of a viscous thread*, Stephen W. Morris, Jonathan H. P. Dawes, Neil M. Ribe, and John R. Dabre
- *Bending-Filament Model for the Buckling and Coiling Instability of Viscous Fluid Rope*, Shin-ichiro Nagahiro, Yoshinori Hayakawa
- *The folding motion of an axisymmetric jet of wormlike-micelles solution*, Matthieu Varagnat, Trushant Majumdar, Will Hartt, Gareth H. McKinley



PROBLEM

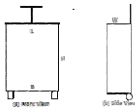
When one pulls along a **two wheeled suitcase**, it can under certain circumstances **wobble so strongly** from side to side that it can **turn over**. Investigate this phenomenon. Can one **suppress or intensify** the effect by **varied packing** of the luggage?

IYPT — AUSTRALIA QUESTION 17: CRAZY SUITCASE

Reporter: Jeong Han Song

TERMS & DEFINITIONS

When one pulls along a **two wheeled suitcase**, it can under certain circumstances **wobble so strongly** from side to side that it can **turn over**. Investigate this phenomenon. Can one **suppress or intensify** the effect by **varied packing** of the luggage?



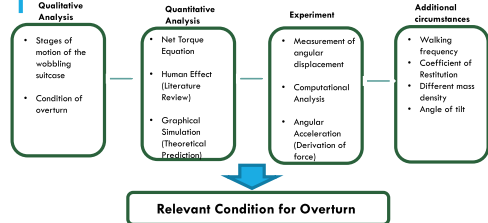
$L = \text{length} = 40 \text{ cm}$
 $H = \text{height} = 50 \text{ cm}$
 $B = \text{distance between two wheels} = 39 \text{ cm}$
 $W = \text{width} = 20 \text{ cm}$

$$\tau = Fd$$

Detachment of both wheels

Different Positions of Centre of mass

FLOW CHART



STAGE 1: INITIAL CONDITION

Titled

Net Force = 0

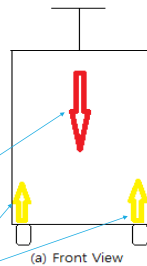
No External Force

Acceleration = 0 (i.e. Constant Speed)

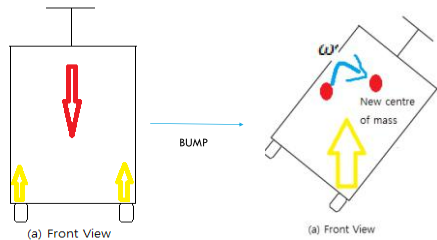
Both wheels are in contact with the floor

Weight of a suitcase

Normal Force by two wheels



STAGE 2: EFFECT OF INITIAL DISTURBANCE



STAGE 3: HUMAN RESPONSE

Human exerts a **periodic force** that **opposes** the disturbance created by the weight.

Opposite direction to the torque created by the weight

Effect of **inertia**

STAGE 2: EFFECT OF INITIAL DISTURBANCE (TORQUE)

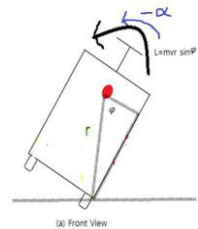
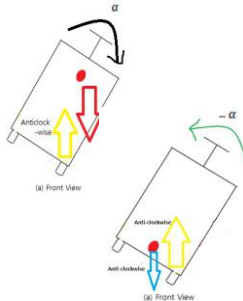
• Different effect depending on the position of weight

• Case 1: **High weight**

- Centre of mass now causes rotation about the supporting wheel
- Torque created by the center of mass
- Angular acceleration is present

• Case 2: **Low weight**

- Low centre of mass acts as a restoring force that opposes the original torque created by the initial disturbance
- Diminishes the wobble effect

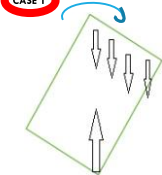


STAGE 4: POINT OF INSTABILITY

The **restoring force overshoots**
Overcompensation of restoring force would lead to the rather sharp increase in the oscillation.

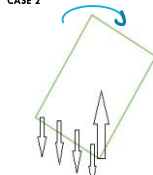
QUALITATIVE MODEL – SUMMARY

CASE 1



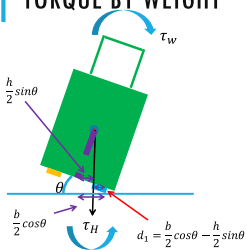
Intensifies the initial disturbance (torque is applied in the same direction as disturbance)

CASE 2



Acts as a restoring force (torque is applied in the opposite direction as disturbance)

TORQUE BY WEIGHT



From basic torque equation

$$\tau = F \times d_1$$

Force is equivalent to the

$$F = mg$$

Shortest distance is:

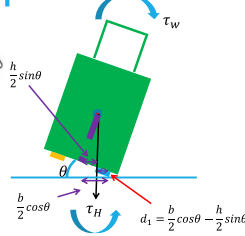
$$\Rightarrow \frac{b}{2} \cos \theta - \frac{h}{2} \sin \theta$$

$$\therefore \tau_w = mg \left(\frac{b}{2} \cos \theta - \frac{h}{2} \sin \theta \right) \dots (1)$$

STAGE 5: REPETITION AND AMPLIFICATION

When a suitcase meets a **critical amplitude** that exceeds what human can counteract, it **overturns**.

TORQUE ANALYSIS



Two Types of torque:

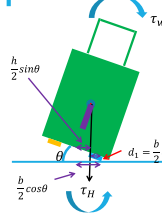
- 1) Torque by Weight
- 2) Torque by Human

Net Torque Equation:

$$I \frac{d^2 \theta}{dt^2} = \tau_H - \tau_w$$

(Opposite Direction)

TORQUE BY HUMAN (SUHERMAN'S MODEL)



Two Fundamental Assumptions:

- 1) Puller's walking motion induces a periodic moment on the handle of suitcase
- 2) Puller exerts additional restoring moment to suppress the wobble proportional to rocking angle

$$\tau_H = q_\theta \sin(\omega t) - k\theta \dots (2)$$

θ : angle of rocking

q_θ : amplitude of excitation torque

ω : walking frequency

k : constant for restoring torque

Periodic Excitation Torque (First Assumption)

Restoring Torque (Second Assumption)

FINAL NET TORQUE EQUATION

Define: $S = +1, \text{ if } \theta > 0$ $S = -1, \text{ if } \theta < 0$

Final Equation:

$$I \frac{d^2 \theta}{dt^2} = q_0 \sin(\omega t) - k\theta - Smg \frac{b}{2} \cos \theta + mg \frac{h}{2} \sin \theta$$

b : bottom length of a suitcase

h : effective height

θ : angle of rocking

q_θ : amplitude of excitation torque

ω : walking frequency

k : constant for restoring torque

As effective height increases, net torque increases!

