Some examples on my historic Geometric Theorem.

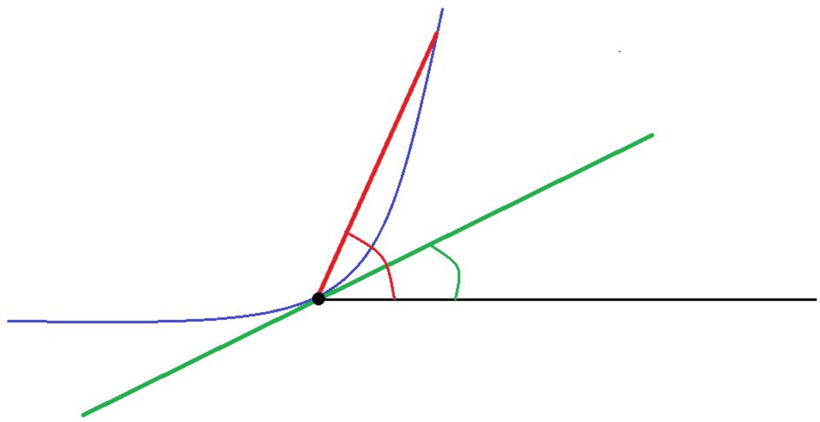
Statement of identity:

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$$

$\frac{f(x+h)-f(x)}{h}$ is the slope of the non-parallel secant line [A]

$f'(x)$ is the slope of the tangent line [B]

$Q(x,h)$ is the difference in slopes [A] and [B]



It should have been pretty obvious to Newton and Leibniz and anyone who came before me that both the non-parallel secant line and tangent line have slopes with respect to the horizontal black line and that $Q(x,h)$ must be the difference. Alas, no one realised this. The theorem and its proof are described here.

Example 1:

$$f(x) = x^4 + 2x$$

$$\rightarrow \frac{f(x+h)-f(x)}{h} = \frac{\left(x^4+4x^3h+6x^2h^2+4xh^3+h^4+2x+2h\right)-\left(x^4+2x\right)}{h}$$

$$\rightarrow \frac{f(x+h)-f(x)}{h} = \frac{4x^3h+6x^2h^2+4xh^3+h^4+2h}{h}$$

$$\rightarrow \frac{f(x+h)-f(x)}{h} = 4x^3 + 2 + 6x^2h + 4xh^2 + h^3$$

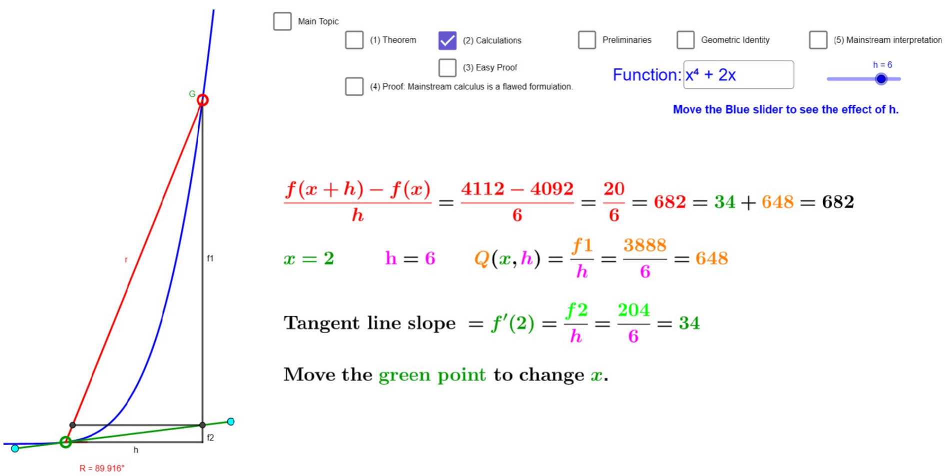
According to the theorem, all the terms without h are the derivative, that is, $4x^3 + 2$. All the terms with a factor of h are the slope difference, that is, $6x^2h + 4xh^2 + h^3$.

If $x = 2$ and $h = 6$,

$$f'(2) = 4(2)^3 + 2 = 34$$

$$Q(2,6) = 6(2)^2(6) + 4(2)(6)^2 + 6^3 = 648$$

See diagram below:



