HIGHER LEVEL

interactive eBook inside

HIGHER LEVEL

Physics

for the IB Diploma Programme

3rd Edition

Pearson

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Contents

Introduction

Authors' introduction

Welcome to your study of IB Diploma Programme (DP) Higher Level (HL) physics! This textbook has been written to match the specifications of the new physics curriculum for first examinations in 2025 and gives comprehensive coverage of the course.

Content

The book covers the content that is common to all DP physics students and the additional material for HL students.

HL The additional HL material is labeled as such, and the sequence of the chapters matches the sequence of the Subject Guide themes, with textbook chapter numbering matching the Guide topic numbering.

Each chapter starts with a caption for the opening image, the Guiding Questions, an introduction (which gives the context of the topic and how it relates to your previous knowledge) and the Understandings for the topic. These will give a sense of what is to come, with the Understandings providing the ultimate checklists for when you are preparing for assessments.

Guiding Questions

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

The text covers the course content using plain language, with all scientific terms explained. We have been careful to apply the same terminology you will see in IB examinations in worked examples and questions.

Linking Questions that relate topics to one another can be found throughout, with a hint as to where the answer might be located. The purpose of Linking Questions is to connect different areas of the subject to one another – between topics and to the Nature of Science (NOS) more generally. These questions will encourage an open mind about the scope of the course during your first read through and will be superb stimuli for revision.

How does the motion of a mass (A.1) in a gravitational field (D.1) compare to the motion of a charged particle in an electric field (D.2)?

Each chapter concludes with Guiding Questions revisited and a summary of the chapter, in which we describe how we sought to present the material and what you should now know, understand and be able to do.

Guiding Questions revisited

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

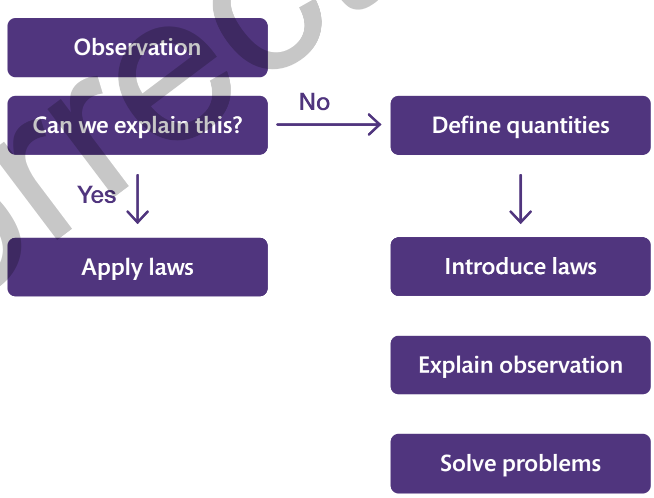
Aims

Using this textbook as part of your course will help you meet these IB DP physics aims to:

- develop conceptual understanding that allows connections to be made between different areas of the subject, and to other DP sciences subjects
- acquire and apply a body of knowledge, methods, tools and techniques that characterize science
- develop the ability to analyze, evaluate and synthesize scientific information and claims
- develop the ability to approach unfamiliar situations with creativity and resilience
- design and model solutions to local and global problems in a scientific context
- develop an appreciation of the possibilities and limitations of science
- develop technology skills in a scientific context
- develop the ability to communicate and collaborate effectively
- develop awareness of the ethical, environmental, economic, cultural and social impact of science.

Nature of physics

Physicists attempt to understand the nature of the Universe. They seek to expand knowledge through testing hypotheses and explaining observations, and by a commitment to checking and re-checking in a bid to set out basic principles. 'Doing physics' involves collecting evidence to reach partial conclusions, creating models to mediate and enable understanding, and using technology.



Physics flowchart.

You will find examples of the nature of physics throughout this book, such as the scattering experiments in E.1, the speed of light in A.5, the relationships between pressure, volume and temperature in B.3, and detecting radiation in E.3.

Nature of Science

Throughout the course, you are encouraged to think about the nature of scientific knowledge and the scientific process as it applies to physics. Examples are given of the evolution of physical theories as new information is gained, the use of models to conceptualize our understanding, and the ways in which experimental work is enhanced by modern technologies. Ethical considerations, environmental impacts, the importance of objectivity and the responsibilities regarding scientists' code of conduct are also considered here. The emphasis is not on memorization, but rather on appreciating the broader conceptual themes in context. We have included some examples but hope that you will come up with your own as you keep these ideas at the forefront of your learning.

The following table provides a comprehensive list of the elements of the Nature of Science that you should become familiar with.

Element	Details
Making observations	Using the human senses, or instruments, and identifying new fields for exploration.
Identifying patterns and trends	Using inductive reasoning (from specific cases to more general laws) and classification of bodies (in overlapping ways), and distinguishing between correlation (relationships between two variables) and causation (when one variable has an effect on another).
Suggesting and testing hypotheses	Provisional qualitative and quantitative relationships with explanations before experimentation is carried out, which can then be tested and evaluated.
Experimentation	The process of obtaining data, testing hypotheses, controlling variables, deciding the appropriate quantity of data, and developing technology that requires creativity and imagination.
Measuring	Recognizing limitations in precision and accuracy, carrying out repeats for reliability, and accepting the existence of and quantifying the random errors that lead to imprecision and uncertainty and the systematic errors that lead to inaccuracy.
Using models	Artificial representations of natural phenomena that are useful when direct observation is difficult, and simplifications of complex systems in the form of physical representations, abstract diagrams, mathematical equations or algorithms, which have inherent limitations.
Collecting evidence	Used to evaluate scientific claims to support or refute scientific knowledge.
Proposing and using theories	Understanding theories (general explanations with wide applicability), deductive reasoning (from the general to the specific) when testing for corroboration or falsification of the theory, paradigm shifts (new and different ways of thinking), and laws (that allow predictions without explanation).
Falsification	Accepting that evidence can refute a claim but cannot prove truth with certainty.
Perceiving science as a shared endeavor	Making use of agreed conventions, common terminology and peer review in the spirit of global communication and collaboration.
Commitment to global impact	Assessing risk to ensure that no harm is done and the ethical, environmental, political, social, cultural, economic and unintended consequences that work may have through compliance with ethics boards, and by communicating honestly and clearly with the public.

Learning physics

Approaches to learning

The IB aspires for all students to become more skilled in thinking, communicating, social activities, research and self-management.

In physics, thinking might include:

- being curious about the natural world
- asking questions and framing hypotheses based upon sensible scientific rationale
- designing procedures and models
- reflecting on the credibility of results
- providing a reasoned argument to support conclusions
- evaluating and defending ethical positions
- combining different ideas in order to create new understandings
- applying key ideas and facts in new contexts
- engaging with, and designing, linking questions
- experimenting with new strategies for learning
- reflecting at all stages of the assessment and learning cycle.

High-quality communication looks like:

- practicing active listening skills
- evaluating extended writing in terms of relevance and structure
- applying interpretive techniques to different forms of media
- reflecting on the needs of the audience when creating engaging presentations
- clearly communicating complex ideas in response to open-ended questions
- using digital media for communicating information
- using terminology, symbols and communication conventions consistently and correctly
- presenting data appropriately
- delivering constructive criticism.

The learning you will do socially could involve:

- working collaboratively to achieve a common goal
- assigning and accepting specific roles during group activities
- appreciating the diverse talents and needs of others
- resolving conflicts during collaborative work
- actively seeking and considering the perspective of others
- reflecting on the impact of personal behavior or comments on others
- constructively assessing the contribution of peers.

You will carry out research, in particular during the Internal Assessment, that includes:

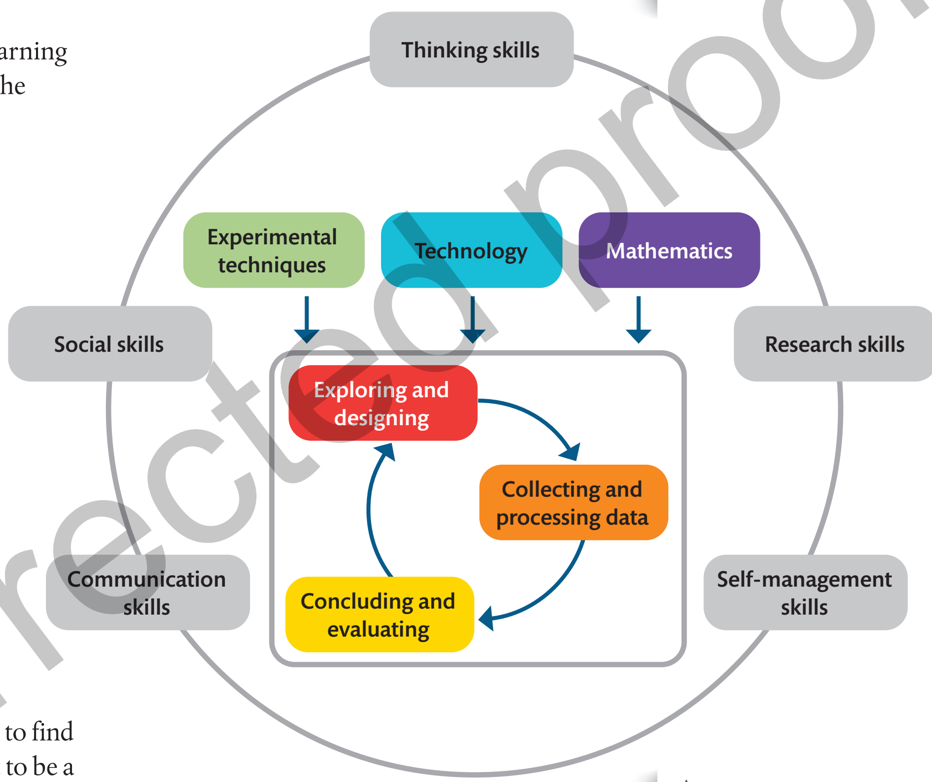
- evaluating information sources for accuracy, bias, credibility and relevance
- explicitly discussing the importance of academic integrity and full acknowledgement of the ideas of others
- using a single, standard method of referencing and citation
- comparing, contrasting and validating information
- using search engines and libraries effectively.

And remember that a significant component of learning comes from you. Maybe you have even reflected on your skills while reading these bullet points! How competent are you at these self-management skills?

- breaking down major tasks into a sequence of stages
- being punctual and meeting deadlines
- taking risks and regarding setbacks as opportunities for growth
- avoiding unnecessary distractions
- drafting, revising and improving academic work
- setting learning goals and adjusting them in response to experience
- seeking and acting on feedback.

Inquiry

Combining the approaches to learning above will facilitate your use of the tools in physics: experimental techniques, technology and mathematics. The next chapter specifically highlights some of these tools; the rest can be found throughout the book.



▲ Tools for physics.

In turn, these tools will enable you to thrive in the inquiry process, which involves exploring and designing, collecting and processing data, and concluding and evaluating. There are opportunities to practice the inquiry process in this book, and the Internal Assessment and Extended Essay chapters include eBook links to exemplar work. You are also sure to find the collaborative sciences project to be a highlight, with its:

- inclusion of real-world problems
- integration of factual, procedural and conceptual knowledge through study of scientific disciplines
- understanding of interrelated systems, mechanisms and processes
- solution-focused strategies
- critical lens for evaluation and reflection
- global interconnectedness (regional, national and local)
- appreciation of collective action and international cooperation.

Learner profile

There is an abundance of ways in which your physics course will support your all-round growth as an IB learner.

Learning attribute	Advice on how to develop
Inquirer	• Be curious, conduct research and try to become more independent. • Ask questions about the world, search for answers and experiment. • Extend your scientific knowledge and engage with existing research.
Knowledgeable	• Explore concepts, ideas and issues, and seek to deepen your understanding of facts and procedures. • Access a variety of resources. • Apply your knowledge to unfamiliar contexts.
Thinker	• Solve complex problems while reflecting on your strategies. • Analyze methods critically and embrace creativity when seeking solutions. • Practice reasoning and critical thinking (testing assumptions, formulating hypotheses, interpreting data and drawing conclusions from evidence).
Communicator	• Accept opportunities to collaborate. • Step out of your comfort zone during group work, for example, by opening discussions or using scientific language. • Listen to others and share your ideas.
Principled	• Take responsibility for your work, promoting shared values and acting in an ethical manner. • Acknowledge the work of others, cite your sources and reduce waste. • To show integrity during data collection, consider all data, including that which does not match your hypothesis.
Open-minded	• Be aware of the existence of different perspectives and models. • Reject or refine your models due to reasoning, deduction or falsification. • Challenge perspectives and ideas.
Caring	• Protect your environment and aim to improve the lives of others. • Choose sustainable practices. • Connect topics to global challenges (like healthcare, energy supply, food production).
Risk-taker	• Seek opportunities for learning and challenge. • Recognize your freedom to try different techniques or methods of learning. • Collect experimental data in a bid to falsify (not just validate) ideas.
Balanced	• Look holistically at your own development and consider how attentive you are to your tasks. • Have a balanced perspective on scientific issues. • Organize your time to avoid negative impacts on the emotional or social aspects of your life.
Reflective	• Consider why and how success is achieved, and how you might change your approach when learning becomes difficult. • Review your strategies, methods, techniques and approaches, for example, using success criteria. • Reflect on your internal network of knowledge.

How to use this book

The book is written according to the following approach, in which we use electric fields as an example.

Observation	Explanation	Laws	Solving problems
• Two rubbed balloons move apart	• Acceleration • Unbalanced forces • Charge	• Coulomb's law • Work done and potential energy	• Exploring field strength and potential gradient • Interpreting further observations of attraction or current electricity • Anticipating the model of the atom

Observation

The aim of the course is to be able to model the physical Universe, so first we must consider a physical process.

A student observes two rubbed balloons moving apart and wonders why they repel. They realize that there must be an unbalanced force. That's the beauty of physical laws; they are always right. The student recognizes a similarity with gravity, which is related to the mass of a body. But gravitational forces are weak and only attractive.

So what is the key property of the body and what is the force? The student does not know, so they have to add something to their model of the Universe.

Explanation

Having studied mechanics and particles, the student has some knowledge of the fundamentals of physics. They know that a body will only accelerate if there is an unbalanced force. We could stop there if this was enough to explain everything, but it is not.

The student reads about a new property, charge. Using what they know about gravitational fields, they expect to learn about field strength (in this case, electric) and wonder if electric forces follow an inverse square law. They carry out an experiment to confirm this.

Laws

Some research reveals that electric forces (like all forces) are vectors, that Coulomb's law applies to point charges, and that moving a charge in an electric field requires a force (so work is done).

They then become curious about the energies involved and read about electric potential energy. They know, using the tool of mathematics, that the area of a graph is the integral of the function and that the reverse of integration is differentiation, so the gradient of a graph of potential energy vs position could be force.

The student is unclear about how field strength can be zero when potential energy is non-zero. They use a simulation and apply the definitions of field strength and potential to a point midway between two equal charges to explore these ideas.

Solving problems

The student makes two further observations. The first is of the attraction between a balloon and a sweater. What might they determine from this? The student then observes their teacher demonstrating a simple electric circuit. What is the connection between the balloons and the circuit?

Based on observations, physicists define quantities and make up a series of rules and laws that fit the observations. They then use these laws to explain further observations, make predictions and solve problems. And it goes on! Having added to their knowledge, the student could now use what they know about mechanics and electricity to develop an understanding of atomic structure.

This example shows how the structure of the book connects factual, procedural and metacognitive knowledge and recognizes the importance of connecting learning with conceptual understanding. Learning physics is a non-linear, ongoing process of adding new knowledge, evolving understanding and identifying misconceptions.

Key to boxes

A popular feature of the book is the different colored boxes interspersed through each chapter. These are used to enhance your learning as explained below.

Nature of Science

This is an overarching theme in the course to promote concept-based learning. Through the book, you should recognize some similar themes emerging across different topics. We hope they help you to develop your own skills in scientific literacy.

Nature of Science

The principle of conservation of momentum is a consequence of Newton's laws of motion applied to the collision between two bodies. If this applies to two isolated bodies, we can generalize that it applies to any number of isolated bodies. Here we will consider colliding balls but it also applies to collisions between microscopic particles such as atoms.

Global context

The impact of the study of physics is global, and includes environmental, political and socio-economic considerations. Examples of this are given here to help you to see the importance of physics in an international context.

Dynamic friction is less than static friction so once a car starts to skid on a corner it will continue. This is also why it is not a good idea to spin the wheels of a car while going round a corner.

Interesting fact

These give background information that will add to your wider knowledge of the topic and make links with other topics and subjects. Aspects such as historic notes on the life of scientists and origins of names are included here.

Negative time does not mean going back in time – it means the time before you started the clock.

Skills

These indicate links to SNSEb eBook resources that include ideas for experiments, technology and mathematics that will support your learning in the course, and help you prepare for the Internal Assessment. Look out for the grey eBook icons.

SKILLS

To find the decay constant and hence half-life of short-lived isotopes, the change in activity can be measured over a period of time using a GM tube.

Theory of Knowledge

These stimulate thought and consideration of knowledge issues as they arise in context. Each box contains open questions to help trigger critical thinking and discussion.

TOK

Color is perceived but wavelength is measured.

Key fact

These key facts are drawn out of the main text and highlighted in bold. This will help you to identify the core learning points within each section. They also act as a quick summary for review.

$$\textbf{velocity} = \frac{\textbf{displacement}}{\textbf{time}}$$

Hint

These give hints on how to approach questions, and suggest approaches that examiners like to see. They also identify common pitfalls in understanding, and omissions made in answering questions.

It is very important to realize that Newton's third law is about two bodies. Avoid statements of this law that do not mention anything about there being two bodies.