NET TORQUE EQUATION

Combining Equation 1 & 2:

$$I \frac{d^2 \theta}{dt^2} = \tau_H - \tau_w$$

$$I \frac{d^2 \theta}{dt^2} = q_0 \sin(\omega t) - k\theta - mg \left(\frac{b}{2} \cos \theta - \frac{h}{2} \sin \theta \right) \dots \theta > 0$$

Or

$$I \frac{d^2 \theta}{dt^2} = q_0 \sin(\omega t) - k\theta + mg \left(\frac{b}{2} \cos \theta + \frac{h}{2} \sin \theta \right) \dots \theta < 0$$

THEORETICAL PREDICTION – WOBBLE

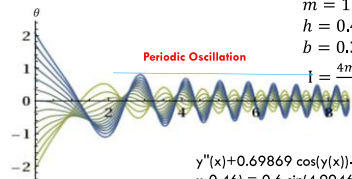
Sample Graph from our computer simulation (wolfram alpha) using values below:

$$m = 12 \text{ kg}$$

$$h = 0.458 \text{ m}$$

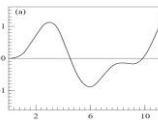
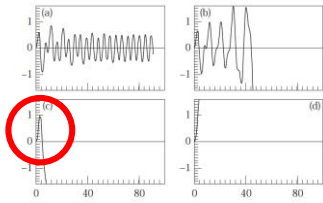
$$b = 0.320 \text{ m}$$

$$I = \frac{4m(h^2 + b^2)}{3} = 4.996 \text{ kgm}^2$$



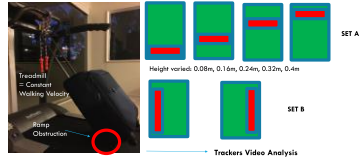
$$y''(x) + 0.69869 \cos(y(x)) - \sin(y(x)) + y \times (2.3232 x - 0.46) = 0.6 \sin(4.9946 \times 2.3232 x + 0.426)$$

LITERATURE REVIEW: SUHERMAN'S RESULTS OVERTURN OF A SUITCASE



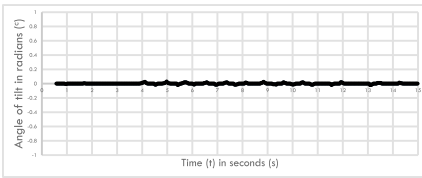
ROCKING ANGLE v TIME
OVERTURN ($\theta > \frac{\pi}{2}$)

EXPERIMENTAL SETUP



- Sharp change in angular displacement
- No wobble during the overturn

RESULT 1: 0.08M EFFECTIVE HEIGHT OF CM



- Very minimal angular displacement
- Inaccurate measurement (interval: 0.1s)
- Hard to observe trend or data



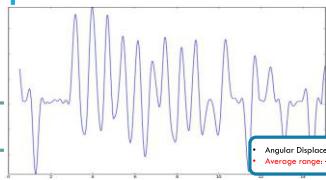
Fast Fourier Transform

SIMULATING THE EQUATION

```

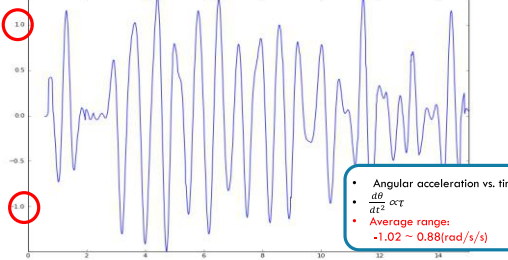
In [12]: import numpy
from pylab import *
g=9.81
m=0.25
L=0.15
theta=0
omega=0
dt=0.001
t=0
theta_max=0
omega_max=0
while t<3:
    theta=theta+omega*dt
    omega=omega-g*L*cos(theta)/m/L*dt
    if abs(theta)>theta_max:
        theta_max=abs(theta)
    if abs(omega)>omega_max:
        omega_max=abs(omega)
    t=t+dt
theta_max
omega_max
    
```

ORIGINAL GRAPH (MAGNIFIED VERSION)



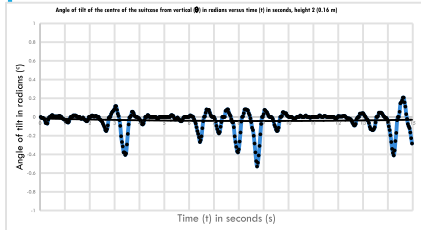
- Angular Displacement vs Time
- Average range: -0.04 ~ 0.01 (rad)

SECOND DERIVATIVE (ANGULAR ACCELERATION)

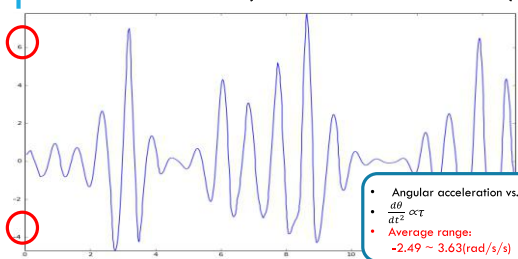


- Angular acceleration vs. tin
- $\frac{d^2\theta}{dt^2} \propto \tau$
- Average range: -1.02 ~ 0.88(rad/s/s)

RESULT 2: 0.16M EFFECTIVE HEIGHT OF CM

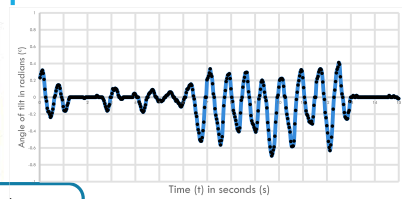


SECOND DERIVATIVE (ANGULAR ACCELERATION)

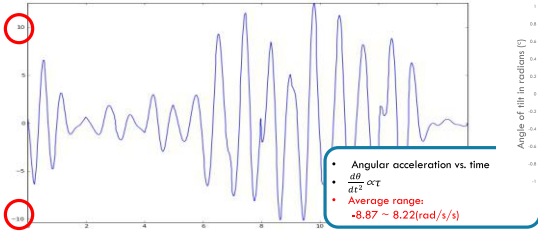


- Angular acceleration vs. time
- $\frac{d^2\theta}{dt^2} \propto \tau$
- Average range: -2.49 ~ 3.63(rad/s/s)