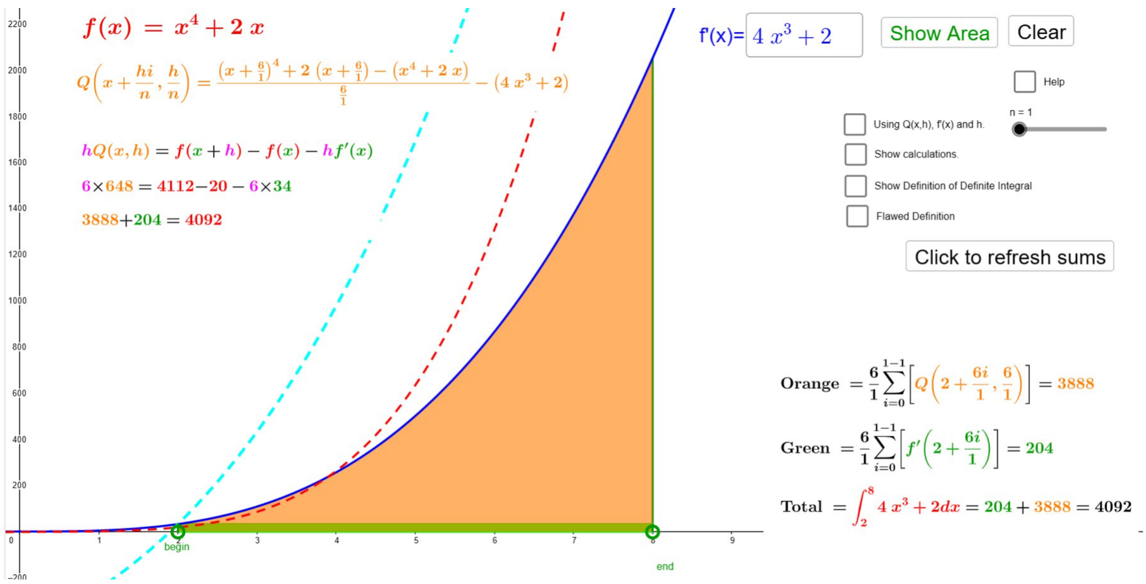
If we wanted to find the area under the curve of

$f'(x) = 4x^3 + 2$ we would simply take the sum of the areas:

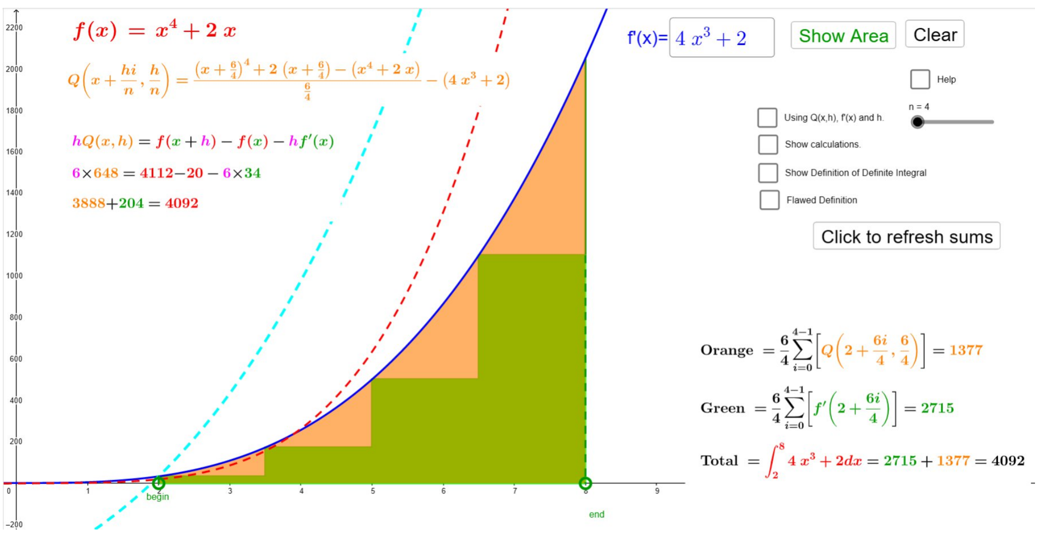
$$34 \times 6 + 648 \times 6 = 4092$$

$$\int_2^6 4x^3 + 2 \; dx = 4092$$

See diagram below:



Recalculating the integral with $n = 6$.



The green coloured areas are always generated by the derivative and the orange coloured areas by the difference function $Q(x,h)$.

Example 2.