Challenge yourself

These boxes contain open questions that encourage you to think about the topic in more depth, or to make detailed connections with other topics. They are designed to be challenging and to make you think.

Challenge yourself

A projectile is launched perpendicular to a 30° slope at 20 m s^{-1}. Calculate the distance between the launching position and landing position.

Toward the end of the book, there are four appendix chapters: Theory of Knowledge as it relates to physics, and advice on the Extended Essay, External Assessment and Internal Assessment.

eBook

In the eBook you will also find the following:

- answers and worked solutions to all exercises in the book
- lab and activity worksheets
- interactive quizzes
- links to videos
- and links to simulations.

Questions

In addition to the Guiding and Linking Questions, there are three types of problems in this book.

1. Worked examples with solutions

These appear at intervals in the text and are used to illustrate the concepts covered. They are followed by the solution, which shows the thinking and the steps used in solving the problem.

Worked example

A body with a constant acceleration of $-5\,m\,s^{-2}$ is traveling to the right with a velocity of $20\,m\,s^{-1}$. What will its displacement be after 20 s?

Solution

$s = ?$

$u = 20\,m\,s^{-1}$

$v = ?$

$a = -5\,m\,s^{-2}$

$t = 20\,s$

To calculate s, we can use the equation: $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \frac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\,m$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped, and then gone backward.

2. Exercises

Exercise questions are found throughout the text. They allow you to apply your knowledge and test your understanding of what you have just been reading. The answers to these are accessed via icons in the eBook next to the questions.

Exercise

Q1. Convert the following speeds into $m\,s^{-1}$:

(**a**) a car traveling at $100\,km\,h^{-1}$

(**b**) a runner running at $20\,km\,h^{-1}$.

3. Practice questions

These questions are found at the end of each chapter. They are mostly taken from previous years' IB examination papers. The mark schemes used by examiners when marking these questions are given in the eBook next to the questions.

Practice questions

1. Police car P is stationary by the side of a road. Car S passes car P at a constant speed of 18 m s^{-1}. Car P sets off to catch car S just as car S passes car P. Car P accelerates at 4.5 m s^{-2} for 6.0 s and then continues at a constant speed. Car P takes *t* seconds to draw level with car S.

(**a**) State an expression, in terms of *t*, for the distance car S travels in *t* seconds. (1)

(**b**) Calculate the distance traveled by car P during the first 6.0 s of its motion. (1)

Worked solutions

Full worked solutions to all exercises and practice questions can also be found in the eBook using the grey icons next to the questions.

We hope you enjoy your study of IB physics.

Chris Hamper and Emma Mitchell

Skills

A vernier caliper is a device that relates to all three aspects of the tools in physics: experimental techniques, technology and mathematics.

As discussed in the Introduction, an excellent IB physicist should be aware of the course aims, appreciate the nature of physics (and science more broadly), and know how to learn and how to inquire.

The skills associated with inquiry have already been discussed and will be referred to once again in the Internal Assessment and Extended Essay chapters. In this chapter, you will find out about the three tools that physicists benefit most from: experimental techniques, technology and mathematics.

Read this chapter before embarking on your studies and continue to refer back to the skills addressed, as almost all elements could be required in any of the topics that follow. When preparing for External Assessment (in particular Paper 1B), you may wish to attempt the practice questions that are located in the eBook.

Tool 1: Experimental techniques

Physics is about modeling the physical Universe so that we can predict outcomes, but before we can develop models, we need to define quantities and measure them. To measure a quantity, we first need to invent a measuring device and define a unit. When measuring, we should try to be as accurate as possible but we can never be exact – measurements will always have uncertainties. This could be due to the instrument or the way we use it, or it might be that the quantity we are trying to measure is changing.

Making observations

Before we can try to understand the Universe, we have to observe it. Imagine you are a cave person looking up into the sky at night. You would see lots of bright points scattered about (assuming it is not cloudy). The points are not the same but how can you describe the differences between them? One of the main differences is that you have to move your head to see different examples. This might lead you to define their position. Occasionally, you might notice a star flashing so would realize that there are also differences not associated with position, leading to the concept of time. If you shift your attention to the world around you, you will be able to make further close-range observations. Picking up rocks, you notice some are easy to pick up while others are more difficult, some are hot and some are cold, and different rocks are different colors. These observations are just the start: to be able to understand how these quantities are related, you need to measure them, and before you do that, you need to be able to count.

Figure 1 Making observations came before science.

Standard notation

In this course, we will use some numbers that are very big and some that are very small. 602 000 000 000 000 000 000 000 is a commonly used number, as is 0.000 000 000 000 000 000 16. To make life easier, we write these in standard form. This means that we write the number with only one digit to the left of the decimal place and represent the number of zeros with powers of 10.

It is also acceptable to use a prefix to denote powers of 10.

Prefix	Value
T (tera)	10^{12}
G (giga)	10^{9}
M (mega)	10^{6}
k (kilo)	10^{3}
c (centi)	10^{-2}
m (milli)	10^{-3}
μ (micro)	10^{-6}
n (nano)	10^{-9}
p (pico)	10^{-12}
f (femto)	10^{-15}

If you set up your calculator properly, it will always give your answers in standard form.

Realization that the speed of light in a vacuum is the same no matter who measures it led to the speed of light being the basis of our unit of length.

The meter was originally defined in terms of several pieces of metal positioned around Paris. This was not very accurate so now one meter is defined as the distance traveled by light in a vacuum in $\frac{1}{299\,792\,458}$ of a second.

So:

602 000 000 000 000 000 000 000 = 6.02×10^{23} (decimal place must be shifted right 23 places)

0.000 000 000 000 000 000 16 = 1.6×10^{-19} (decimal place must be shifted left 19 places).

A number's order of magnitude is the closest whole power of ten. 10^{-2}, 10^{-1}, 10^{0}, 10^{1}, 10^{2} and so on are all orders of magnitude.

Exercise

Q1. Write the following in standard form.

(a) 48 000

(b) 0.000 036

(c) 14 500

(d) 0.000 000 48

Measuring variables

We have seen that there are certain fundamental quantities that define our Universe from which all other quantities can be derived or explained. These include position, time and mass.

Length and distance

Before we take any measurements, we need to define the quantity. The quantities that we use to define the position of different objects are **length** and **distance**. To measure distance, we need to make a scale and to do that we need two fixed points. We take our fixed points to be two points that never change position, for example, the ends of a stick. If everyone used the same stick, we will all end up with the same measurement. However, we cannot all use the same stick so we make copies of the stick and assume that they are all the same. The problem is that sticks are not all the same length, so our unit of length is now based on one of the few things we know to be the same for everyone: the speed of light in a vacuum. Once we have defined the unit, in this case, the meter, it is important that we all use it (or at least make it very clear if we are using a different one). There is more than one system of units but the one used in this course is the Système International d'Unités (SI units). Here are some examples of distances measured in meters:

distance from the Earth to the Sun = 1.5×10^{11} m

diameter of a grain of sand = 2×10^{-4} m

the distance to the nearest star = 4×10^{16} m

radius of the Earth = 6.378×10^{6} m

Exercise

Q2. Convert the following into meters (m) and write in standard form:

(**a**) Distance from London to New York = 5585 km

(**b**) Height of Einstein = 175 cm

(**c**) Thickness of a human hair = 25.4 μm

(**d**) Distance to furthest part of the observable Universe = 100 000 million million million km.

Time

When something happens, we call it an **event**. To distinguish between different events, we use time. The time between two events is measured by comparing to some fixed value, the second. Time is also a fundamental quantity.

Some examples of times:

time between beats of a human heart = 1 s

time for the Moon to go around the Earth = 1 month

time for the Earth to go around the Sun = 1 year

Exercise

Q3. Convert the following times into seconds (s) and write in standard form:

(**a**) 85 years, how long Newton lived

(**b**) 2.5 ms, the time taken for a mosquito's wing to go up and down

(**c**) 4 days, the time it took Apollo 11 to travel to the Moon

(**d**) 2 hours 52 min 59 s, the time it took for Concord to fly from London to New York.

Mass

If we pick up different objects, we find another difference. Some objects are easy to lift up and others are difficult. This seems to be related to how much matter the objects consist of. To quantify this, we define mass measured by comparing different objects to the standard kilogram.

Some examples of mass:

approximate mass of a human = 75 kg

mass of the Earth = 5.97×10^{24} kg

mass of the Sun = 1.98×10^{30} kg

Exercise

Q4. Convert the following masses to kilograms (kg) and write in standard form:

(**a**) The mass of an apple = 200 g

(**b**) The mass of a grain of sand = 0.00001 g

(**c**) The mass of a family car = 2 tonnes.

i

The second was originally defined as a fraction of a day but today's definition is 'the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom'.

TOK

If nothing ever happened, would there be time?

TOK

The kilogram was the last fundamental quantity to be based on an object kept in Paris. It is now defined using Planck's constant. What are the benefits of using physical constants instead of physical objects?

Area and volume

The 2-dimensional space taken up by an object is defined by the area and the 3-dimensional space is volume. Area is measured in square meters (m^2) and volume is measured in cubic meters (m^3). Area and volume are not fundamental units since they can be split into smaller units (m × m or m × m × m). We call units like these derived units.

A list of useful area and volume equations is located in your data booklet.

Exercise

Q5. Calculate the volume of a room of length 5 m, width 10 m and height 3 m.

Q6. Using the information from pages xviii-xix, calculate:

(**a**) the volume of a human hair of length 20 cm

(**b**) the volume of the Earth.

Density

By measuring the mass and volume of many different objects, we find that if the objects are made of the same material, the ratio $\frac{\text{mass}}{\text{volume}}$ is the same. This quantity is called the **density**. The unit of density is $kg\,m^{-3}$. This is another derived unit.

Examples include:

$$\text{density of water} = 1.0 \times 10^3\,kg\,m^{-3}$$
$$\text{density of air} = 1.2\,kg\,m^{-3}$$
$$\text{density of gold} = 1.93 \times 10^4\,kg\,m^{-3}$$

Exercise

Q7. Calculate the mass of air in a room of length 5 m, width 10 m and height 3 m.

Q8. Calculate the mass of a gold bar of length 30 cm, width 15 cm and height 10 cm.

Q9. Calculate the average density of the Earth.

Displacement

So far, all that we have modeled is the position of objects and when events take place, but what if something moves from one place to another? To describe the movement of a body, we define the quantity **displacement**. This is the distance moved in a particular direction.

The unit of displacement is the same as length: the meter.

Referring to the map in Figure 2:
If you move from B to C, your displacement will be 5 km north.
If you move from A to B, your displacement will be 4 km west.

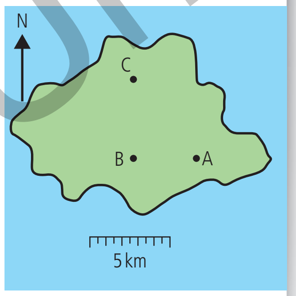


Figure 2 Displacements on a map.

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Angle

When two straight lines join, an angle is formed. The size of the angle can be increased by rotating one of the lines about the point where they join (the vertex) as shown in Figure 3. To measure angles, we often use degrees. Taking the full circle to be 360° is very convenient because 360 has many whole number factors so it can be divided easily by e.g. 4, 6, and 8. However, it is an arbitrary unit not related to the circle itself.

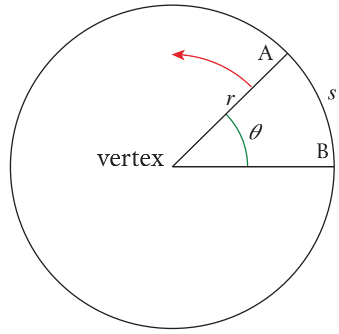


Figure 3 The angle between two lines.

If the angle is increased by rotating line A, the arc lengths will also increase. So for this circle, we could use the arc length as a measure of angle. The problem is that if we take a bigger circle, then the arc length for the same angle will be greater. We therefore define the angle by using the ratio $\frac{s}{r}$, which will be the same for all circles. This unit is the radian.

For one complete circle, the arc length is the circumference = $2\pi r$ so the angle 360° in radians = $\frac{2\pi r}{r} = 2\pi$.

So 360° is equivalent to 2π.

Since the radian is a ratio of two lengths, it has no units.

Summary – Tool 1: Experimental techniques

So far, you will have become familiar with a range of experimental techniques, including measurements of:

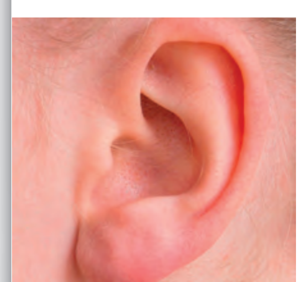
- length
- time
- mass
- volume
- angle.

These tools are prescribed in your Subject Guide.

There are others still to come throughout the textbook. These include measurements of:

- force (A.2)
- temperature (B.1)
- electric current (B.5)
- electric potential difference (B.5)
- sound intensity (C.2)
- light intensity (C.2).

You should also be aware of how to recognize and address safety, ethical and environmental issues. Try to spot these throughout the textbook, such as the risks of high-temperature fluids (B.3) or ionizing radiation (E.3), or the environmental impact of using electricity (B.5) or water (C.2) for experimentation.



The ear is an example of a sensor. Look out for human-made sensors throughout this book.

Tool 2: Technology

Technology and physics are closely linked. Technology enables the advancement of physics, and the pursuit of scientific understanding stimulates improvements in technology. The fields impacted are as wide-ranging as communication, medicine and environmental sustainability.

Every measurement requires an instrument, which is itself inherently technological. Technology facilitates collaboration, which is to the benefit of international teams of scientists and IB physicists alike. Technology makes the processes carried out by physicists much faster, for example, when collecting data or performing calculations.

A model is a representation of reality. It can be as concise as a single word (e.g. the brain is like a 'computer') or an equation (e.g. speed is the ratio of distance traveled to time taken). Technology supports physicists in forming new models during exploratory experimental work (e.g. by making it easy to compare the 'fit' of a range of mathematical relationships) and in creating simulations that enable experimentation without need for a lab.

TOK

Humans can sense light intensity, temperature, sounds, smells, tastes and applied pressure. How might technology replicate or improve upon these senses? What else does technology enable us to measure?

▲ Algodoo® is software that enables the simulation of ideas that may or may not be possible in the lab. Gravity can be altered (or removed altogether) and materials or any desired properties can be tested.

Summary – Tool 2: Technology

Technology can be used to good effect in physics. The Tool 3: Mathematics section of this chapter will reveal that technology can be used to display graphs for representing data. In the remainder of the textbook, you can expect to learn about:

- using sensors (A.2, B.1, B.3, C.1, C.4)
- models and simulations for generation of data (B.2, C.4)
- spreadsheets for manipulation of data (B.5)
- computer modeling for processing data (C.1)
- image analysis of motion (C.5, E.1)
- databases for data extraction (C.5, E.5)
- video analysis of motion (E.3)

TOK

If the system of numbers had been totally different, would our models of the Universe be the same?

Tool 3: Mathematics

When counting apples, we can say there are exactly 6 apples, but if we measure the length of a piece of paper, we cannot say that it is exactly 21 cm wide. All measurements have an associated uncertainty and it is important that this is also quoted with the value. Uncertainties cannot be avoided, but by carefully using accurate instruments, they can be minimized. Physics is all about relationships between different quantities. If the uncertainties in measurement are too big, then relationships are difficult to identify. Throughout the practical part of this course, you will be trying to find out what causes the uncertainties in your measurements. Sometimes, you will be able to reduce them and at other times not. It is quite alright to have big uncertainties but completely unacceptable to manipulate data so that the numbers appear to fit a predicted relationship.

In physics experiments, we always quote the uncertainties in our measurements. Shops also have to work within given uncertainties and could be prosecuted if they overestimate the weight of something. An approximation is similar, but not exactly equal, to something else (for example, a rounded number). An estimate is a simplification of a quantity (such as assuming that an apple has a mass of 100 g).

Summary of SI units

The SI system of units is the set of units that are internationally agreed to be used in science. It is still OK to use other systems in everyday life (miles, pounds, Fahrenheit), but in science, we must always use SI. There are seven fundamental (or base) quantities.

Base quantity	Quantity symbol	Unit	Unit symbol
length	x or l	meter	m
mass	m	kilogram	kg
time	t	second	s
electric current	I	ampere	A
thermodynamic temperature	T	kelvin	K
amount of substance	n	mole	mol
luminous intensity	I	candela	cd

Table 1

All other SI units are derived units. These are based on the fundamental units and will be introduced and defined where relevant. So far we have come across just three.

The candela will not be used in this course.

Derived quantity	Symbol	Base units
area	m^2	m × m
volume	m^3	m × m × m
density	kg m^{-3}	$\frac{kg}{m \times m \times m}$

Table 2

By breaking down the units of derived quantities into base quantity units, it is possible to check whether an equation could be correct. This technique is an informal version of dimensional analysis, in which the 'powers of' quantities are compared on either side of an equation. Note, however, that dimensional analysis provides no insights into the constant of proportionality.

Processing uncertainties

The SI system of units is defined so that we all use the same sized units when building our models of the physical world. However, before we can understand the relationship between different quantities, we must measure how big they are. To make measurements, we use a variety of instruments. To measure length, we can use a ruler and to measure time, a clock. If our findings are to be trusted, then our measurements must be accurate, and the accuracy of our measurement depends on the instrument used and how we use it. Consider the following examples.

Even this huge device at CERN has uncertainties.

When using a scale such as a ruler, the uncertainty in the reading is half of the smallest division. In this case, the smallest division is 1 mm so the uncertainty is 0.5 mm. When using a digital device such as a balance, we take the uncertainty as the smallest digit. So if the measurement is 20.5 g, the uncertainty is ±0.1 g.