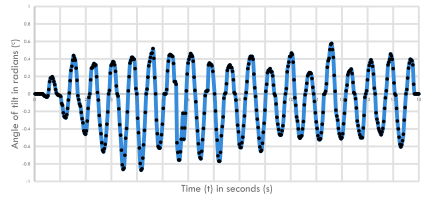
RESULT 3: 0.24M EFFECTIVE HEIGHT OF CM



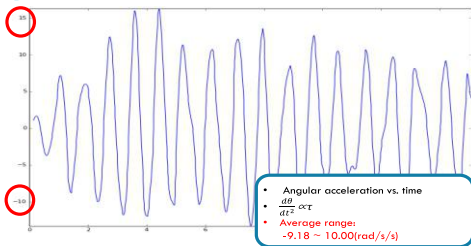
SECOND DERIVATIVE (ANGULAR ACCELERATION)



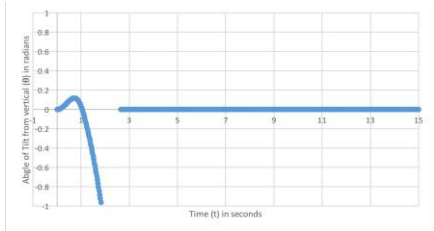
RESULT 4: 0.32M EFFECTIVE HEIGHT OF CM



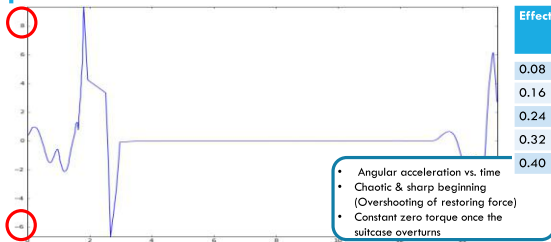
SECOND DERIVATIVE (ANGULAR ACCELERATION)



RESULT 5: 0.40M EFFECTIVE HEIGHT OF CM (OVERTURN)



SECOND DERIVATIVE (ANGULAR ACCELERATION)



SUMMARY OF EXPERIMENTAL RESULT (SET A)

| Effective Height (m) | Range of angular acceleration (rad/s/s) | Average angular acceleration (rad/s/s) |
|----------------------|---|--|
| 0.08 | -1.02 ~ 0.88 | 0.95 |
| 0.16 | -2.49 ~ 3.63 | 3.06 |
| 0.24 | -8.87 ~ 8.22 | 8.54 |
| 0.32 | -9.18 ~ 10.00 | 9.59 |
| 0.40 | Overturn | N/A |

As effective height increases:
 =Increase in angular acceleration
 e in torque
 e in wobble
 LIKELY TO OVERTURN

COMPARISON TO THEORETICAL PREDICTION

Theory (Oscillation)

Experiment (Oscillation)

Theory (Overturn)

Experiment (Overturn)

✔ Verified

SUMMARY OF EXPERIMENTAL RESULT (SET B)

| Wobble Out | Speed (m/s) | Angle | Wobbles | Turns | Wobble Out | Speed (m/s) | Angle | Wobbles | Turns |
|-------------|-------------|-------|---------|-------------------|------------|-------------|-------|---------|-------|
| Plaid wheel | 20° | 1 | 1 | Constructed wheel | 20° | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

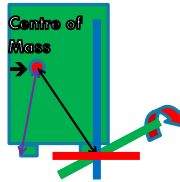
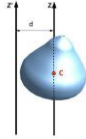
Potential outlier @ 50° for opposite wheel

RESULTS DISCUSSION (SET B)

The closer the centre of mass to the obstructed wheel the less wobbles/overturns the suitcase will have (Parallel axis Theorem).

Having the centre of mass closer to the pivot wheel causes more instability.

$$I = I_{CM} + md^2$$



FURTHER CIRCUMSTANCES OF OVERTURN

- Walking speed
- Angle of tilt
- Mass density
- Coefficient of Restitution

1) Angle of tilt vs Walking speed

| Degree of tilt | 0.81 m/s | 0.92 m/s | 1.04 m/s |
|----------------|----------|----------|----------|
| 70° | O | O | O |
| 60° | N | O | O |
| 50° | N | O | O |
| 40° | N | N | N |



APP FOR WALKING SPEED



ANGLE OF TILT

At 0.81 m/s, stable until high angles.
At 1.04 m/s, stability reduced (high variations in τ_w)

2) LOW WEIGHT:

Mass density vs Walking speed

| No added mass | 0.92 m/s |
|---------------|----------|
| 70° | O |
| 60° | O |
| 50° | N |
| 40° | N |
| 1.5 kg mass | 0.92 m/s |
| 70° | O |
| 60° | O |
| 50° | N |
| 40° | N |
| 3 kg mass | 0.92 m/s |
| 70° | O |
| 60° | N |
| 50° | N |

3) HIGH WEIGHT:

Mass density vs Walking speed

| No added mass | 0.92 m/s |
|---------------|----------|
| 70° | O |
| 60° | O |
| 50° | O |
| 40° | N |
| 1.5 kg mass | 0.92 m/s |
| 70° | O |
| 60° | N |
| 50° | N |
| 40° | N |
| 3 kg mass | 0.92 m/s |
| 70° | N |
| 60° | N |
| 50° | N |

For low weight:

As mass is increased, stability increases.

$$\tau_w \downarrow$$

For high weight:

As mass is increased, stability decreases.

$$\tau_w \uparrow$$

4) COR vs Walking speed

| Indoor (COR 0.35) | 0.81 m/s | 0.92 m/s | 1.04 m/s | Outdoor (COR 0.68) | 0.81 m/s | 0.92 m/s | 1.04 m/s |
|-------------------|----------|----------|----------|--------------------|----------|----------|----------|
| 70° | O | O | O | 70° | O | O | O |
| 60° | N | O | O | 60° | O | O | O |
| 50° | N | O | O | 50° | O | O | O |
| 40° | N | N | N | 40° | O | - | - |

- At lower walking frequency, suitcase was less stable for trials outdoors, due to higher COR and less energy loss.
- At higher walking frequency, energy loss is less significant due to balancing effect of higher walking frequency.

CONCLUSION

Investigated circumstances when suitcase wobbles and turns over.