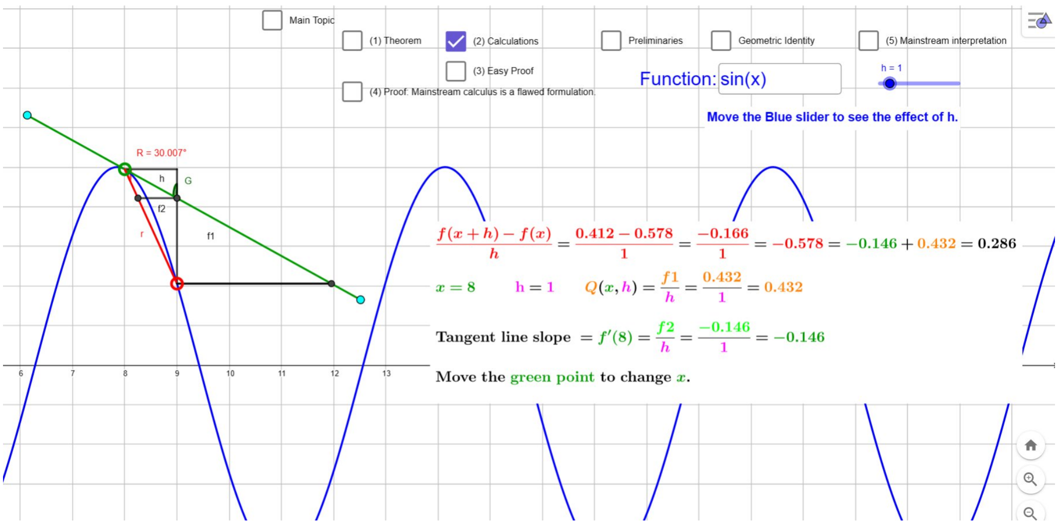
$f\,(x) = \sin\,(x)$

If $x = 8$ (radians) and $h = 1$,

$f'(8) = \cos\,(8) = -\,0.146$

$Q(8,1) = \ 0.432$

$\rightarrow \frac{f(8+1)-f(8)}{1} = -\,0.578 = \cos\,(x) \ + \ Q(x,h) = -\,0.146 + \ 0.432$



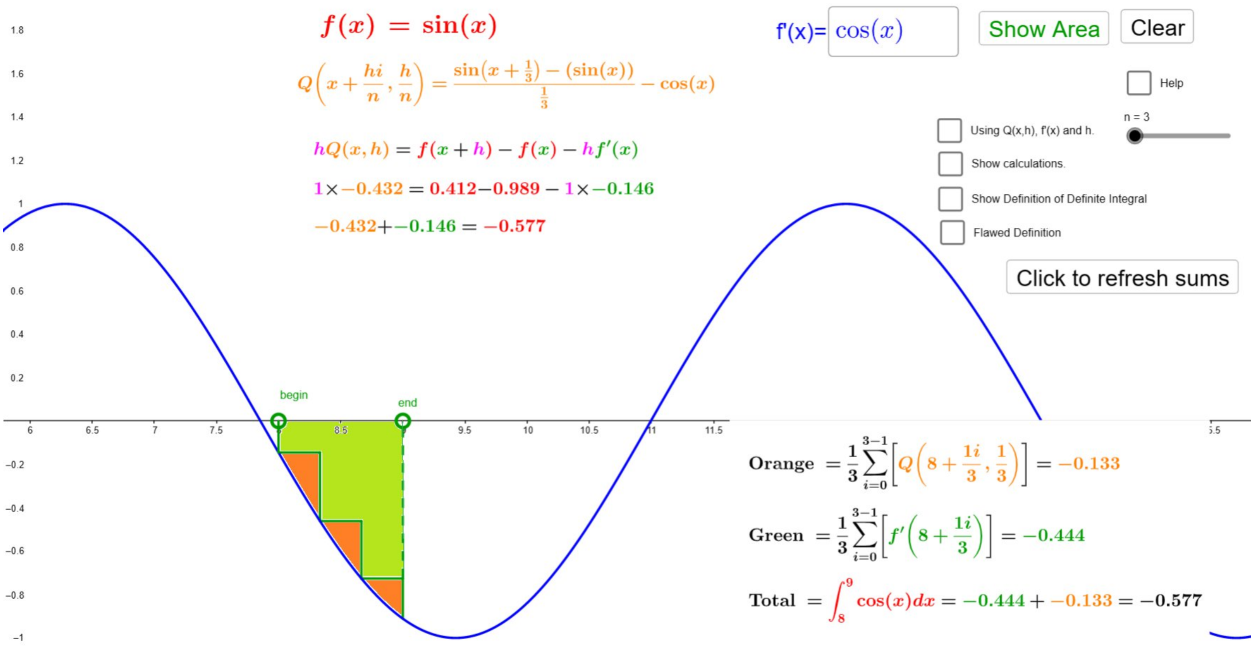
If we wanted to find the area under the curve of

$f'\,(x) = \cos\,(x)$ we would simply take the sum of the areas:

$$(-\,0.146 \times 1) \ + \ (-\,0.432 \times 1) \ = -\,0.578$$

$$\int_8^9 \cos\,(x)\ dx = \ -\,0.578$$

See diagram below:



The geometric theorem will work for **ANY SMOOTH** function and shows you how to **differentiate and integrate** any given function.

Links to applets:

[Derivative]() through Geometry.

[Integral]() through Geometry.

I am the great John Gabriel, discoverer of the New Calculus, the first rigorous formulation of calculus in human history. More advanced alien civilisations may already know of it.