In Examples 1 and 2, we are assuming that there is no uncertainty at the 'zero' end of the ruler because it might be possible to line up paper with the long ruler marking.

In reality, the uncertainty for Example 1 may be ±0.1 cm, which is the combination of the 0.05 cm uncertainties at each end of the length.

Notice that uncertainties are generally quoted to one significant figure. The uncertainty then dictates the number of decimal places to which the measurement is written.

Measuring length using a ruler

Example 1

A good straight ruler marked in mm is used to measure the length of a rectangular piece of paper as in Figure 4.

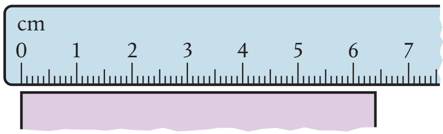


Figure 4 Length = 6.40 ± 0.05 cm.

The ruler measures to within 0.5 mm (we call this the **uncertainty** in the measurement) so the length in cm is quoted to 2 d.p. This measurement is precise and accurate. This can be written as 6.40 ± 0.05 cm, which tells us that the actual value is somewhere between 6.35 and 6.45 cm.

Example 2

Figure 5 shows how a ruler with a broken end is used to measure the length of the same piece of paper. When using the ruler, you might fail to notice the end is broken and think that the 0.5 cm mark is the zero mark.

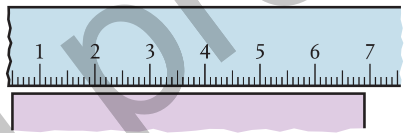


Figure 5 Length ≠ 6.90 ± 0.05 cm.

This measurement is precise since the uncertainty is small but is not accurate since the value 6.90 cm is wrong.

Example 3

A ruler marked only in $\frac{1}{2}$ cm is used to measure the length of the paper as in Figure 6.

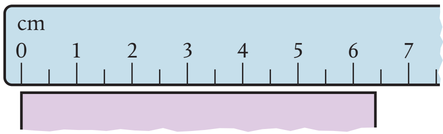


Figure 6 Length = 6.5 ± 0.3 cm.

These measurements are precise and accurate, but the scale is not very sensitive.

Example 4

In Figure 7, a ruler is used to measure the maximum height of a bouncing ball. The ruler has more markings, but it is very difficult to measure the height of the bouncing ball. Even though you can use the scale to within 0.5 mm, the results are not precise (the base of the ball may be at about 4.2 cm). However, if you do enough runs of the same experiment, your final answer could be accurate.

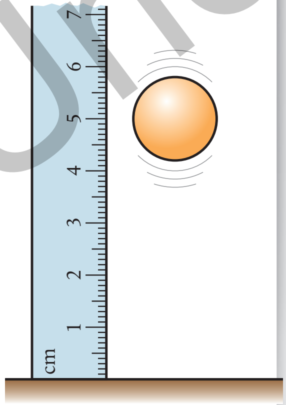


Figure 7
Height = 4.2 ± 0.2 cm.

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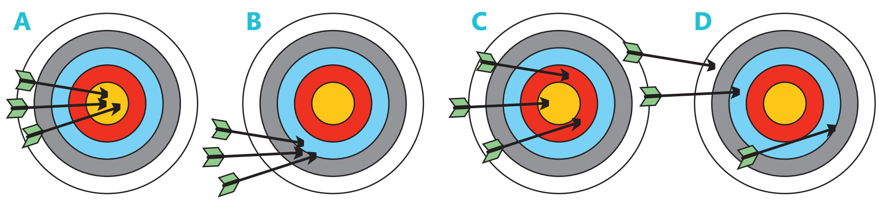
Precision and accuracy

To help understand the difference between precision and accuracy, consider the four attempts to hit the center of a target with three arrows shown in Figure 8.



Precise and accurate | Precise but not accurate | Not precise but accurate | Not precise and not accurate

Figure 8 Precise or accurate?

A The arrows were fired accurately at the center with great precision.

B The arrows were fired with great precision as they all landed near one another, but not very accurately since they are not near the center.

C The arrows were not fired very precisely since they were not close to each other. However, they were accurate since they are evenly spread around the center. The average of these would be quite good.

D The arrows were not fired accurately and the aim was not precise since they are far from the center and not evenly spread.

So **precision** is how close to each other a set of measurements are (related to the resolution of the measuring instrument) and the **accuracy** is how close they are to the actual value (often based on an average).

If you measure the same thing many times and get the same value, then the measurement is precise. If the measured value is close to the expected value, then the measurement is accurate. If a football player hits the post 10 times in a row when trying to score a goal, you could say the shots are precise but not accurate.

Errors in measurement

There are two types of measurement error – random and systematic.

Random error

If you measure a quantity many times and get lots of slightly different readings, then this called a random error. For example, when measuring the bounce of a ball, it is very difficult to get the same value every time even if the ball is doing the same thing.

Systematic error

A systematic error is when there is something wrong with the measuring device or method. Using a ruler with a broken end can lead to a 'zero error' as in Example 2 on page xxiv. Even with no random error in the results, you would still get the wrong answer.

TOK

It is not possible to measure anything exactly. This is not because our instruments are not exact enough but because the quantities themselves do not exist as exact quantities. What measurements could you make in the space around you? What might makes these quantities inexact?

Reducing errors

To reduce random errors, you can repeat your measurements. If the uncertainty is truly random, your measurements will lie either side of the true reading and the mean of these values will be close to the actual value. To reduce a systematic error, you need to find out what is causing it and correct your measurements accordingly. A systematic error is not easy to spot by looking at the measurements, but is sometimes apparent when you look at the graph of your results or the final calculated value.

Adding uncertainties

If two values are added together, then the uncertainties also add. For example, if we measure two lengths, $L_1 = 5.0 \pm 0.1$ cm and $L_2 = 6.5 \pm 0.1$ cm, then the maximum value of L_1 is 5.1 cm and the maximum value of L_2 is 6.6 cm, so the maximum value of $L_1 + L_2 = 11.7$ cm. Similarly, the minimum value is 11.3 cm. We can therefore say that $L_1 + L_2 = 11.5 \pm 0.2$ cm.

If $\quad y = a \pm b \quad$ then $\quad \Delta y = \Delta a + \Delta b$

If you multiply a value by a constant, then the uncertainty is also multiplied by the same number.

So $\quad 2L_1 = 10.0 \pm 0.2$ cm $\quad$ and $\quad \frac{1}{2}L_1 = 2.50 \pm 0.05$ cm.

Example of measurement and uncertainties

▲ Ball of modeling clay measured with a ruler.

Let us consider an experiment to measure the mass and volume of a piece of modeling clay. To measure mass, we can use a top pan balance so we take a lump of clay and weigh it. The result is 24.8 g. We can repeat this measurement many times and get the same answer. There is no variation in the mass so the uncertainty in this measurement is the same as the uncertainty in the scale. The smallest division on the balance used is 0.1 g so the uncertainty is ±0.1 g.

So: $\quad$ mass = 24.8 ±0.1 g

To measure the volume of the modeling clay, we first need to mold it into a uniform shape: let us roll it into a sphere. To measure the volume of the sphere, we measure its diameter from which we can calculate its radius ($V = \frac{4\pi r^3}{3}$).

Making an exact sphere out of the modeling clay is not easy. If we do it many times, we will get different-shaped balls with different diameters so let us try rolling the ball five times and measuring the diameter each time with a ruler.

Using the ruler, we can only judge the diameter to the nearest mm so we can say that the diameter is 3.5 ± 0.1 cm. It is actually even worse than this since we also have to line up the zero at the other end, so 3.5 ± 0.2 cm might be a more reasonable estimate. If we turn the ball round, we get the same value for *d*. If we squash the ball and make a new one, we might still get a value of 3.5 ± 0.2 cm. This is not because the ball is a perfect sphere every time but because our method of measurement is not **sensitive** enough to measure the difference.

Let us now try measuring the ball with a vernier caliper.

A vernier caliper has sliding jaws, which are moved so they touch both sides of the ball.

The vernier caliper can measure to the nearest 0.002 cm. Repeating measurements of the diameter of the same lump of modeling clay might give the results in Table 3.

Table 3

Diameter/cm								
3.640	3.450	3.472	3.500	3.520	3.520	3.530	3.530	3.432
3.540	3.550	3.550	3.560	3.560	3.570	3.572	3.582	3.582

The reason these measurements are not all the same is because the ball is not perfectly uniform and, if made several times, will not be exactly the same. We can see that there is a spread of data from 3.400 cm to 3.570 cm, with most lying around the middle. This can be shown on a graph but first we need to group the values as in Table 4.

Range/cm	No. of values within range
3.400–3.449	1
3.450–3.499	2
3.500–3.549	6
3.550–3.599	8
3.600–3.649	1

Table 4

Distribution of measurements

Even with this small sample of measurements, you can see in Figure 9 that there is a spread of data: some measurements are too big and some too small but most are in the middle. With a much larger sample, the shape would be closer to a 'normal distribution' as in Figure 10.

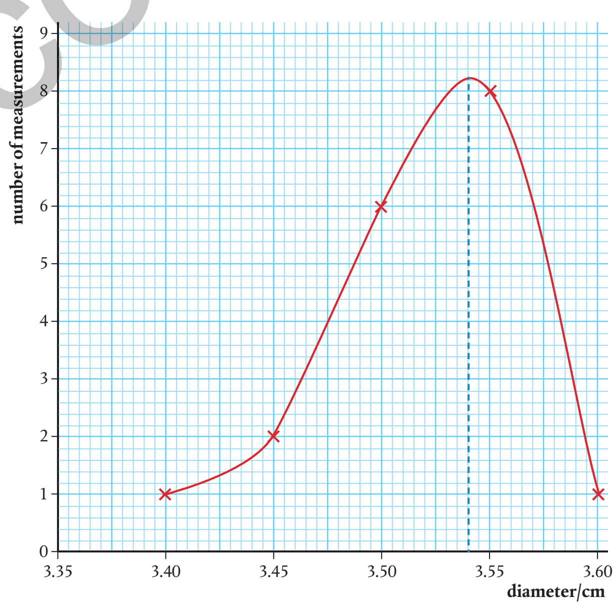


Figure 9 Distribution of measurements of diameter.

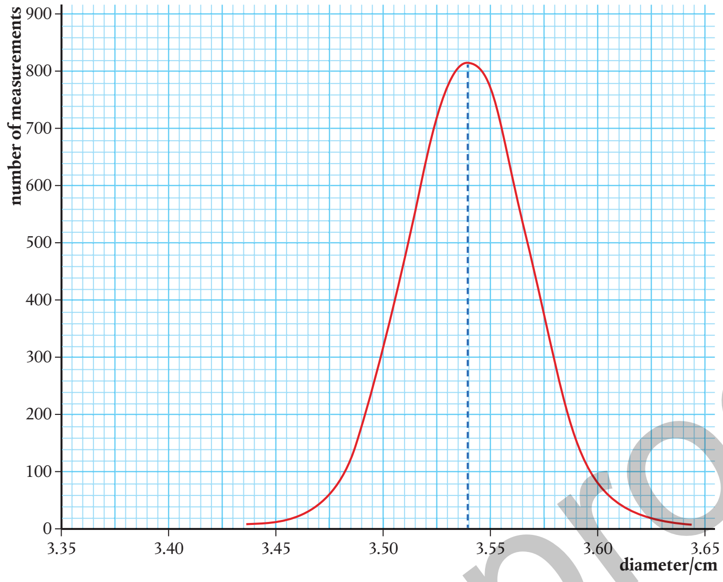


Figure 10 Normal distribution curve.

The mean

At this stage, you may be wondering what the point is of trying to measure something that does not have a definite value. Well, we are trying to find the volume of the modeling clay using the formula $V = \frac{4\pi r^3}{3}$. This is the formula for the volume of a perfect sphere. The problem is we cannot make a perfect sphere. It is probably more like the shape of an egg, so depending on which way we measure it, sometimes the diameter will be too big and sometimes too small. It is, however, just as likely to be too big as too small, so if we take the mean of all our measurements, we should be close to the 'perfect sphere' value which will give us the correct volume of the modeling clay.

The mean or average is found by adding all the values and dividing by the number of values. In this case, the mean = 3.537 cm. This is the same as the peak in the distribution. We can check this by measuring the volume in another way, for example, sinking it in water and measuring the volume displaced. Using this method gives a volume = 23 cm³. Rearranging the formula gives: $r = \sqrt[3]{\frac{3V}{4\pi}}$

Substituting for V gives d = 3.53 cm, which is fairly close to the mean. Calculating the mean reduces the random error in our measurement.

There is a very nice example of this that you might like to try. Fill a jar with jelly beans and get your classmates to guess how many there are. Assuming that they really try to make an estimate rather than randomly saying a number, the guesses are just as likely to be too high as too low. So, if after you collect all the data you find the average value, it should be quite close to the actual number of beans.

If the data follows a normal distribution, 68% of the values should be within one standard deviation of the mean.

Knowing the mean of data enables a calculation of the standard deviation to be performed. Standard deviation gives an idea of the spread of the data.

Smaller samples

You will be collecting a lot of different types of data throughout the course but you will not often have time to repeat your measurements enough to get a normal distribution. With only four values, the uncertainty is not reduced significantly by taking the mean so *half* the range of values is used instead. This often gives a slightly exaggerated value for the uncertainty – for the example above, it would be ± 0.1 cm – but it is an approach accepted by the IB.

Relationships

In physics, we are very interested in the relationships between two quantities, for example, the distance traveled by a ball and the time taken. To understand how we represent relationships by equations and graphs, let us consider a simple relationship regarding fruit.

Linear relationships

Let us imagine that all apples have the same mass, 100 g. To find the relationship between number of apples and their mass, we would need to measure the mass of different numbers of apples. These results could be put into a table as in Table 5.

Number (N)	Mass (m)/g
1	100
2	200
3	300
4	400
5	500
6	600

Table 5

In this example, we can clearly see that the mass of the apples increases by the same amount every time we add an apple. We say that the mass of apples is **proportional** to the number. If we draw a graph of mass vs number, we get a straight line passing through the origin as in Figure 11.

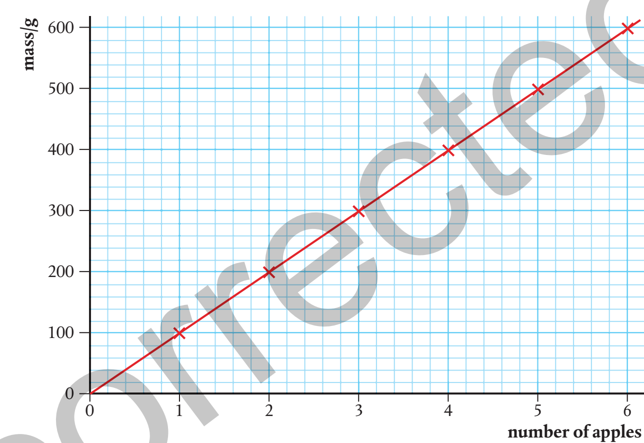


Figure 11 Graph of mass vs number of apples.

The gradient of this line is given by $\frac{\Delta y}{\Delta x}$ = 100 g/apple. The fact that the line is straight and passing through the origin can be used to test if two quantities are proportional to each other.

The equation of the line is $y = mx$, where m is the gradient, so in this case $y = 100x$ and $m = 100\text{ g apple}^{-1}$.

This equation can be used to calculate the mass of any given number of apples. This is a simple example of what we will spend a lot of time doing in this course.

To make things a little more complicated, let us consider apples in a basket with mass 500 g. The table of masses is shown in Table 6.

Number (N)	Mass (m)/g
1	600
2	700
3	800
4	900
5	1000
6	1100

Table 6

The slope in Figure 12 is still 100 g/apple, indicating that each apple still has a mass of 100 g, but the intercept is no longer (0, 0). We say that the mass is linearly related to the number of apples but they are *not* directly proportional.

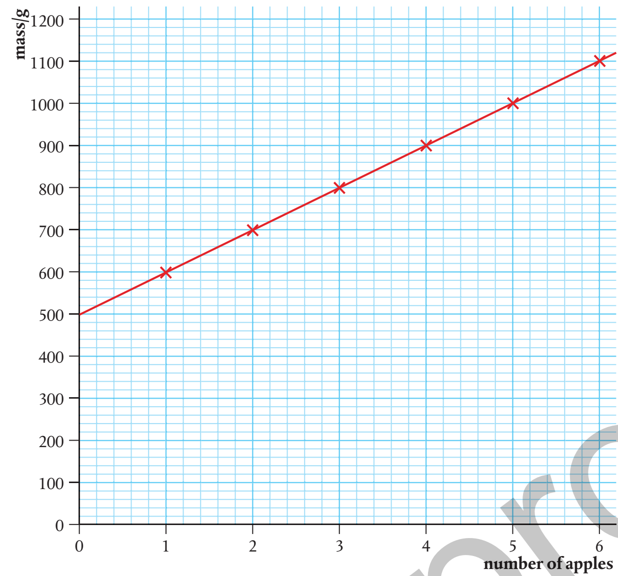


Figure 12 Graph of mass vs number of apples in a basket.

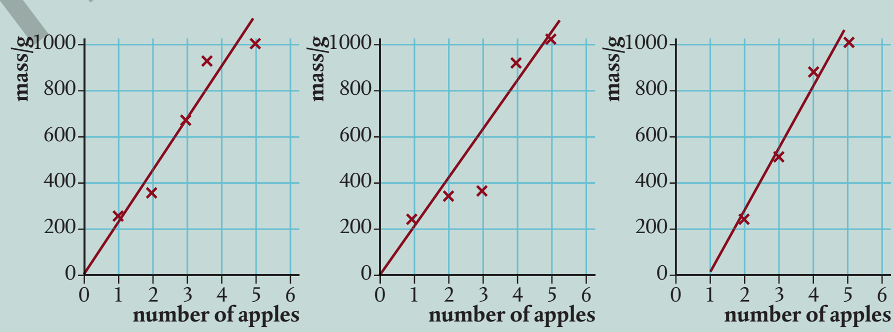
It is much easier to plot data from an experiment without processing it but this will often lead to curves that are very difficult to draw conclusions from. Linear relationships are much easier to interpret so are worth the time spent processing the data.