1. Developed mathematical simulation to model the wobble (FFT)
2. Simulation verified -> Comparison to theory
3. Studied effect of varied packing of the luggage
 - A. Significance of effective height of CM
 - B. Effect of other positions of CM vs. Wheel Position
4. Considered other factors that influence the wobble
 - A. Walking Frequency
 - B. Angle of tilt
 - C. Coefficient of Restitution
 - D. Different mass density

REFERENCE

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Invent Yourself

Boris Vavřík Katarína Tureková Kamil Součková

Paper

standard officepaper

- areal density 80 g/m²
- thickness 0.1 mm
- A4: 210 mm × 297 mm



BRIDGE OPTIMIZATION

How a Simple Bridge Breaks



Bridge Designs



accordion



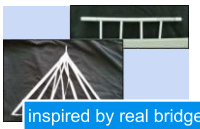
tube



accordion



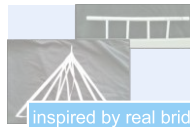
tube



inspired by real bridge



paperunique

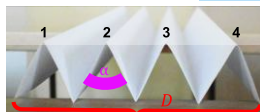


inspired by real bridge



paperunique

Accordion: Parameters



- number of dents V
- width of accordion D

1 Invent Yourself

It is more difficult to bend a paper sheet, if it is

Using a single A4 sheet and a small amount of glue, if required, construct a bridge spanning a gap of 280 mm .

Introduce parameters to describe the strength of your bridge, and optimize some or all of them.

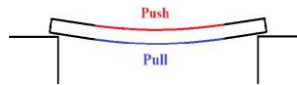
Definition of Strength

strength \equiv maximal load the bridge can hold in its center

- load placed over constant width of the bridge (12 cm)



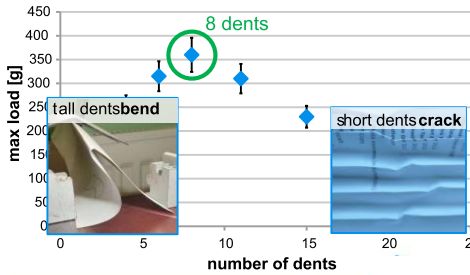
What Happened



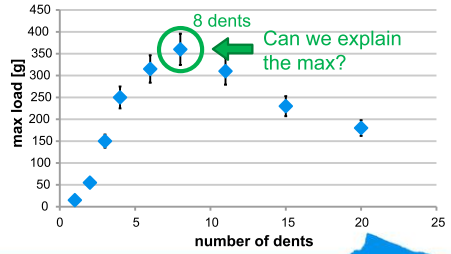
paper: strong in pull, weaker in push



Max Load vs. # of Dents for $D = 8$ cm

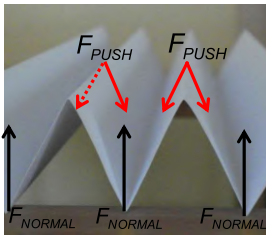


Max Load vs. # of Dents for $D = 8$ cm



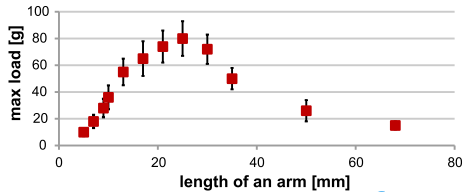
Force analysis

strength of bridge = strength of single dent N



Accordion: Explaining the Max

strength of bridge = strength of single dent N



Accordion: Explaining the Max

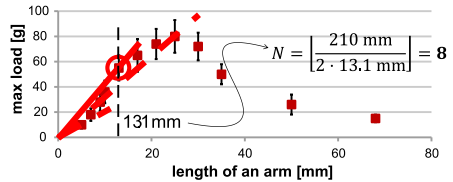
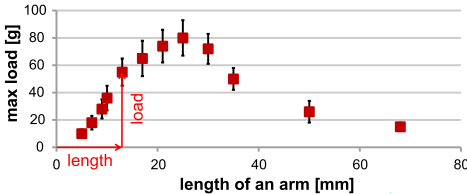
strength of bridge = strength of single dent N

max: highest load/length ratio ($N \propto \frac{1}{\text{length}}$)

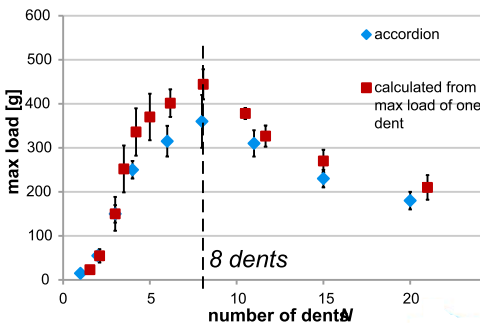
Accordion: Explaining the Max

strength of bridge = strength of single dent N

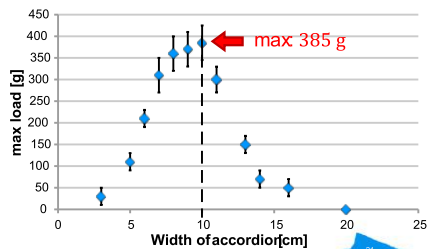
max: highest load/length ratio ($N \propto \frac{1}{\text{length}}$)



Measured Max Load vs N Single Dent



Max Load vs. Accordion Width for 8 dents



Accordion: Optimal Parameters



- max:
- 8 dents
 - $D = 10$ cm

385 g

Modifying the Accordion



max load:
180 g

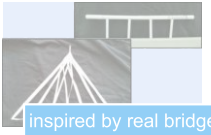
low stability crashes near the center
useless



accordion



tube



inspired by real bridge



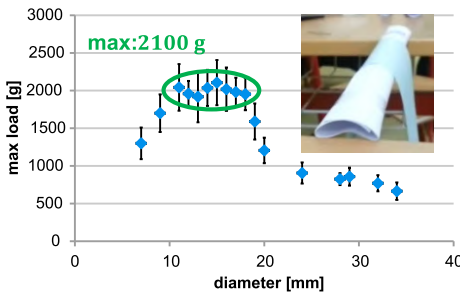
paperunique

Tube Parameters

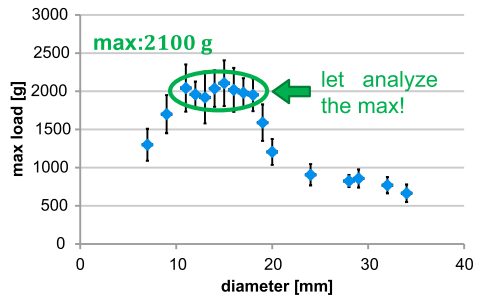


- radius $r \Rightarrow$ number of layers $n = \frac{210 \text{ mm}}{2\pi r}$

Max Load vs. Radius



Max Load vs. Radius



Analysis of Variables

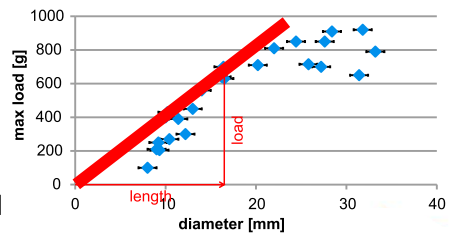
need to separate the effect of

- radius
- number of layers

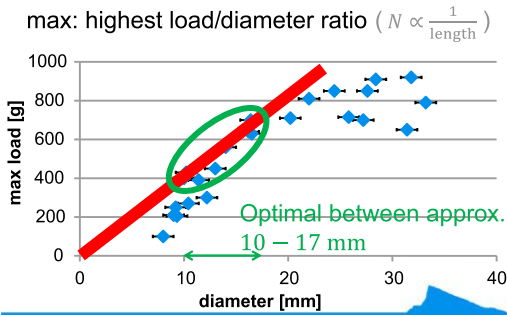
change radius but not number of |
experiment with **double layer tubes**

Max Load of Double Layer Tubes

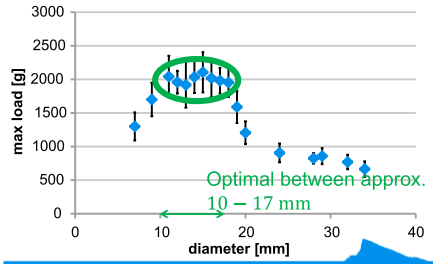
max: highest load/diameter ratio ($N \propto \frac{1}{\text{length}}$)



Max Load of DoubleLayer Tubes

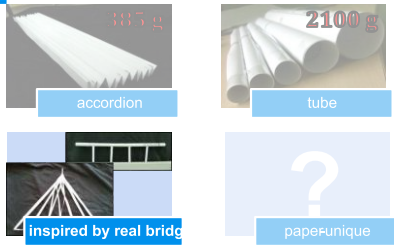


Max Load vs. Radius



Tube: Optimal Parameters

Maximal strength: with 15 mm diameter

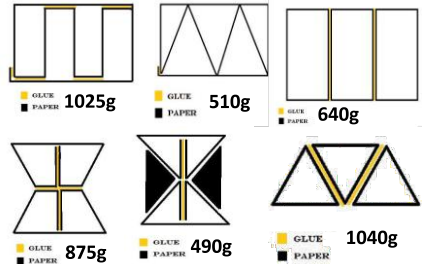


Bridges Inspired by Reality

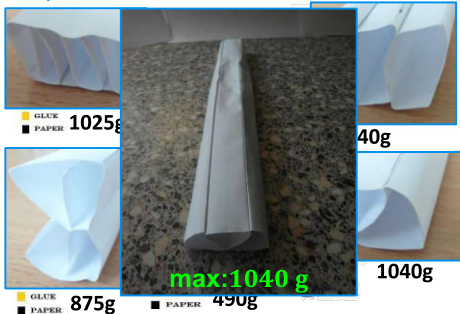
- Different girders
- Suspension bridge
- Pier bridge



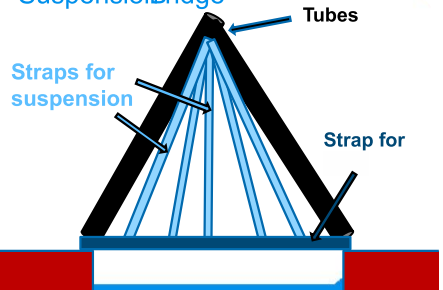
Experiments Different Girders/Profiles



Experiments Different Girders/Profiles



Suspension Bridge



Suspension Bridge



limiting factor:
strength of

- supporting tubes
- suspension straps

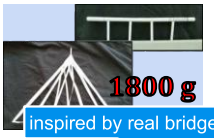
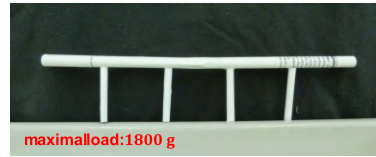
results

| | max load | |
|-----------------|----------|-------------|
| narrower straps | 1000 g | straps tore |
| thinner tubes | 900 g | tubes bent |

Pier bridge

relevant parameter

strength of the tube under bending and pressure

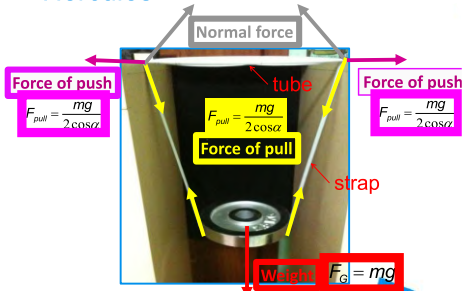


Applicability of Components

| bending | push | pull |
|---------|-------|-------|
| 0 N | 0 N | 570 N |
| 21 N | 100 N | 570 N |
| 11 N | 60 N | 570 N |

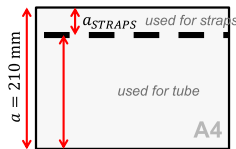
Emojis: weak (sad face), strong (happy face), complex (neutral face)

Hercules



Hercules: Free Parameters

- % of paper used for straps/tube



- angle of straps α



Hercules: Optimizing the Design

straps

$$\frac{mg}{2 \cos \alpha} \leq wtG$$

tube

$$\frac{mg}{2 \cos \alpha} \leq N_{LAYERS} K$$

- t: paper thickness
w: width of the strap
G: paper tensile strength
K: strength of a single tube layer



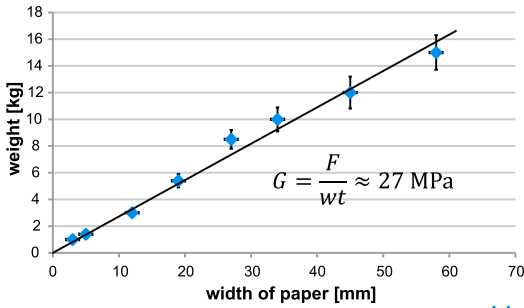
Tensile Strength of Paper Straps

no precise material constants known

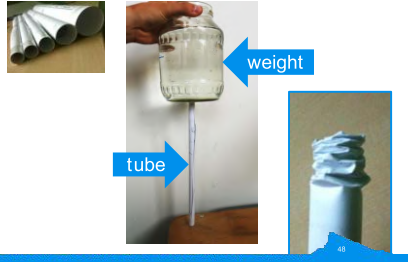
experiment
changing strap width,
measuring max load