The equation of this line is $y = mx + c$, where m is the gradient and c the intercept on the y-axis. The equation in this case is therefore $y = 100x + 500$.

Finding the equation that relates two quantities can be useful for interpolation and extrapolation. Both techniques involve inserting a value for one of the quantities into the equation to find the corresponding value of the other. Interpolation can be performed with good confidence as it is done within the range of collected data. Extrapolation is more risky as the values are beyond the range of collected data; you are making a prediction.

Exercise

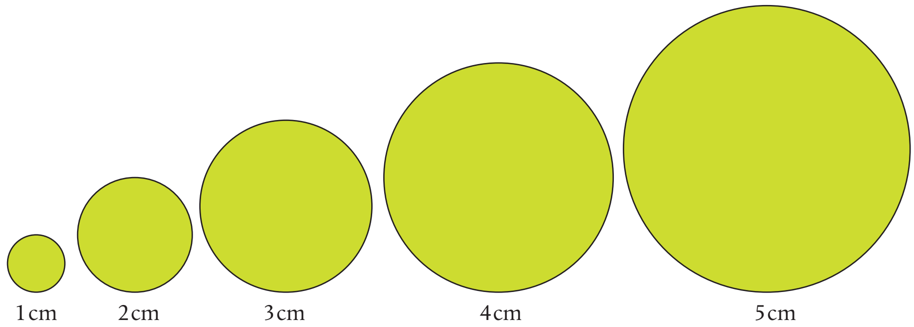
Q10. The data displayed in the graphs below all show examples of correlation. What other conclusions can you make?



Non-linear relationships

Let us now consider the relationship between radius and the area of circles of paper as shown in Figure 13.

Figure 13 Five circles of green paper. XXX



The results are recorded in Table 7.

If we now graph the area vs the radius, we get the graph shown in Figure 14.

Radius/m	Area/m^2
1	3.14
2	12.57
3	28.27
4	50.27
5	78.54

Table 7

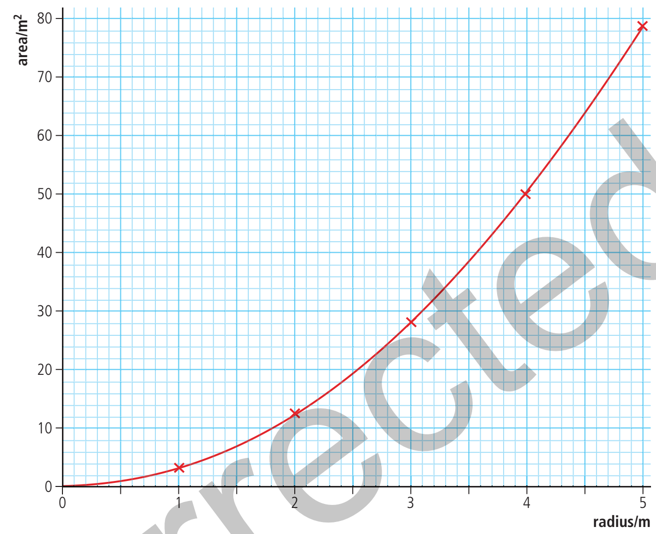


Figure 14 Graph of area of green circles vs radius.

This is not a straight line so we cannot deduce that area is linearly related to radius. However, you may know that the area of a circle is given by $A = \pi r^2$, which would mean that A is proportional to r^2. To test this, we can calculate r^2 and plot a graph of area vs r^2. The calculations are shown in Table 8.

Radius/m	r^2/m^2	Area/m^2
1	1	3.14
2	4	12.57
3	9	28.27
4	16	50.27
5	25	78.54

Table 8

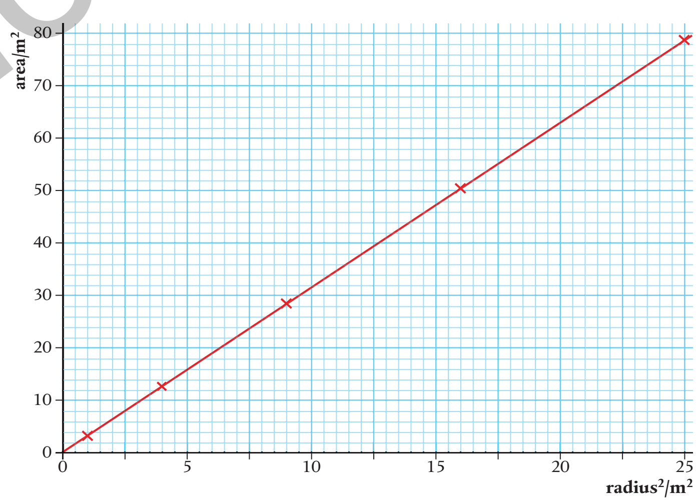


Figure 15 Graph of area of green circles vs radius^2.

This time, the graph is linear, confirming that the area is indeed proportional to the radius². The gradient of the line is 3.1, which is π to two significant figures. So the equation of the line is $A = \pi r^2$ as expected.

Using logs

Logs can be useful in your practical work. In the previous exercise, we knew that $A = \pi r^2$, but if we had not known this, we could have found the relationship by plotting a log graph. Let us pretend that we did not know the relationship between A and r, only that they were related. So it could be $A = kr^2$ or $A = kr^3$ or even $A = k\sqrt{r}$.

We can write all of these in the form: $A = kr^n$

Now if we take logs of both sides of this equation, we get: $\log A = \log kr^n = \log k + n\log r$

This is of the form $y = mx + c$, where log A is y and log r is x.

So if we plot log A vs log r, we should get a straight line with gradient n and intercept log k. This is all quite easy to do using a spreadsheet, resulting in Table 9 and the graph in Figure 16.

Radius/m	Area/m²	log (A/m²)	log (r/m)
1	3.14	0.4969	0.0000
2	12.57	1.0993	0.3010
3	28.27	1.4513	0.4771
4	50.27	1.7013	0.6021
5	78.54	1.8951	0.6990

Table 9

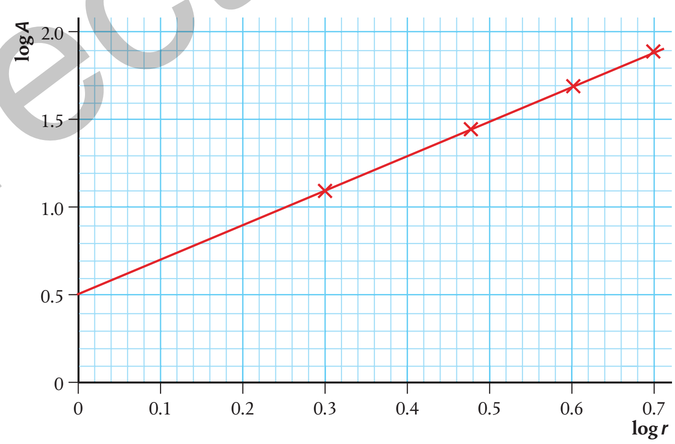


Figure 16 log A vs log r for the green paper discs.

This has gradient = 2 and intercept = 0.5, so if we compare it to the equation of the line:

$$\log A = \log k + n\log r$$

we can deduce that: $n = 2$ and $\log k = 0.5$

The inverse of log k is 10^k so $k = 10^{0.5} = 3.16$, which is quite close to π.

Substituting into our original equation $A = kr^n$, we get $A = \pi r^2$.

A	B
1.1	0.524
3.6	0.949
4.2	1.025
5.6	1.183
7.8	1.396
8.6	1.466
9.2	1.517
10.7	1.636

Table 10

Exercise

Q11. Use a log–log graph to find the relationship between A and B in Table 10.

Relationship between the diameter of a modeling clay ball and its mass

So far, we have only measured the diameter and mass of one ball of modeling clay. If we want to know the relationship between the diameter and mass, we should measure many balls of different sizes. This is limited by the amount of modeling clay we have, but should be from the smallest ball we can reasonably measure up to the biggest ball we can make.

Table 11

Mass/g ± 0.1 g	Diameter/cm ± 0.002 cm			
	1	2	3	4
1.4	1.296	1.430	1.370	1.280
2.0	1.570	1.590	1.480	1.550
5.6	2.100	2.130	2.168	2.148
9.4	2.560	2.572	2.520	2.610
12.5	2.690	2.840	2.824	2.720
15.7	3.030	2.980	3.080	2.890
19.1	3.250	3.230	3.190	3.204
21.5	3.490	3.432	3.372	3.360
24.8	3.550	3.560	3.540	3.520

In Table 11, the uncertainty in diameter d is given as 0.002 cm. This is the uncertainty in the vernier caliper: the actual uncertainty in diameter is *more* than this as is revealed by the spread of data which you can see in the first row, which ranges from 1.280 to 1.430, a difference of 0.150 cm. Because there are only four different measurements, we can use the approximate method using $\Delta d = \frac{(d_{max} - d_{min})}{2}$. This gives an uncertainty in the first measurement of ±0.08 cm. Table 12 includes the uncertainties and the mean.

Table 12

Mass/g ± 0.1 g	Diameter/cm ± 0.002 cm					
	1	2	3	4	d_{mean}/cm	Uncertainty Δd/cm
1.4	1.296	1.430	1.370	1.280	1.34	0.08
2.0	1.570	1.590	1.480	1.550	1.55	0.06
5.6	2.100	2.130	2.168	2.148	2.14	0.03
9.4	2.560	2.572	2.520	2.610	2.57	0.04
12.5	2.690	2.840	2.824	2.720	2.77	0.08
15.7	3.030	2.980	3.080	2.890	3.00	0.10
19.1	3.250	3.230	3.190	3.204	3.22	0.03
21.5	3.490	3.432	3.372	3.360	3.41	0.07
24.8	3.550	3.560	3.540	3.520	3.54	0.02

Now, to reveal the relationship between the mass m and diameter d, we can draw a graph of m vs d as shown in Figure 17. However, since the values of m and d have uncertainties, we do not plot them as points but as lines. The length of the lines equals the uncertainty in the measurement. These are called **uncertainty bars**.

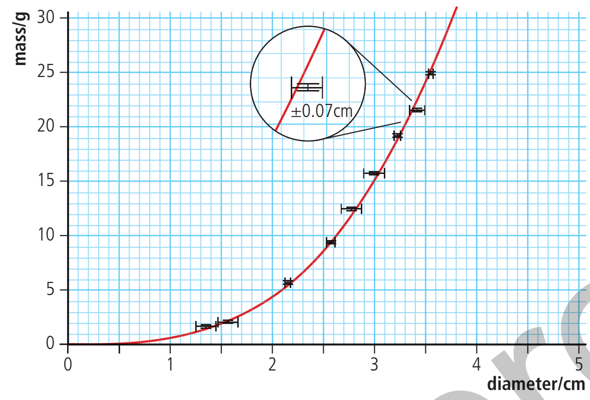


Figure 17 Graph of mass of modeling clay ball vs diameter with error bars.

A worksheet with full details of how to carry out this experiment is available on this page of your eBook.

This curve is the *best fit* for the data collected.

The curve is quite a nice fit but very difficult to analyze. It would be more convenient if we could manipulate the data to get a straight line. This is called **linearizing**. To do this, we must try to deduce the relationship using physical theory and then test the relationship by drawing a graph. In this case, we know that density, $\rho = \frac{\text{mass}}{\text{volume}}$ and the volume of a sphere = $\frac{4\pi r^3}{3}$, where r = radius.

So: $$\rho = \frac{3m}{4\pi r^3}$$

Rearranging this equation gives: $$r^3 = \frac{3m}{4\pi\rho}$$

But: $$r = \frac{d}{2} \text{ so } \frac{d^3}{8} = \frac{3m}{4\pi\rho}$$

$$d^3 = \frac{6m}{\pi\rho}$$

Since $\frac{6}{\pi\rho}$ is a constant, this means that d^3 is proportional to m. So, a graph of d^3 vs m should be a straight line with gradient = $\frac{6}{\pi\rho}$. To plot this graph, we need to find d^3 and its uncertainty. The uncertainty can be found by calculating the difference between the maximum and minimum values of d^3 and dividing by 2: $\frac{(d_{max}{}^3 - d_{min}{}^3)}{2}$. This has been done in Table 13.

Table 13

Mass/g ± 0.1 g	Diameter/cm ± 0.002 cm						
	1	2	3	4	d_{mean}/cm	d^3_{mean}/cm³	$d^3_{unc.}$/cm³
1.4	1.296	1.430	1.370	1.280	1.34	2.4	0.4
2.0	1.570	1.590	1.480	1.550	1.55	3.7	0.4
5.6	2.100	2.130	2.168	2.148	2.14	9.8	0.5
9.4	2.560	2.572	2.520	2.610	2.57	17	1
12.5	2.690	2.840	2.824	2.720	2.77	21	2
15.7	3.030	2.980	3.080	2.890	3.00	27	3
19.1	3.250	3.230	3.190	3.204	3.22	33	1
21.5	3.490	3.432	3.372	3.360	3.41	40	2
24.8	3.550	3.560	3.540	3.520	3.54	44	1

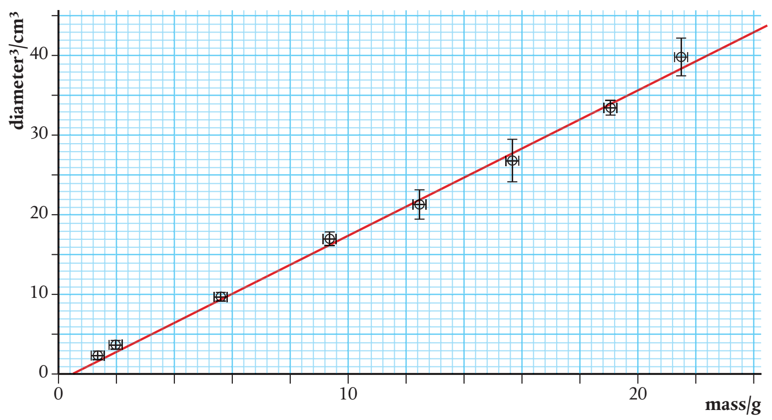


Figure 18 Graph of diameter^3 of a modeling clay ball vs mass.

The best fit of these points is now a straight line. Over time, you will learn how to judge whether data is best represented by a linear or non-linear fit, perhaps based on the theory behind an experiment or the positions of the points.

Looking at the line in Figure 18, we can see that due to random errors in the data, the points are not exactly on the line but close enough. What we expect to see is the line touching all of the error bars, which is the case here. The error bars should reflect the random scatter of data. In this case, they are slightly bigger, which is probably due to the approximate way that they have been calculated. Notice how the points furthest from the line have the biggest error bars.

According to the formula, d^3 should be directly proportional to m; the line should therefore pass through the origin. Here we can see that the y-intercept is $-0.3\,\text{cm}^3$, which is quite close to the origin and is probably just due to the random errors in d. If the intercept had been more significant, then it might have been due to a **systematic error** in mass. For example, if the balance had not been zeroed properly and instead of displaying zero with no mass on the pan, it read 0.5 g, then each mass measurement would be 0.5 g too big. The resulting graph would be as in Figure 19.

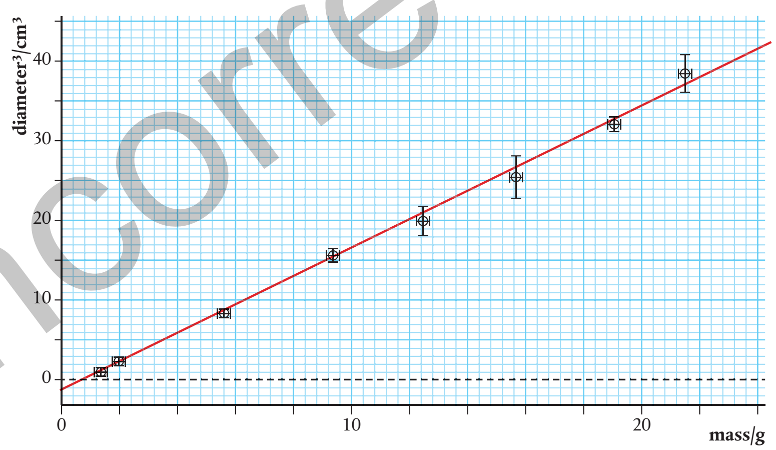


Figure 19 Graph of diameter^3 of a modeling clay ball vs mass with a systematic error.

A systematic error in the diameter would not be so easy to see. Since diameter is cubed, adding a constant value to each diameter would cause the line to become curved.

Outliers

Sometimes a mistake is made in one of the measurements. This is quite difficult to spot in a table but will often lead to an outlier on a graph. For example, one of the measurements in Table 14 is incorrect.

Table 14

Mass/g ± 0.1 g	Diameter/cm ± 0.002 cm			
	1	2	3	4
1.4	1.296	1.430	1.370	1.280
2.0	1.570	1.590	1.480	1.550
5.6	2.100	2.130	2.148	3.148
9.4	2.560	2.572	2.520	2.610
12.5	2.690	2.840	2.824	2.720
15.7	3.030	2.980	3.080	2.890
19.1	3.250	3.230	3.190	3.204
21.5	3.490	3.432	3.372	3.360
24.8	3.550	3.560	3.540	3.520

This is revealed in the graph in Figure 20.