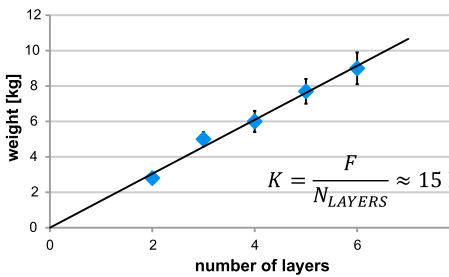
Tensile Strength of Paper Straps



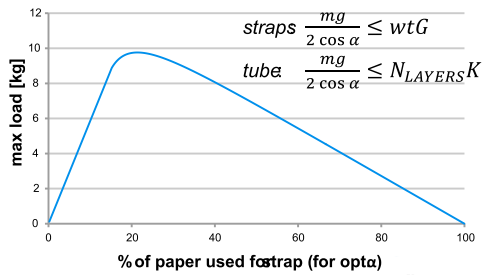
Strength of a Single Layer Tube



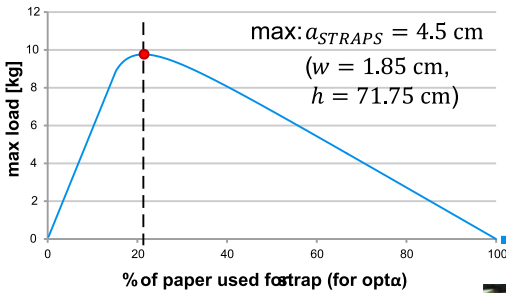
Strength of a Single Layer Tube



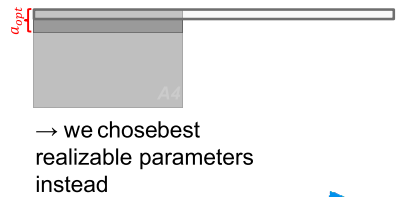
Hercules Strength Prediction



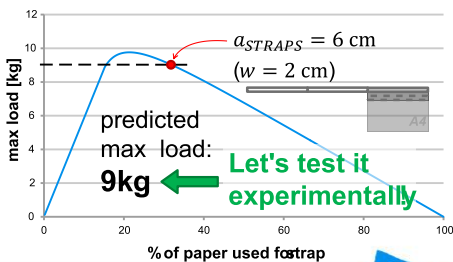
Hercules Strength Prediction



How to make a $1.85 \times 71.75 \text{ cm}$ strap out of a $4.5 \times 29.5 \text{ cm}$ paper?



Hercules Strength Prediction



Hercules: Summary

Applicability Components

bending push pull

maxload

0 50 100

%used fostrap

8.75 kg

9 kg

Conclusion

385 g

2100 g

1800 g

1000 g

1040 g

simpletheoretically explored, max explained

Hercules

maxload

0 50 100

%used fostrap

inspired by reality

385 g

2100 g

1800 g

1000 g

1040 g

simpletheoretically explored, max explained

Hercules

maxload

0 50 100

%used fostrap

inspired by reality



15. Boiled egg

Rozhina Sedigh

15. Boiled egg

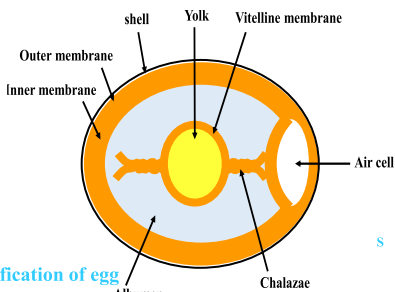
Suggest **non-invasive methods** to detect the degree to which a **hen's egg is cooked by boiling**. Investigate the **sensitivity** of your methods.



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Anatomy of an egg

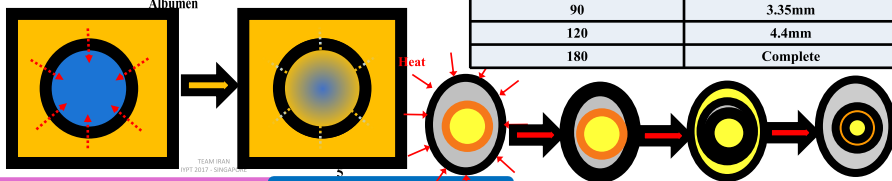


| | Water [%] | Proteins [%] | Lipids [%] | Carbohydrates [%] | Mineral [%] |
|-----------|-----------|--------------|------------|-------------------|-------------|
| Egg white | 88 | 10 | 0.03 | 0.9 | 0.6 |
| Yolk | 49 | 16 | 33 | 1 | 1.0 |
| Shell | 1.6 | 3.3 | | | 95 |

| Protein | Protein fraction present in egg white [%] | Denaturation temperature [°C] |
|-----------|---|-------------------------------|
| Ovalbumin | 58 | 77 |

| Boiling time (sec. In 100°C water) | Thickness of solidified albumen |
|------------------------------------|---------------------------------|
| 30 | 1.9mm |
| 60 | 2.4mm |
| 90 | 3.35mm |
| 120 | 4.4mm |
| 180 | Complete |

solidification of egg

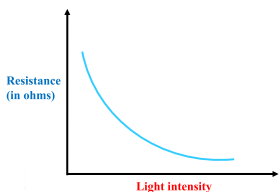


$$t = \frac{M^{3/2} c \rho^{1/3}}{K \pi^2 (4\pi/3)^{2/3}} \ln \left[0.76 \frac{T_{egg} - T_{water}}{T_{yolk} - T_{water}} \right]$$

t : cooking time (min)
 r : radius of egg (mm)
 D : thermal diffusivity
 c : specific heat
 T : egg temperature (C)
 T_c : cooking temperature (C)

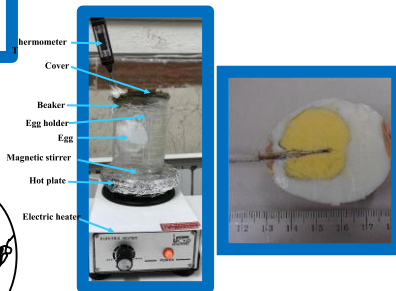
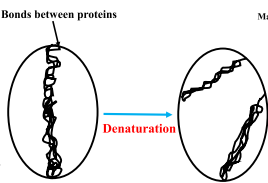
$$\dot{t} \propto \frac{r^2}{D^2 c^2 [\log(T - T_c)]^2}$$

Resistance



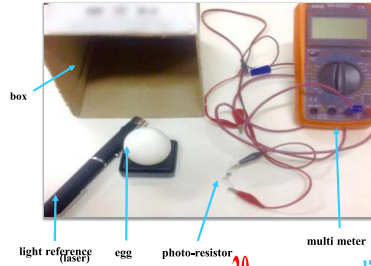
$$R = \frac{v}{I}$$

Denaturation



- Egg Weight \cong 60 g
- Surface \cong 53 cm²
- Shell thickness \cong 0.3 mm
- Shell Weight \cong 6 g
- Egg white weight \cong 34 g
- yolk weight \cong 19 g

Measuring the transparency between different parts of the egg



The light scattering method

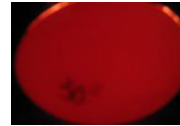


*results from Imagej
 **all pictures taken by Canon DIGITAL IXUS 100 IS, exposure time 1.40 sec, ISO- 250, f/3.2 (aperture time)