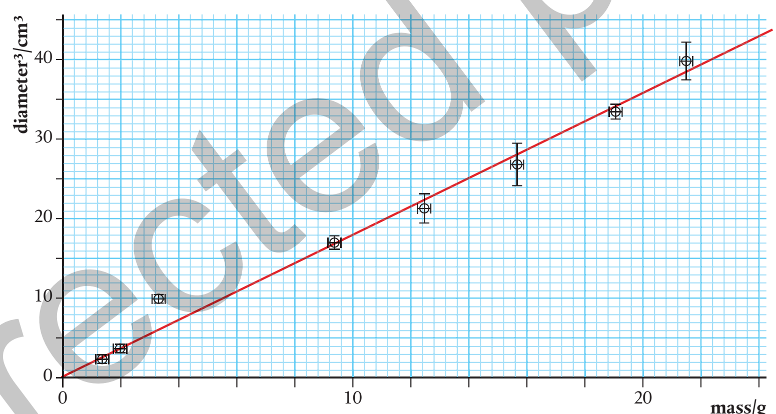


Figure 20 Graph of diameter³ of a modeling clay ball vs mass with outlier.

If asked for a sketch graph, you should consider what shape it will have and where it will cross the axes. Scales are not required.

When you find an outlier, you need to do some detective work to try to find out why the point is not closer to the line. Taking a close look at the raw data sometimes reveals that one of the measurements was incorrect. This can then be removed and the line plotted again. However, you cannot simply leave out the point because it does not fit. A sudden decrease in the level of ozone over the Antarctic was originally left out of the data since it was an outlier. Later investigation of this 'outlier' led to a significant discovery.

Uncertainty in the gradient

The general equation for a straight-line graph passing through the origin is $y = mx$. In this case, the equation of the line is $d^3 = \frac{6m}{\pi\rho}$, where d^3 is y and m is x and the gradient is $\frac{6}{\pi\rho}$. You can see that the unit of the gradient is cm³/g. This is consistent with it representing $\frac{6}{\pi\rho}$.

From the graph, we see that gradient $= 1.797\,\text{cm}^3\,\text{g}^{-1} = \frac{6}{\pi\rho}$ so $\rho = \frac{6}{1.797\pi}$

$\frac{6}{1.797\pi} = 1.063\,\text{g}\,\text{cm}^{-3}$ but what is the uncertainty in this value?

There are several ways to estimate the uncertainty in a gradient. One of them is to draw the steepest and least steep lines through the error bars as shown in Figure 21.

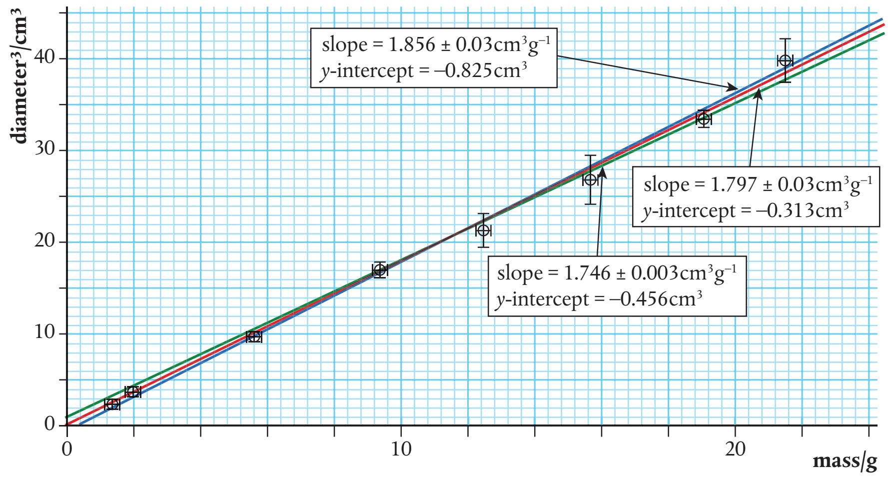


Figure 21 Graph of diameter³ of a modeling clay ball vs mass showing steepest and least steep lines.

This gives a maximum gradient = 1.856 $cm^3\,g^{-1}$ and minimum gradient = 1.746 $cm^3\,g^{-1}$.

So: uncertainty in the gradient = $\frac{(1.856 - 1.746)}{2} = 0.06\,cm^3\,g^{-1}$

Note that the program used to draw the graph (LoggerPro®) gives an uncertainty in the gradient of ±0.03 $cm^3\,g^{-1}$. This is a more correct value but the steepest and least steep lines method is accepted in IB assessments.

The steepest and least steep gradients give maximum and minimum values for the density of:

$$\rho_{max} = \frac{6}{1.746\pi} = 1.094\,g\,cm^{-3}$$

$$\rho_{min} = \frac{6}{1.856\pi} = 1.029\,g\,cm^{-3}$$

So the uncertainty is: $\frac{(1.094 - 1.029)}{2} = 0.03\,g\,cm^{-3}$

The density can now be written as: 1.06 ± 0.03 $g\,cm^{-3}$

Fractional uncertainties

So far, we have dealt with uncertainty as $\pm\Delta x$. This is called the **absolute uncertainty** in the value. Uncertainties can also be expressed as fractions. This has some advantages when processing data.

In the previous example, we measured the diameter of modeling clay balls then cubed this value in order to linearize the data. To make the sums simpler, let us consider a slightly bigger ball with a diameter of 10 ± 1 cm.

So the measured value d = 10 cm and the absolute uncertainty Δd = 1 cm.

The fractional uncertainty = $\frac{\Delta d}{d} = \frac{1}{10} = 0.1$ (or, expressed as a percentage, 10%).

During the processing of the data, we found $d^3 = 1000\,cm^3$.

The uncertainty in this value is not the same as in d. To find the uncertainty in d^3, we need to know the biggest and smallest possible values of d^3. These we can calculate by adding and subtracting the absolute uncertainty:

$$\text{maximum } d^3 = (10 + 1)^3 = 1331\,cm^3$$

$$\text{minimum } d^3 = (10 - 1)^3 = 729\,cm^3$$

If the y-intercept was of more importance, then constructing steepest and least steep lines would also allow maximum and minimum intercept values to be read off.

A value obtained from an experiment can be compared with a 'known' value by seeing if the known value lies within the uncertainty range.
Additionally, you could use percentage difference. Find the difference between the experimental and known values and then divide this difference by the known value.

So the range of values is: $(1331 - 729) = 602\,\text{cm}^3$

The uncertainty is therefore $\pm 301\,\text{cm}^3$, which rounded down to one significant figure gives $\pm 300\,\text{cm}^3$.

This is not the same as $(\Delta d)^3$, which would be $1\,\text{cm}^3$.

The fractional uncertainty in $d^3 = \frac{300}{1000} = 0.3$. This is the same as 3 × the fractional uncertainty in d. This leads to an alternative way of finding uncertainties when raising data to the power 3.

If $\frac{\Delta x}{x}$ is the fractional uncertainty in x, then the fractional uncertainty in $x^3 = \frac{3\Delta x}{x}$.

More generally, if $\frac{\Delta x}{x}$ is the fractional uncertainty in x, then the fractional uncertainty in $x^n = \frac{n\Delta x}{x}$.
So if you square a value, the fractional uncertainty is 2 × bigger.

Another way of writing this would be that, if $\frac{\Delta x}{x}$ is the fractional uncertainty in x, then the fractional uncertainty in $x^2 = \frac{\Delta x}{x} + \frac{\Delta x}{x}$. This can be extended to any multiplication.

So if $\frac{\Delta x}{x}$ is the fractional uncertainty in x and $\frac{\Delta y}{y}$ is the fractional uncertainty in y, then the fractional uncertainty in $xy = \frac{\Delta x}{x} + \frac{\Delta y}{y}$.

It seems strange but, when dividing, the fractional uncertainties also add. So if $\frac{\Delta x}{x}$ is the fractional uncertainty in x and $\frac{\Delta y}{y}$ is the fractional uncertainty in y, then the fractional uncertainty in $\frac{x}{y} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$.

If you divide a quantity by a constant with no uncertainty, then the fractional uncertainty remains the same.

This is all summarized in the Data Booklet as:

If $y = \frac{ab}{c}$ then $\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c}$

And if $y = a^n$ then $\frac{\Delta y}{y} = n\frac{\Delta a}{a}$

Challenge yourself

1. When a solid ball rolls down a slope of height h, its speed at bottom v is given by the equation:

   $$v = \sqrt{\frac{10}{7}gh}$$

   where g is the acceleration due to gravity.

   In an experiment to determine g, the following results were achieved:

   Distance between two markers at the bottom of the slope $d = 5.0 \pm 0.2\,\text{cm}$
   Time taken to travel between markers $t = 0.06 \pm 0.01\,\text{s}$
   Height of slope $h = 6.0 \pm 0.2\,\text{cm}$.

   Given that the speed $v = \frac{d}{t}$, find a value for g and its uncertainty. How might you reduce this uncertainty?

Example

If the length of the side of a cube is quoted as 5.00 ± 0.01 m, what are its volume and the uncertainty in the volume?

fractional uncertainty in length = $\frac{0.01}{5}$ = 0.002

$$\text{volume} = 5.00^3 = 125\ \text{m}^3$$

When a quantity is cubed, its fractional uncertainty is 3 × bigger so the fractional uncertainty in volume = 0.002 × 3 = 0.006.

The absolute uncertainty is therefore 0.006 × 125 = 0.75 (approximately 1) so the volume is 125 ±1 m^3.

Exercise

Q12. The length of the sides of a cube and its mass are quoted as:
length = 0.050 ± 0.001 m
mass = 1.132 ± 0.002 kg

Calculate the density of the material and its uncertainty.

Q13. The distance around a running track is 400 ± 1 m. If a person runs around the track four times, calculate the distance traveled and its uncertainty.

Q14. The time for 10 swings of a pendulum is 11.2 ± 0.1 s. Calculate the time for one swing of the pendulum and its uncertainty.

Nature of Science

We have seen how we can use numbers to represent physical quantities. By representing those quantities by letters, we can derive mathematical equations to define relationships between them, then use graphs to verify those relationships. Some quantities cannot be represented by a number alone so a whole new area of mathematics needs to be developed to enable us to derive mathematical models relating them.

Vector and scalar quantities

So far we have dealt with six different quantities: length, time, mass, volume, density, displacement.

All of these quantities have a size, but displacement also has a direction. Quantities that have size and direction are **vectors** and those with only size are **scalars**. All quantities are either vectors or scalars. It will be apparent why it is important to make this distinction when we add displacements together.

Example

Consider two displacements one after another as shown in Figure 22.

Starting from A, walk 4 km west to B, then 5 km north to C.

Scalar
A quantity with magnitude only.
Vector
A quantity with magnitude and direction.

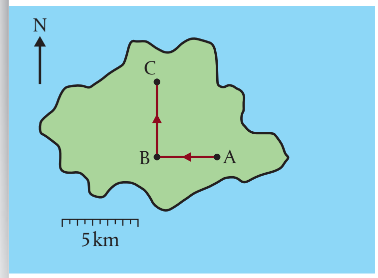


Figure 22 Displacements shown on a map.

The total displacement from the start is not 5 + 4 but can be found by drawing a line from A to C on a scale diagram.

We will find that there are many other vector quantities that can be added in the same way.

Addition of vectors

Vectors can be represented by drawing arrows. The *length* of the arrow is proportional to the magnitude of the quantity and the *direction* of the arrow is the direction of the quantity. The arrow commences at the point of application, the significance of which will become clearer in A.2.

To add vectors, the arrows are simply arranged so that the point of one touches the tail of the other. The resultant vector is found by drawing a line joining the free tail to the free point.

Example

Figure 22 is a map illustrating the different displacements. We can represent the displacements by the vectors in Figure 23.

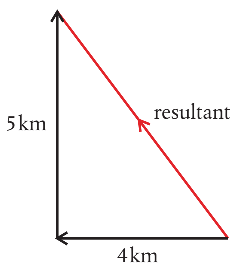


Figure 23 Vector addition.

Calculating the resultant:

If the two vectors are at right angles to each other, then the resultant will be the hypotenuse of a right-angled triangle. This means that we can use simple trigonometry to relate the different sides.

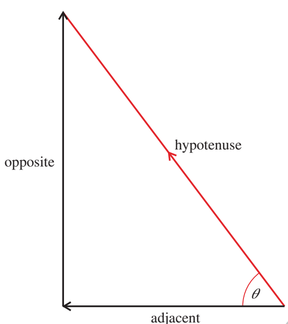


Figure 24 Triangle key terms.

Some simple trigonometry

You will find **cos**, **sin** and **tan** buttons on your calculator. These are used to calculate unknown sides of right-angled triangles.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \rightarrow \text{opposite} = \text{hypotenuse} \times \sin\theta$$

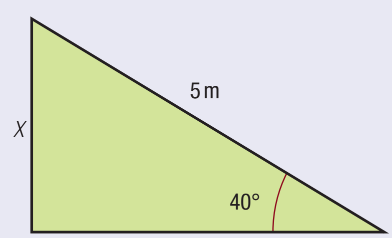
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \rightarrow \text{adjacent} = \text{hypotenuse} \times \cos\theta$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

To show that a quantity is a vector, we can write it in a special way. In textbooks, this is often in bold (**A**) but when you write, you can put an arrow on the top. In physics texts, the vector notation is often left out. This is because if we know that the symbol represents a displacement, then we know it is a vector and do not need the vector notation to remind us.

Worked example

Find the side *X* of the triangle.



Solution

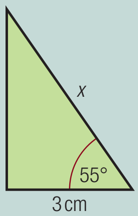
Side *X* is the opposite so: $X = 5 \times \sin 40°$

$\sin 40° = 0.6428$ so $X = 3.2$ m

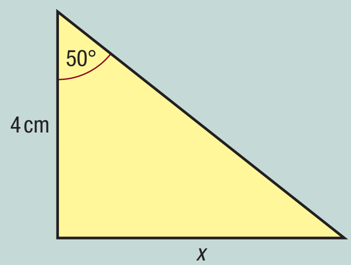
Exercise

Q15. Use your calculator to find x in each triangle.

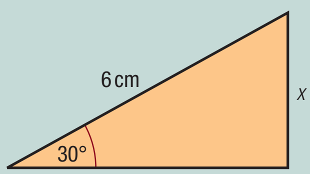
(a)



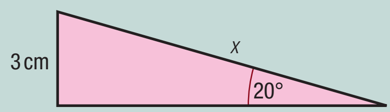
(b)



(c)



(d)



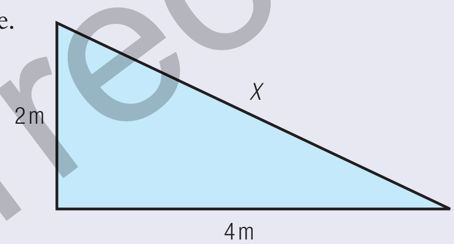
Pythagoras

The most useful mathematical relationship for finding the resultant of two perpendicular vectors is Pythagoras' theorem:

$$\text{hypotenuse}^2 = \text{adjacent}^2 + \text{opposite}^2$$

Worked example

Find the side X on the triangle.



Solution

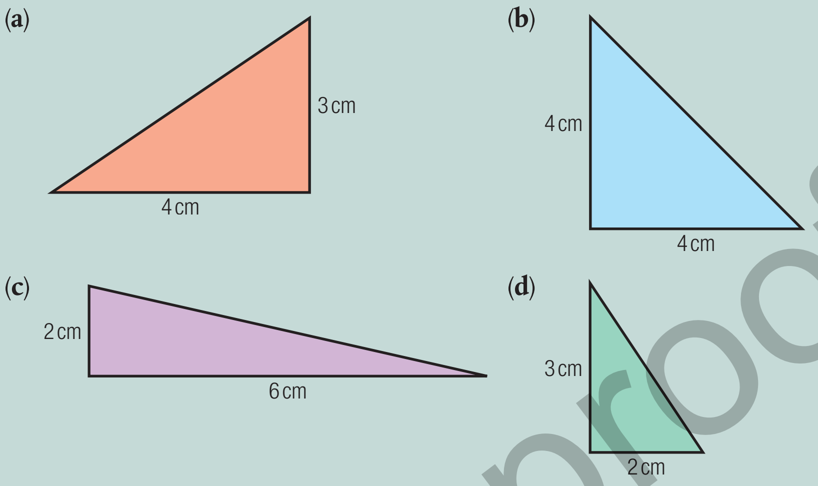
Applying Pythagoras:

$$X^2 = 2^2 + 4^2$$

So: $X = \sqrt{2^2 + 4^2} = \sqrt{20} = 4.5\text{m}$

Exercise

Q16. Use Pythagoras' theorem to find the hypotenuse in each triangle.



Using trigonometry to solve vector problems

Once the vectors have been arranged point to tail, it is a simple matter of applying the trigonometrical relationships to the triangles that you get.

Exercise

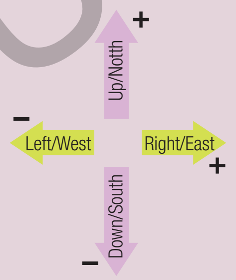
Draw the vectors and solve the following problems using Pythagoras' theorem.

Q17. A boat travels 4 km west followed by 8 km north. What is the resultant displacement?

Q18. A plane flies 100 km north then changes course to fly 50 km east. What is the resultant displacement?

Vectors in one dimension

You can decide for yourself which you want to be positive but generally we follow the convention below.



In this course, we will often consider the simplest examples where the motion is restricted to one dimension, for example, a train traveling along a straight track. In examples like this, there are only two possible directions – forward and backward. To distinguish between the two directions, we give them different signs (forward + and backward –). Adding vectors is now simply a matter of adding the magnitudes, with no need for complicated triangles.

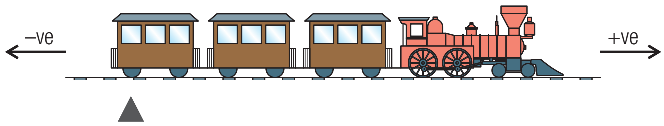


Figure 25 The train can only move forward or backward.

Worked example

If a train moves 100 m forward along a straight track then 50 m back, what is its final displacement?

Solution

The vector diagram is as follows.

The resultant is 50 m forward.

Subtracting vectors

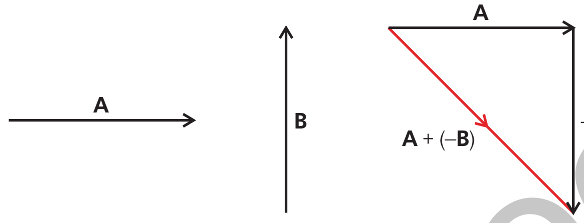


Figure 27 Subtracting vectors.

When a vector is multiplied by a scale factor, its alignment is unchanged. If the scale factor is negative, the vector is in the opposite direction. The magnitude is increased by the magnitude of the scale factor.

Now we know that a negative vector is simply the opposite direction to a positive vector, we can subtract vector **B** from vector **A** by changing the direction of vector **B** and adding it to **A**.

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Taking components of a vector

Consider someone walking up the hill in Figure 28. They walk 5 km up the slope but want to know how high they have climbed rather than how far they have walked. To calculate this, they can use trigonometry.

$$\text{height} = 5 \times \sin 30°$$

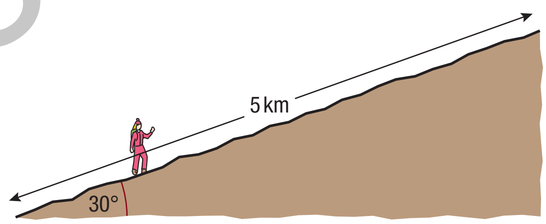


Figure 28 5 km up the hill but how high?

The height is called the vertical component of the displacement.

The horizontal displacement can also be calculated.

$$\text{horizontal displacement} = 5 \times \cos 30°$$

This process is called taking components of a vector and is often used in solving physics problems.

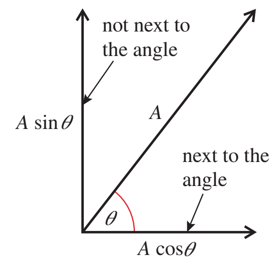


Figure 29 An easy way to remember which is cos is to say that 'it is becos it is next to the angle'.

Exercise

Q19. If a boat travels 10 km in a direction 30° to the east of north, how far north has it traveled?

Q20. On his way to the South Pole, Amundsen travelled 8 km in a direction that was 20° west of south. What was his displacement south?

Q21. A mountaineer climbs 500 m up a slope that is inclined at an angle of 60° to the horizontal. How high has he climbed?

SKILLS

You will find practice questions and solutions on this page of your eBook.

Summary – Tool 3: Mathematics

In terms of mathematics, you should now be aware of:

- scientific notation
- SI prefixes and units
- orders of magnitude
- area and volume
- fundamental units
- derived units in terms of SI units
- approximation and estimation
- dimensional analysis of units for checking expressions
- the significance of uncertainties in raw and processed data
- recording uncertainties in measurements as a range to appropriate precision
- expressing measurement and processed uncertainties to appropriate significant figures or precision
- expressing values to appropriate significant figures or decimal places
- mean and range
- extrapolate and interpolate graphs
- linear and non-linear graphs with appropriate scales and axes
- linearizing graphs
- drawing and interpreting uncertainty bars
- drawing lines or curves of best fit
- constructing maximum and minimum gradient lines by considering all uncertainty bars
- determining uncertainty in gradients and intercepts
- percentage change and percentage difference
- percentage error and percentage uncertainty
- propagation of uncertainties
- scalars and vectors
- scale diagrams
- drawing and labeling vectors
- vector addition and subtraction
- decimals, fractions, percentages, ratios, reciprocals, exponents and trigonometric ratios
- multiplication of vectors by a scalar
- resolving vectors.

There are some mathematical tools that have been introduced here and will be continued, including:

- arithmetic and algebra (see worked example calculations throughout)
- tables and graphs for raw and processed data (see also Sankey diagrams (A.3) and greenhouse gas spectra (B.2))
- direct and inverse proportionality, and positive and negative relationships or correlations (A.1, A.2, B.1, B.2, B.5, C.1, D.1, D.2, D.3)
- interpreting graph features (A.1)
- logarithmic graphs (E.5)
- sketch graphs (labeled but unscaled axes) to qualitatively describe trends (A.1).

The mathematical skills listed in the Guide that will be addressed in the textbook content more generally are:

- symbols from the Guide and Data Booklet (throughout)
- selection and manipulation of equations (throughout)
- effect of changes to variables on other variables (throughout)
- use of units (throughout)
- rates of change (A.1, A.3, HL D.4)
- neglecting effects and explaining why (A.1)
- free-body diagrams (A.2)
- derivations of equations (B.3, HL C.5, HL D.1)
- continuous and discrete variables (E.1)
- logarithmic and exponential functions (HL E.3).

THEME A Space, time and motion

A Space, time, and motion

Fireworks displays encapsulate a lot of physics, including thermal energy, light and sound waves, their behavior in the Earth's gravitational field and the effects generated by particular types of atom. As you will see, they also relate to space, time and motion.

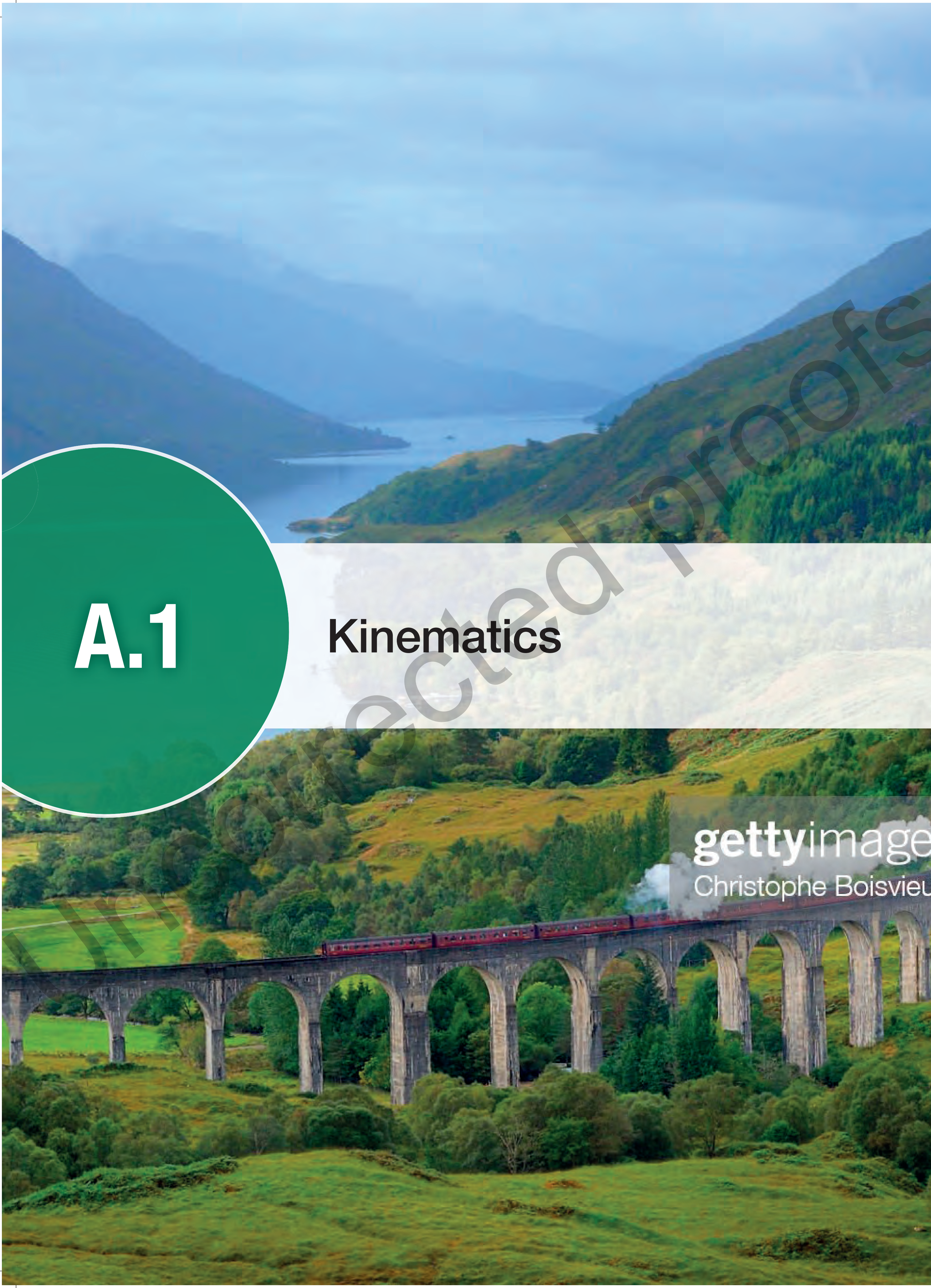
There are lots of words that can be used to describe something's motion: distance and displacement, speed and velocity, and acceleration. If you know everything about a body's motion at a specific position and time, then as an IB physicist you will be able to predict its state of motion at another position or time. This is kinematics. When simplified, the equations that govern motion horizontally and vertically can be treated separately. Fireworks are not so simple; they experience air resistance and continue to combust their fuel mid-flight.

The burning of fuels to generate changes in motion relates to forces and momentum. Isaac Newton articulated three laws that describe how a lack of resultant force means there is no change in velocity, a resultant force leads to a change in momentum and the force of one body 'A' on another 'B' means that the same type and size of force must be being exerted by 'B' on 'A' in precisely the opposite direction. There are types of force to contend with, and of course not all forces act in the same direction that the body is already moving in; circular motion results from perpendicular forces and velocities and has its own set of governing equations.

If kinematics is the study of the journey, energy is the study of the before and after. Energy, along with momentum, is a conserved quantity that can be changed only if work is done. Power is another term still; it is the rate at which energy is changed or work is done. Balancing a 100 g apple in your hand requires a force of about 1 N. Lifting it vertically to arm's reach requires you to provide about 1 J of work. Doing so repeatedly every second represents 1 W of power, irrespective of how long in total you do it for.

Some bodies rotate, and you will study how the angles and angular velocities of circular motion can be linked to the kinematics equations in the Rigid Body Mechanics chapter (A.4). Other bodies have velocities similar in magnitude to the speed of light, which leads to relativistic effects like time dilation and length contraction. But fear not. In the first case you will always be solving problems with real-world connections. In the latter, you will get to know about the experimental evidence for these effects as well as how to use the Lorentz transformation and space–time diagrams to your advantage.

A.1 Kinematics



◀ When can you think of a steam train as a particle?

Guiding Questions

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

How can the analysis of motion in one and two dimensions be used to solve real-life problems?

The photograph at the start of this chapter shows a train, but we will not be dealing with complicated systems like trains in their full complexity. In physics, we try to understand everything on the most basic level. Understanding a physical system means being able to predict its final conditions given its initial conditions. To do this for a train, we would have to calculate the position and motion of every part – and there are a lot of parts. In fact, if we considered all the particles that make up all the parts, then we would have a huge number of particles to deal with.

In this course, we will be dealing with one particle of matter at a time. This is because the ability to solve problems with one particle makes us able to solve problems with many particles. We may even pretend a train is one particle.

The initial conditions of a particle describe where it is and what it is doing. These can be defined by a set of numbers, which are the results of measurements. As time passes, some of these quantities might change. What physicists try to do is predict their values at any given time in the future. To do this, they use mathematical models.

Nature of Science

From the definitions of velocity and acceleration, we can use mathematics to derive a set of equations that predict the position and velocity of a particle at any given time. We can show by experiment that these equations give the correct result for some examples, then make the generalization that the equations apply in all cases.

Students should understand:

that the motion of bodies through space and time can be described and analyzed in terms of position, velocity, and acceleration
velocity is the rate of change of position, and acceleration is the rate of change of velocity
the change in position is the displacement
the difference between distance and displacement
the difference between instantaneous and average values of velocity, speed and acceleration, and how to determine them

the equations of motion for solving problems with uniformly accelerated motion as given by: $s = \frac{u+v}{2}t$ $v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$
motion with uniform and non-uniform acceleration
the behavior of projectiles in the absence of fluid resistance, and the application of the equations of motion resolved into vertical and horizontal components
the qualitative effect of fluid resistance on projectiles, including time of flight, trajectory, velocity, acceleration, range and terminal speed.

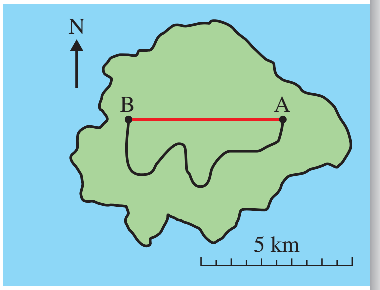
Further information about the fluid resistance force can be found in A.2.

Nature of Science

In the Tools chapter, we observed that things move and now we are going to mathematically model that movement. Before we do that, we must define some quantities.

Displacement and distance

It is important to understand the difference between distance traveled and displacement. To explain this, consider the route marked out on the map shown in Figure 1.



A.1 Figure 1

Displacement is the shortest path moved in a particular direction.

The unit of displacement is the meter (m). Displacement is a vector quantity.

On the map, the displacement is the length of the straight line from A to B, which is a distance of 5 km west.

Distance is how far you have traveled from A to B.

The unit of distance is also the meter (m). Distance is a scalar quantity.

In this example, the distance traveled is the length of the path taken, which is about 10 km.

Note: since displacement is a vector, you should always say what the direction is.

Sometimes, this difference leads to a surprising result. For example, if you run all the way round a running track, you will have traveled a distance of 400 m but your displacement will be 0 m.

In everyday life, it is often more important to know the distance traveled. For example, if you are going to travel from Paris to Lyon by road, you will want to know that the distance by road is 450 km, not that your final displacement will be 336 km SE. However, in physics, we break everything down into its simplest parts, so we start by considering motion in a straight line only. In this case, it is more useful to know the displacement, since that also has information about which direction you have traveled in.

Velocity and speed

Both speed and velocity are a measure of how fast a body is moving.

Velocity is defined as the rate of change of position. Since 'change of position' is displacement and 'rate of change' requires division by time taken:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

The unit of velocity is m s^{-1}.

Velocity is a vector quantity.

Speed is defined as the distance traveled per unit time:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The unit of speed is also m s^{-1}.

Speed is a scalar quantity.

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Exercise

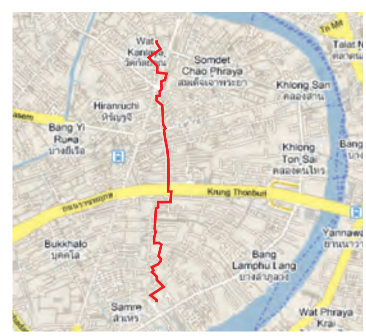
Q1. Convert the following speeds into m s^{-1}:

(**a**) a car traveling at $100\,\text{km h}^{-1}$

(**b**) a runner running at $20\,\text{km h}^{-1}$.

Average velocity and instantaneous velocity

Consider traveling by car from the north of Bangkok to the south – a distance of about 16 km. If the journey takes 4 hours, you can calculate your velocity to be $\frac{16}{4} = 4\,\text{km h}^{-1}$ in a southward direction. This does not tell you anything about the journey, just the difference between the beginning and the end (unless you managed to travel at a constant speed in a straight line). The value calculated is the **average velocity** and in this example it is quite useless. If we broke the trip down into lots of small pieces, each lasting only one second, then for each second the car could be considered to be traveling in a straight line at a constant speed. For these short stages, we could quote the car's **instantaneous velocity** – which is how fast it is going at that moment in time and in which direction.



A.1 Figure 2 It is not possible to take this route across Bangkok with a constant velocity.

The bus in the photo has a constant velocity for a very short time.

Exercise

Q2. A runner runs once around a circular track of length 400 m with a constant speed in 96 s. Calculate:

(**a**) the average speed of the runner

(**b**) the average velocity of the runner

(**c**) the instantaneous velocity of the runner after 48 s

(**d**) the displacement after 24 s.

Constant velocity

What does Newton's first law tell us about the forces on a body traveling at constant velocity? (A.2)

If the velocity is constant, then the instantaneous velocity is the same all the time so:

$$\text{instantaneous velocity} = \text{average velocity}$$

Since velocity is a vector, this also implies that the direction of motion is constant.

Measuring a constant velocity

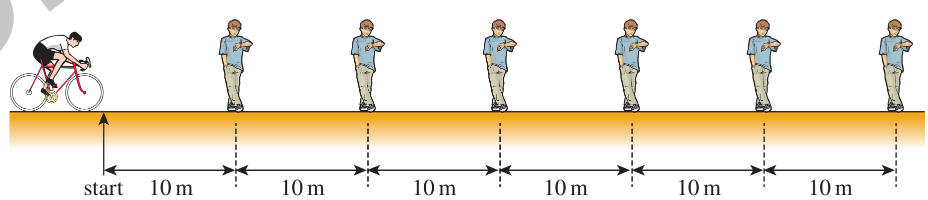
From the definition of velocity, we see that:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

Rearranging this gives:

$$\text{displacement} = \text{velocity} \times \text{time}$$

So, if velocity is constant, displacement is proportional to time. To test this relationship and find the velocity, we can measure the displacement of a body at different times. To do this, you either need a lot of clocks or a stop clock that records many times. This is called a **lap timer**. In this example, a bicycle was ridden at constant speed along a straight road past six students standing 10 m apart, each operating a stop clock as in Figure 3. The clocks were all started when the bike, already moving, passed the start marker and stopped as the bike passed each student.