30 sec egg green light scattering & light intensity

| | bottom | middle | upside |
|-------------|--------|---------|--------|
| Raw | 16.2 Ω | 154.5 Ω | 22.3 Ω |
| Soft-boiled | 2.6 Ω | 59.9 Ω | 8.6 Ω |
| Hard-boiled | 65.6 Ω | 167.7 Ω | 16.6 Ω |

30 sec boiled egg



Mean intensity for 30 sec of boiling



Mean intensity of green light for 30 sec of boiling



Count: 804492 Min: 46
 Mean: 57.171 Max: 93
 StDev: 1.949 Mode: 57 (185192)

*results from Imagej
 **all pictures taken by Canon DIGITAL IXUS 100 IS, exposure time 1.40 sec, ISO- 250, f/3.2 (aperture time)

1 min egg green light scattered & light intensity

*results from Imagej
 **all pictures taken by Canon DIGITAL IXUS 100 IS, exposure time 1.40 sec, ISO- 250, f/3.2 (aperture time)



Mean intensity for 1 min of boiling



Count: 588500 Min: 44
 Mean: 51.102 Max: 67
 StDev: 2.335 Mode: 50 (101697)

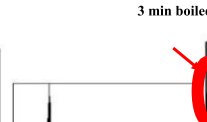
Mean intensity of green light for 1 min of boiling



3 min egg green light scattered & light intensity



Mean intensity for 3 min of boiling



Count: 684456 Min: 31
 Mean: 47.960 Max: 58
 StDev: 2.293 Mode: 48 (185513)

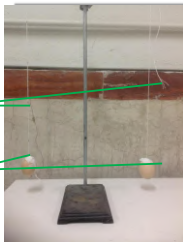
Mean intensity of green light for 3 min of boiling



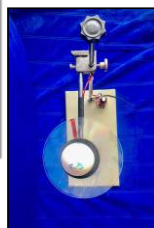
Pendulum method



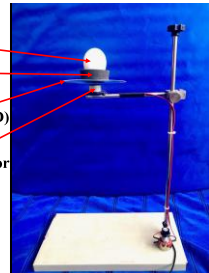
Beaker
 2 strings
 2 eggs



Snipping method



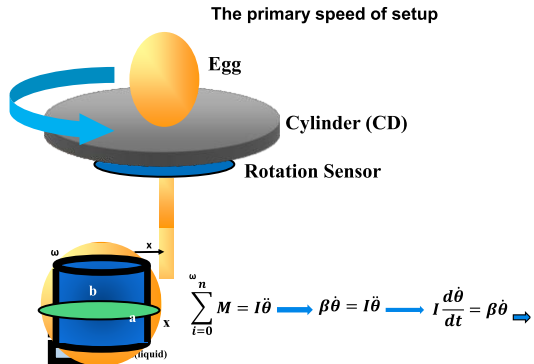
Egg
 Egg holder
 Cylinder(CD)
 Rotation Sensor



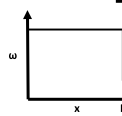
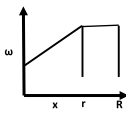
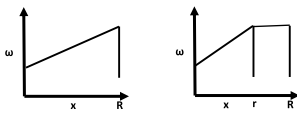
Measuring the speed of rotation



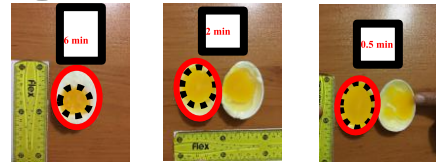
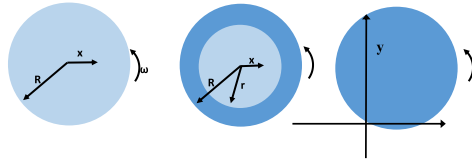
5 min boiled egg



The primary speed of setup



$$\sum_{i=0}^n M = I\ddot{\theta} \rightarrow \beta\dot{\theta} = I\ddot{\theta} \rightarrow I \frac{d\dot{\theta}}{dt} = \beta\dot{\theta} \rightarrow I \frac{d\dot{\theta}}{\dot{\theta}} = \beta dt \rightarrow I \int \frac{d\dot{\theta}}{\dot{\theta}} = \beta \int_0^t dt \rightarrow I \ln|\dot{\theta}| = \beta t \rightarrow \ln|\dot{\theta}| = \frac{\beta}{I} t \rightarrow \dot{\theta} = \frac{I}{\beta} (e^{\frac{\beta t}{I}} - 1)$$



$$F_{vis} = A\mu\omega$$

$$F = \mu \frac{d\theta}{dy}$$

$$A = 4\pi xy$$

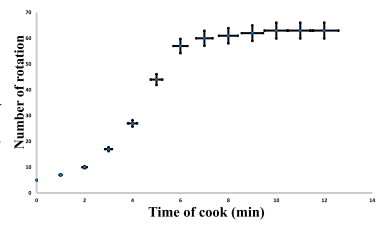
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\int_0^b F_{vis} = \frac{4}{3}\pi a^2 b \mu \dot{\theta}$$

$$x = a \sqrt{1 - \frac{y^2}{b^2}}$$

$$F = 4\pi\mu\dot{\theta} \int_0^a y \sqrt{1 - \frac{y^2}{b^2}} dy$$

$$F_{vis} = 4\pi\mu \left[\frac{1}{3}a^2 - \frac{1}{3} \frac{a^2 - b^2}{a^2} (a^2 - b^2) \right]$$



The photo resistance method

- High error rate because of the multi meter.
- Only can predict the resistance transfer between 2-3 eggs.
- Not reliable relations between the parameters shown in the charts.(charts 1 & 2).

The light scattering method

- Each color-light scattered could be analyzed separately in the soft wares.
- The results could be announced with a high sensitivity degree. (the histograms show it as well).
- As the boiling time increases, the intensity of the light glinted decreases.
- As the boiling time increases, the intensity of the green light scatters, drop.

PROBLEM NO.16 SINKING BUBBLES

When a container of liquid (e.g. water) oscillates vertically, it is possible that bubbles in the liquid move downwards instead of rising. Investigate this phenomenon.

Reporter

Bahar Aghazadeh



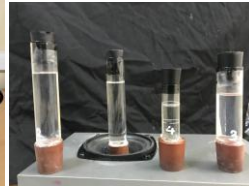
Approach

- What happens?
- Set up
- Acceleration
- Volume of the bubble
- Forces affecting the bubble
- Added mass
- Differential equation
- Motions of the bubble
- Viscosity and sinking
- Density and sinking
- Conclusion

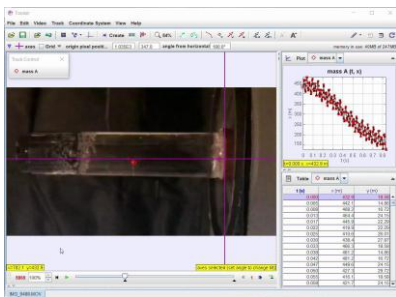
WHAT HAPPENS?