A.1 Figure 3 Measuring the time for a bike to pass.

The results achieved are shown in Table 1.

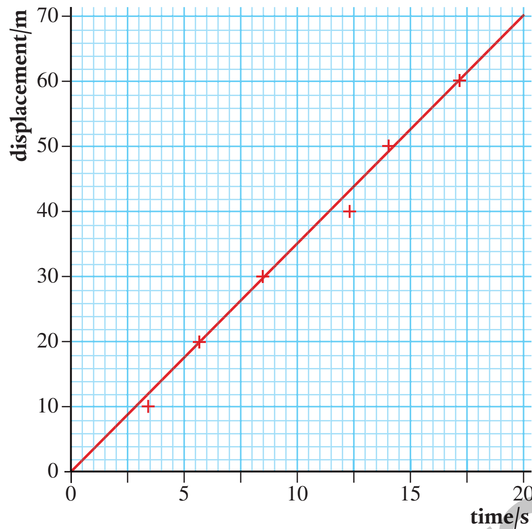
Displacement/m ±0.1 m	Time/s ±0.02 s
10.0	3.40
20.0	5.62
30.0	8.55
40.0	12.31
50.0	14.17
60.0	17.21

A.1 Table 1

The uncertainty in displacement is given as 0.1 m since it is difficult to decide exactly when the bike passed the marker.

The digital stop clock has a scale with 2 decimal places, so the uncertainty is 0.01 s. However, the uncertainty given is 0.02 s since the clocks all had to be started at the same time.

Since displacement (*s*) is proportional to time (*t*), then a graph of *s* vs *t* should give a straight line with gradient = velocity as shown in Figure 4.



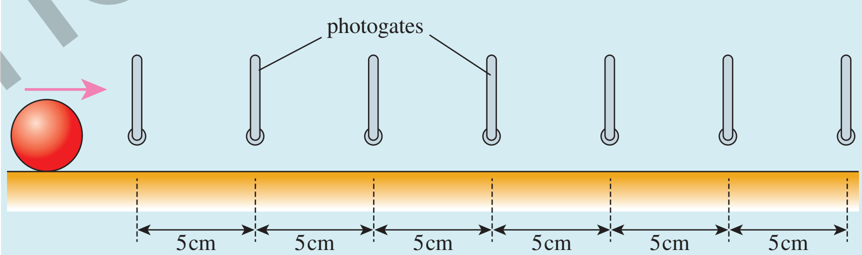
A.1 Figure 4 Graph of displacement vs time for a bike.

Notice that in this graph the line does not pass through all the points. This is because the uncertainty in the measurement in time is almost certainly bigger than the uncertainty in the clock (±0.02 s) due to the reaction time of the students stopping the clock. To get a better estimate of the uncertainty, we would need to have several students standing at each 10 m position. Repeating the experiment is not possible in this example since it is very difficult to ride at the same velocity several times.

The gradient indicates that: velocity = 3.5 m s^{-1}

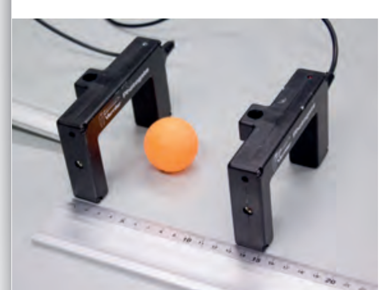
SKILLS

Most school laboratories are not large enough to ride bikes in so when working indoors, we need to use shorter distances. This means that the times are going to be shorter so hand-operated stop clocks will have too great a percentage uncertainty. One way of timing in the lab is by using photogates. These are connected to a computer via an interface and record the time when a body passes in or out of the gate. So, to replicate the bike experiment in the lab using a ball, we would need seven photogates as in Figure 5, with one extra gate to represent the start.



A.1 Figure 5 How to measure the time for a rolling ball if you have seven photogates.

This would be quite expensive so we compromise by using just two photogates and a motion that can be repeated. An example could be a ball moving along a horizontal section of track after it has rolled down an inclined plane. Provided the ball starts from the same point, it should have the same velocity. So, instead of using seven photogates, we can use two – one is at the start of the motion and the other is moved to different positions along the track as in Figure 6.



A.1 Figure 6 The ball interrupts the infrared light transmitted across each gate as it passes through them. The times of these interruptions are measured and recorded by a data logger.

A.1 Figure 7 Measuring the velocity of a ball with two photogates.

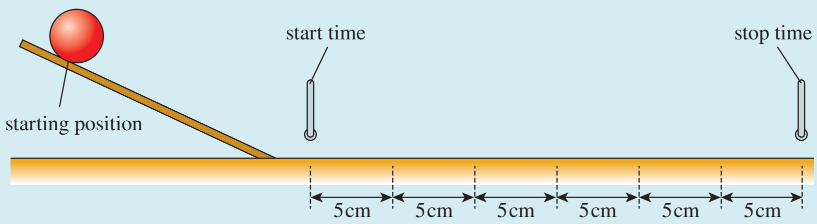
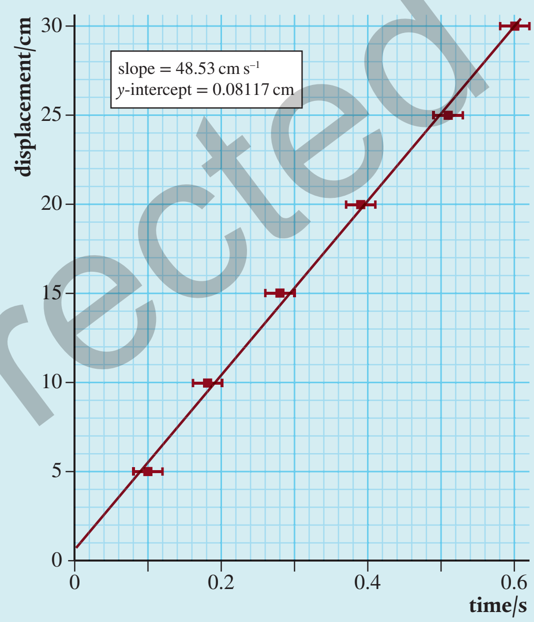


Table 2 shows the results obtained using this arrangement.

A.1 Table 2

Displacement/ cm ± 0.1 cm	Time(t)/s ± 0.0001 s					Mean t/s	Δt/s
5.0	0.0997	0.0983	0.0985	0.1035	0.1040	0.101	0.003
10.0	0.1829	0.1969	0.1770	0.1824	0.1825	0.18	0.01
15.0	0.2844	0.2800	0.2810	0.2714	0.2779	0.28	0.01
20.0	0.3681	0.3890	0.3933	0.3952	0.3854	0.39	0.01
25.0	0.4879	0.5108	0.5165	0.4994	0.5403	0.51	0.03
30.0	0.6117	0.6034	0.5978	0.6040	0.5932	0.60	0.01

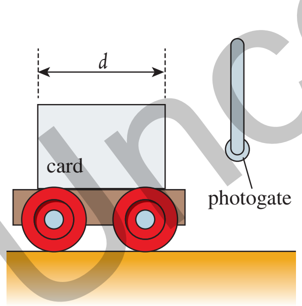
A.1 Figure 8 Graph of displacement vs time for a rolling ball.



Notice that the uncertainty calculated from $\frac{(\max - \min)}{2}$ is much more than the instrument uncertainty.

A graph of displacement vs time gives Figure 8.

From this graph, we can see that within the limits of the experiment's uncertainties the displacement could be proportional to time, so we can conclude that the velocity may have been constant. However, if we look closely at the data, we see that there seems to be a slight curve, indicating that perhaps the ball was slowing down. To verify this, we would have to collect more data.



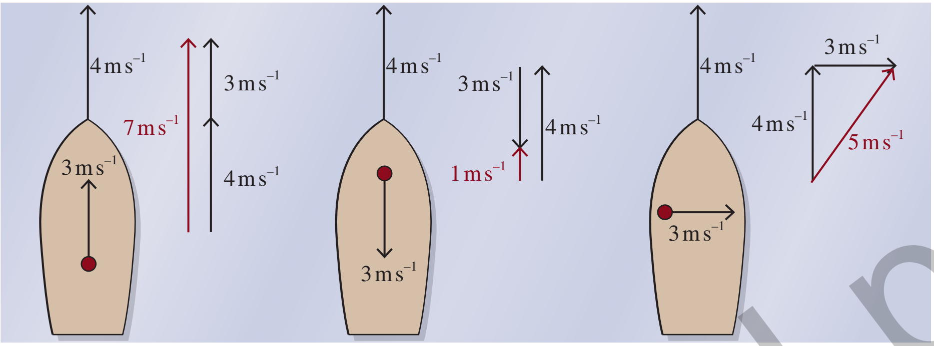
A.1 Figure 9 A card and photogate used to measure instantaneous velocity.

Measuring instantaneous velocity

To measure instantaneous velocity, a very small displacement must be used. This could be achieved by placing two photogates close together or attaching a piece of card to the moving body as shown in Figure 9. The time taken for the card to pass through the photogate is recorded and the instantaneous velocity calculated from: $\frac{\text{length of card}}{\text{time taken}} \left(\frac{d}{t}\right)$

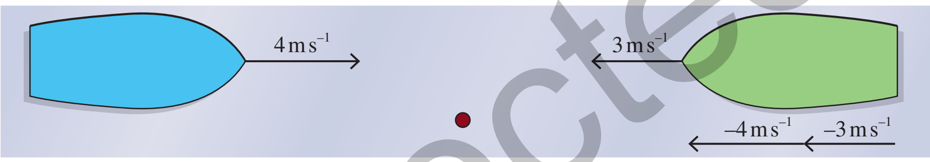
Relative velocity

Velocity is a vector so velocities must be added as vectors. Imagine you are running north at 3 m s^{-1} on a ship that is also traveling north at 4 m s^{-1} as shown in Figure 10. Your velocity relative to the ship is 3 m s^{-1} but your velocity relative to the water is 7 m s^{-1}. If you turn around and run due south, your velocity will still be 3 m s^{-1} relative to the ship but 1 m s^{-1} relative to the water. Finally, if you run toward the east, the vectors add at right angles to give a resultant velocity of magnitude 5 m s^{-1} relative to the water. You can see that the velocity vectors have been added.



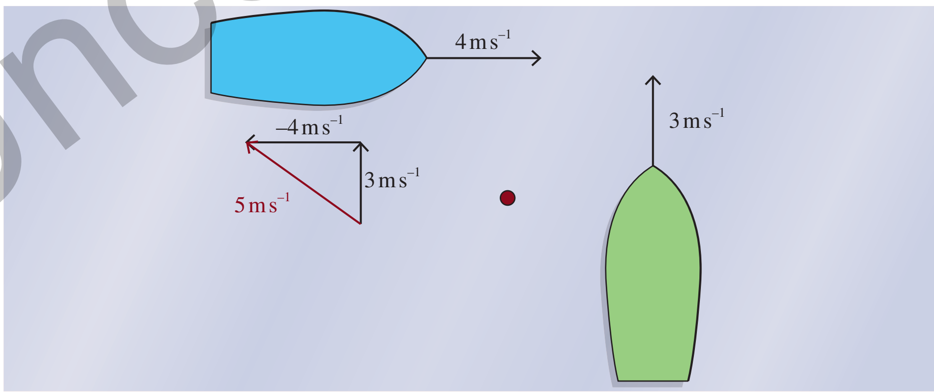
A.1 Figure 10 Running on board a ship.

Imagine that you are floating in the water watching two boats traveling toward each other as in Figure 11.



A.1 Figure 11 Two boats approach each other. The vector addition for the velocity of the green boat from the perspective of the blue boat is shown.

The blue boat is traveling east at 4 m s^{-1} and the green boat is traveling west at −3 m s^{-1}. Remember that the sign of a vector in one dimension gives the direction. So, if east is positive, then west is negative. If you were standing on the blue boat, you would see the water going past at −4 m s^{-1} so the green boat would approach with the velocity of the water plus its velocity in the water: $-4 + -3 = -7$ m s^{-1}. This can also be done in two dimensions as in Figure 12.



A.1 Figure 12 Two boats traveling perpendicular to each other. The vector addition for the velocity of the green boat from the perspective of the blue boat is shown.

According to the swimmer floating in the water, the green boat travels north and the blue boat travels east, but an observer on the blue boat will see the water traveling toward the west and the green boat traveling due north. Adding these two velocities gives a velocity of 5 m s^{-1} in an approximately northwest direction.

What is the relative speed of the light from a star measured by a rocket traveling at 0.5 times the speed of light toward the star? (A.5)

How effectively do the equations of motion model Newton's laws of dynamics? (A.2)

Exercise

Q3. An observer standing on a road watches a bird flying east at a velocity of $10\,m\,s^{-1}$. A second observer, driving a car along the road northward at $20\,m\,s^{-1}$, sees the bird. What is the velocity of the bird relative to the driver?

Q4. A boat travels along a river heading north with a velocity $4\,m\,s^{-1}$ as a woman walks across a bridge from east to west with velocity of $1\,m\,s^{-1}$. Calculate the velocity of the woman relative to the boat.

Acceleration

In everyday usage, the word **accelerate** means to go faster. However, in physics, acceleration is defined as the rate of change of velocity:

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time}}$$

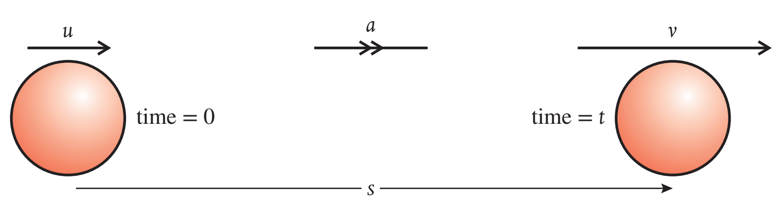
The unit of acceleration is $m\,s^{-2}$.

Acceleration is a vector quantity.

This means that whenever a body changes its velocity, it accelerates. This could be because it is getting faster, slower, or just changing direction. In the example of the journey across Bangkok, the car would have been slowing down, speeding up and going round corners almost the whole time so it would have had many different accelerations. However, this example is far too complicated for us to consider in this course (and probably any physics course). For most of this chapter, we will only consider the simplest example of accelerated motion, which is constant acceleration.

Constant acceleration in one dimension

In one-dimensional motion, acceleration, velocity and displacement are all in the same direction. This means they can be added without having to draw triangles. Figure 13 shows a body that is starting from an initial velocity u and accelerating at a constant rate a to velocity v in t seconds. The distance traveled in this time is s. Since the motion is in a straight line, this is also the displacement.



A.1 Figure 13 A red ball is accelerated at a constant rate.

Using the definitions already stated, we can write equations related to this example.

Average velocity

From the definition, average velocity $= \frac{\text{displacement}}{\text{time}}$

$$\text{average velocity} = \frac{s}{t} \qquad (1)$$

Since the velocity changes at a constant rate from the beginning to the end, we can also calculate the average velocity by adding the initial and final velocities and dividing by two:

$$\text{average velocity} = \frac{(u + v)}{2} \qquad (2)$$

Acceleration

Acceleration is defined as the rate of change of velocity:

$$a = \frac{(v - u)}{t} \qquad (3)$$

We can use these equations to solve any problem involving constant acceleration. However, to make problem solving easier, we can derive two more equations by substituting from one into the other.

Equating equations (1) and (2):

$$\frac{s}{t} = \frac{(u + v)}{2}$$

$$s = \frac{(u + v)\,t}{2} \qquad (4)$$

Rearranging (3) gives: $v = u + at$

If we substitute for v in equation (4), we get: $s = ut + \frac{1}{2}at^2$ (5)

Rearranging (3) again gives: $t = \frac{(v - u)}{a}$

If t is now substituted in equation (4), we get: $v^2 = u^2 + 2as$ (6)

These equations are sometimes known as the *suvat* equations. If you know any three of s, u, v, a, and t, you can find either of the other two in one step.

These equations are known as the *suvat* equations:

$a = \frac{(v - u)}{t}$

$s = \frac{(v + u)t}{2}$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

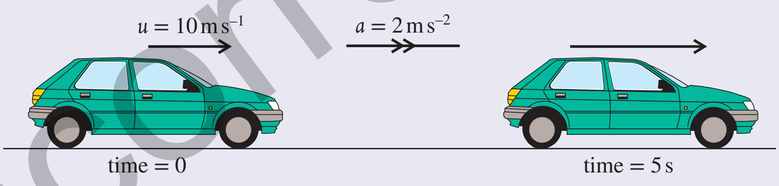
How are the equations for rotational motion related to those for linear motion? (A.4)

Worked example

A car traveling at $10\,m\,s^{-1}$ accelerates at $2\,m\,s^{-2}$ for 5 s. What is its displacement?

Solution

The first thing to do is draw a simple diagram:



This enables you to see what is happening at a glance rather than reading the text. The next stage is to make a list of *suvat*.

$s = ?$

$u = 10\,m\,s^{-1}$

$v = ?$

$a = 2\,m\,s^{-2}$

$t = 5\,s$

To find s, you need an equation that contains *suat*. The only equation with all four of these quantities is: $s = ut + \frac{1}{2}at^2$

Using this equation gives: $s = 10 \times 5 + \frac{1}{2} \times 2 \times 5^2$

$s = 75\,m$

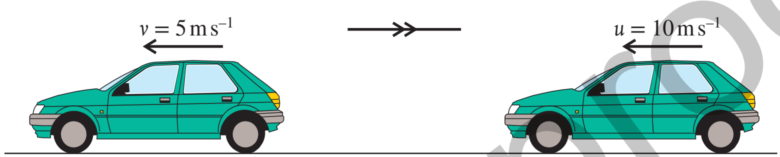
When the units are consistent, you do not need to include units in all stages of a calculation, just in the answer.