Set up



Extracting data



Volume of the bubble

• Ideal gas :

$$P_t V_b = nRT$$

$$P_t V_b = P_0 V_{b0}$$

$$P_t = P_0 + \rho x(g + A\omega^2 \sin \omega t) + \frac{\sigma}{2r}$$

$$V_b = \frac{P_0 V_{b0}}{P_0 + \rho x(g + A\omega^2 \sin \omega t) + \frac{\sigma}{2r}}$$

ACCELERATION

Movement Amplitude \updownarrow

Movement amplitude= A

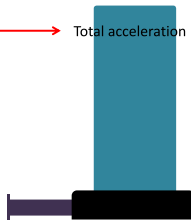
Frequency= ω

$$x = A \sin \omega t \rightarrow \text{Displacement}$$

$$\ddot{x} = A\omega^2 \sin \omega t \rightarrow \text{Sinusoidal acceleration}$$

$$g + A\omega^2 \sin \omega t \rightarrow \text{Total acceleration}$$

Vibrating device



$$V_b = \frac{P_0 V_{b0}}{P_0 + \rho x(g + A\omega^2 \sin \omega t) + \frac{\sigma}{2r}}$$

$$\left(\frac{\rho H_0 (g + A\omega^2)}{P_0} \right)^2 \ll 1$$

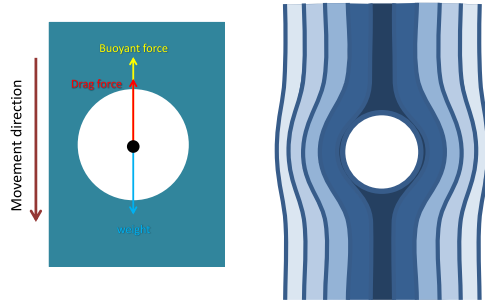
$$V_b = V_{b0} \left(1 - \gamma \frac{x}{H_0} - \gamma \frac{x}{H_0} W \sin \omega t - \frac{\sigma}{2r} \right)$$

FORCES AFFECTING THE BUBBLE

Drag force = $-F(\dot{x})$

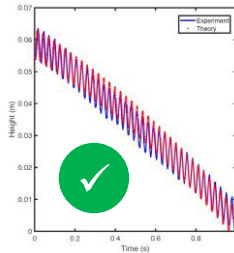
Buoyant force = $\rho g V_b$

$-F(\dot{x}) + (m - \rho V_b)(g + A\omega^2 \sin \omega t) = ma$



Theory vs. experiment

R = 0.001799;
 H0 = 0.06;
 AA = 0.003;
 omega = 2*pi*36;
 rho_f = 1000;
 rho_a = 1;
 P0 = 100000;
 g = 9.8;



DRAG FORCE

$P_{in} - P_{out} = \frac{\sigma}{2r}$

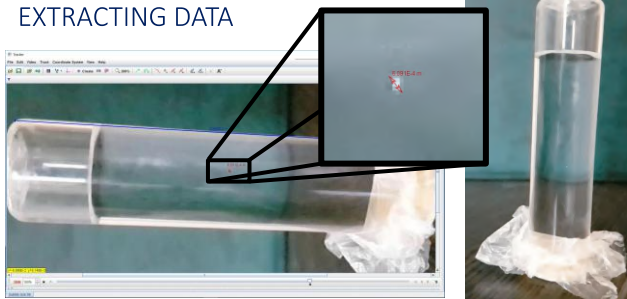
$F_D = 6\pi\mu rV$

$F_D = \frac{1}{2} C_D \rho V^2$

EXTRACTING DATA

Buoyant force $1.14 \times 10^{-6} N$
 Drag force $3 \times 10^{-6} N$

EXTRACTING DATA



Added mass

added mass $m_0 = \rho \bar{x}g$ $\bar{x} = \frac{1}{3}$ of the volume

$m_T = m + m_0$

$F = \frac{d((m + m_0)\dot{x})}{dt}$

$F = (m + m_0)\ddot{x} + \dot{m}_0\dot{x}$



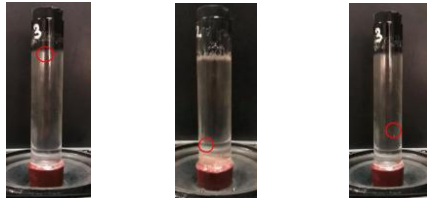
Differential equation

$F = (m + m_0)\ddot{x} + \dot{m}_0\dot{x}$

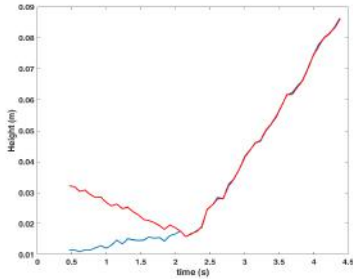
$-F(\dot{x}) + (m - \rho V_b)(g + A\omega^2 \sin \omega t) = ma$

$(m + m_0)\ddot{x} + \dot{m}_0\dot{x} = -F(\dot{x}) + (m - \rho V_b)(g + A\omega^2 \sin \omega t)$

Motions of bubbles



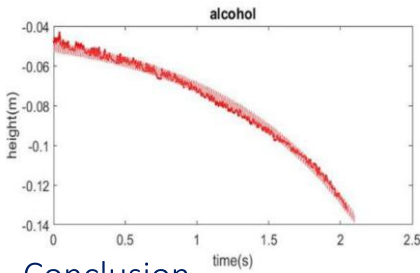
Motions of bubbles



Viscosity and sinking



Density and sinking



Theory
experiment



Conclusion

Volume of the bubble

$$V_b = V_{b0} \left(1 - \gamma \frac{x}{H_0} - \gamma \frac{x}{H_0} W \sin \omega t \right)$$

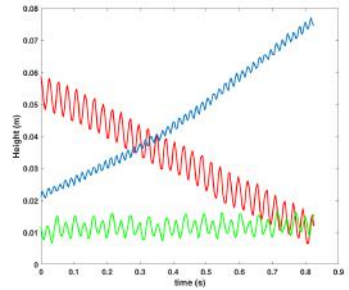
Forces affecting the bubble

1. Weight
2. buoyant force
3. drag force

Mass of the bubble

$$F = (m + m_0)\ddot{x} + \dot{m}_0\dot{x}$$

Different motions of the bubble



Special case (volume increase)