The signs of displacement, velocity, and acceleration

We must not forget that displacement, velocity and acceleration are vectors. This means that they have direction. However, since this is a one-dimensional example, there are only two possible directions, forward and backward. We know which direction the vector is in from its sign.

If we take right to be positive:

- A positive displacement means that the body has moved to the right.
- A positive velocity means the body is moving to the right.
- A positive acceleration means that the body is either moving to the right and getting faster or moving to the left and getting slower. This can be confusing so consider the following example.



A.1 Figure 14 A car moves to the left with decreasing speed

The car is traveling in a negative direction so the velocities are negative.

$$u = -10\,\text{m s}^{-1}$$

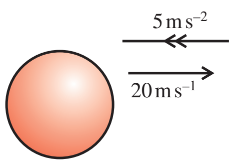
$$v = -5\,\text{m s}^{-1}$$

$$t = 5\,\text{s}$$

The acceleration is therefore given by:

$$a = \frac{(v-u)}{t} = \frac{-5-(-10)}{5} = 1\,\text{m s}^{-2}$$

The positive sign tells us that the acceleration is in a positive direction (right) even though the car is traveling in a negative direction (left).



A.1 Figure 15 The acceleration is negative so points to the left.

The acceleration due to gravity is not constant all over the Earth. 9.81 m s^{-2} is the average value. The acceleration also gets smaller the higher you go. However, we ignore this change when conducting experiments in the lab since labs are not that high.

To make the examples easier to follow, $g = 10\,\text{m s}^{-2}$ is used throughout. However, you should only use this approximate value in exam questions if told to do so.

Worked example

A body with a constant acceleration of $-5\,\text{m s}^{-2}$ is traveling to the right with a velocity of $20\,\text{m s}^{-1}$. What will its displacement be after 20 s?

Solution

$s = ?$

$u = 20\,\text{m s}^{-1}$

$v = ?$

$a = -5\,\text{m s}^{-2}$

$t = 20\,\text{s}$

To calculate s, we can use the equation: $s = ut + \frac{1}{2}at^2$

$$s = 20 \times 20 + \tfrac{1}{2}(-5) \times 20^2 = 400 - 1000 = -600\,\text{m}$$

This means that the final displacement of the body is to the left of the starting point. It has gone forward, stopped, and then gone backward.

Exercise

Q5. Calculate the final velocity of a body that starts from rest and accelerates at $5\,\text{m}\,\text{s}^{-2}$ for a distance of 100 m.

Q6. A body starts with a velocity of $20\,\text{m}\,\text{s}^{-1}$ and accelerates for 200 m with an acceleration of $5\,\text{m}\,\text{s}^{-2}$. What is the final velocity of the body?

Q7. A body accelerates at $10\,\text{m}\,\text{s}^{-2}$ and reaches a final velocity of $20\,\text{m}\,\text{s}^{-1}$ in 5 s. What is the initial velocity of the body?

Free fall motion

Although a car has been used in the previous examples, the acceleration of a car is not usually constant so we should not use the *suvat* equations. The only example of constant acceleration that we see in everyday life is when a body is dropped. Even then, the acceleration is only constant for a short distance.

How does the motion of an object change within a gravitational field? (D.1)

Acceleration of free fall

When a body is allowed to fall freely, we say it is in free fall. Bodies falling freely on the Earth fall with an acceleration of about $9.81\,\text{m}\,\text{s}^{-2}$ (depending where you are). The body falls because of gravity. For that reason, we use the letter g to denote this acceleration. Since the acceleration is constant, we can use the *suvat* equations to solve problems.

If you jump out of a plane (with a parachute on), you will feel the push of the air as it rushes past you. As you fall faster and faster, the air will push upward more and more until you cannot go any faster. At this point, you have reached terminal velocity. We will come back to this after introducing forces.

Exercise

In these calculations, use $g = 10\,\text{m}\,\text{s}^{-2}$.

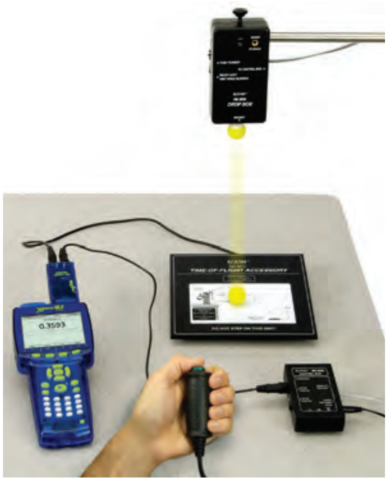
Q8. A ball is thrown upward with a velocity of $30\,\text{m}\,\text{s}^{-1}$. What is the displacement of the ball after 2 s?

Q9. A ball is dropped. What will its velocity be after falling 65 cm?

Q10. A ball is thrown upward with a velocity of $20\,\text{m}\,\text{s}^{-1}$. After how many seconds will the ball return to its starting point?

Measuring the acceleration due to gravity

When a body falls freely under the influence of gravity, it accelerates at a constant rate. This means that time to fall t and distance s are related by the equation: $s = ut + \frac{1}{2}at^2$. If the body starts from rest, then $u = 0$ so the equation becomes: $s = \frac{1}{2}at^2$. Since s is directly proportional to t^2, a graph of s vs t^2 would therefore be a straight line with gradient $\frac{1}{2}g$. It is difficult to measure the time for a ball to pass different markers, but if we assume the ball falls with the same acceleration when repeatedly dropped, we can measure the time taken for the ball to fall from different heights. There are many ways of doing this. All involve some way of starting a clock when the ball is released and stopping it when it hits the ground. Table 3 shows a set of results from a 'ball drop' experiment.



Apparatus for measuring g.

A.1 Table 3

Height(*h*)/m ± 0.001 m	Time(*t*)/s ± 0.001 s					Mean *t*/s	t^2/s²	$\Delta(t^2)$/s²
0.118	0.155	0.153	0.156	0.156	0.152	0.154	0.024	0.001
0.168	0.183	0.182	0.183	0.182	0.184	0.183	0.0334	0.0004
0.218	0.208	0.205	0.210	0.211	0.210	0.209	0.044	0.001
0.268	0.236	0.235	0.237	0.239	0.231	0.236	0.056	0.002
0.318	0.250	0.254	0.255	0.250	0.256	0.253	0.064	0.002
0.368	0.276	0.277	0.276	0.278	0.276	0.277	0.077	0.001
0.418	0.292	0.293	0.294	0.291	0.292	0.292	0.085	0.001
0.468	0.310	0.310	0.303	0.300	0.311	0.307	0.094	0.003
0.518	0.322	0.328	0.330	0.328	0.324	0.326	0.107	0.003
0.568	0.342	0.341	0.343	0.343	0.352	0.344	0.118	0.004

If a parachutist kept accelerating at a constant rate, they would break the sound barrier after about 30 s of flight. By understanding the forces involved, scientists have been able to design wing suits so that base jumpers can achieve forward velocities greater than their rate of falling.

SKILLS

A worksheet with full details of how to carry out this experiment is available in your eBook.

Notice that the uncertainty in t^2 is calculated from: $\dfrac{(t_{max}^2 - t_{min}^2)}{2}$

Notice how the line in Figure 16 is very close to the points and that the uncertainties reflect the actual random variation in the data. The gradient of the line is equal to $\frac{1}{2}g$ so $g = 2 \times$ gradient.

$$g = 2 \times 4.814 = 9.624\ \text{m s}^{-2}$$

The uncertainty in this value can be estimated from the steepest and least steep lines:

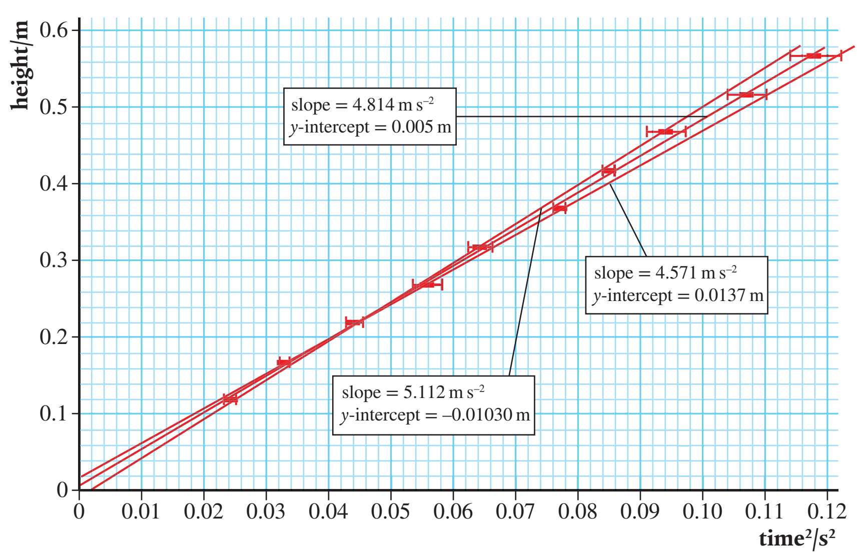
$$g_{max} = 2 \times 5.112 = 10.224\ \text{m s}^{-2}$$

$$g_{min} = 2 \times 4.571 = 9.142\ \text{m s}^{-2}$$

$$\Delta g = \frac{(g_{max} - g_{min})}{2} = \frac{(10224 - 9.142)}{2} = 0.541\ \text{m s}^{-2}$$

So, the final value including uncertainty is $9.6 \pm 0.5\ \text{m s}^{-2}$.

This is in agreement with the accepted average value which is $9.81\ \text{m s}^{-2}$.



A.1 Figure 16 Height vs time² for a falling object.

Why would it not be appropriate to apply the *suvat* equations to the motion of a body falling freely from a distance of 2 times the Earth's radius to the surface of the Earth? (D.1)

Graphical representation of motion

Graphs are used in physics to give a visual representation of relationships. In kinematics, they can be used to show how displacement, velocity and acceleration change with time. Figure 17 shows the graphs for four different examples of motion.

The best way to sketch graphs is to split the motion into sections then plot where the body is at different times. Joining these points will give the displacement–time graph. Once you have done that, you can work out the *v*–*t* and *a*–*t* graphs by looking at the *s*–*t* graph rather than the motion.

You need to be able to:
- work out what kind of motion a body has by looking at the graphs
- sketch graphs for a given motion.

Gradient of displacement–time graph

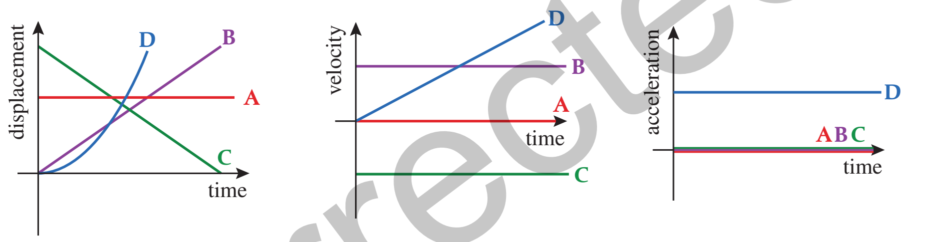
The gradient of a graph is: $\dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\Delta y}{\Delta x}$

In the case of the displacement–time graph, this will give:

$$\text{gradient} = \frac{\Delta s}{\Delta t}$$

This is the same as velocity.

We can represent the motion of a body on displacement–time graphs, velocity–time graphs and acceleration–time graphs. The three graphs of these types shown in Figure 17 display the motion of four bodies, which are labeled A, B, C and D.



A.1 Figure 17 Graphical representation of motion.

Body A

A body that is not moving.
Displacement is always the same.
Velocity is zero.
Acceleration is zero.

Body B

A body that is traveling with a constant positive velocity.
Displacement increases linearly with time.
Velocity is a constant positive value.
Acceleration is zero.

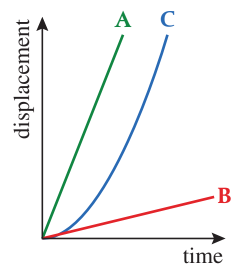
Body C

A body that has a constant negative velocity.
Displacement is decreasing linearly with time.
Velocity is a constant negative value.
Acceleration is zero.

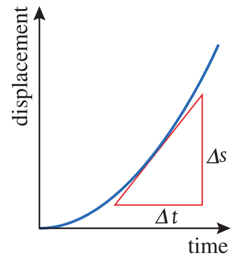
Body D

A body that is accelerating with constant acceleration.
Displacement is increasing at a non-linear rate. The shape of this line is a parabola since displacement is proportional to t^2 ($s = ut + \frac{1}{2}at^2$).
Velocity is increasing linearly with time.
Acceleration is a constant positive value.

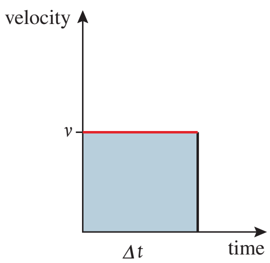
So, the gradient of the displacement–time graph equals the velocity. Using this information, we can see that line A in Figure 18 represents a body with a greater velocity than line B, and that since the gradient of line C is increasing, this must be the graph for an accelerating body.



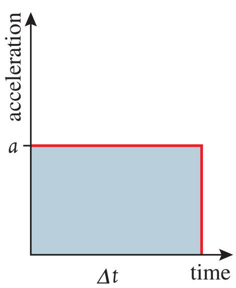
A.1 Figure 18 Three new bodies to compare.



A.1 Figure 19 Finding the gradient of the tangent



A.1 Figure 20 The area is displacement



A.1 Figure 21 The area is change in velocity

How does analyzing graphs allow us to determine other physical quantities? (NOS)

Instantaneous velocity

When a body accelerates, its velocity is constantly changing. The displacement–time graph for this motion is therefore a curve. To find the instantaneous velocity from the graph, we can draw a tangent to the curve and find the gradient of the tangent as shown in Figure 19.

Area under velocity–time graph

The area under the velocity–time graph for the body traveling at constant velocity v shown in Figure 20 is given by:

$$\text{area} = v\Delta t$$

But we know from the definition of velocity that: $v = \dfrac{\Delta s}{\Delta t}$

Rearranging gives $\Delta s = v\Delta t$ so the area under a velocity–time graph gives the displacement.

This is true, not only for simple cases such as this, but for all examples.

Gradient of velocity–time graph

The gradient of the velocity–time graph is given by $\dfrac{\Delta v}{\Delta t}$. This is the same as acceleration.

Area under acceleration–time graph

The area under the acceleration–time graph in Figure 21 is given by $a\Delta t$. But we know from the definition of acceleration that: $a = \dfrac{(v - u)}{t}$

Rearranging this gives $v - u = a\Delta t$ so the area under the graph gives the change in velocity.

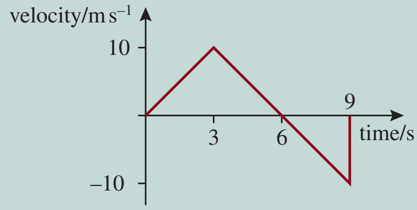
If you have covered calculus in your mathematics course, you may recognize these equations:

$$v = \frac{ds}{dt},\ a = \frac{dv}{dt} = \frac{d^2s}{d^2t} \text{ and } s = \int v dt,\ v = \int a dt$$

Exercise

Q11. Sketch a velocity–time graph for a body starting from rest and accelerating at a constant rate to a final velocity of $25\,\text{m s}^{-1}$ in 10 seconds. Use the graph to find the distance traveled and the acceleration of the body.

Q12. Describe the motion of the body whose velocity–time graph is shown. What is the final displacement of the body?

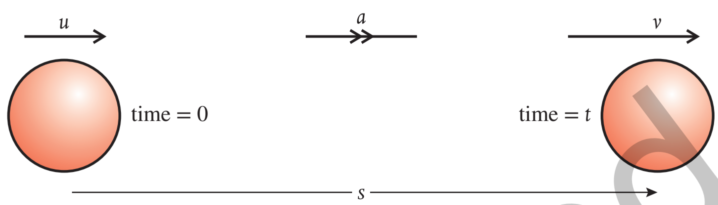


Q13. A ball is released from rest on the hill in the figure below. Sketch the *s*–*t*, *v*–*t*, and *a*–*t* graphs for its horizontal motion.

Q14. A ball rolls along a table then falls off the edge, landing on soft sand. Sketch the *s*–*t*, *v*–*t*, and *a*–*t* graphs for its vertical motion.

Example 1: The *suvat* example

As an example, let us consider the motion we looked at when deriving the *suvat* equations.

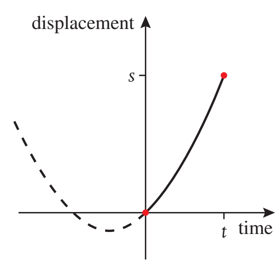


A.1 Figure 22 A body with constant acceleration.

Negative time does not mean going back in time – it means the time before you started the clock.

Displacement–time

The body starts with velocity *u* and travels to the right with constant acceleration *a* for a time *t*. If we take the starting point to be zero displacement, then the displacement–time graph starts from zero and rises to *s* in *t* seconds. We can therefore plot the two points shown in Figure 23. The body is accelerating so the line joining these points is a parabola. The whole parabola has been drawn to show what it would look like – the reason it is offset is because the body is not starting from rest. The part of the curve to the left of the origin tells us what the particle was doing before we started the clock.

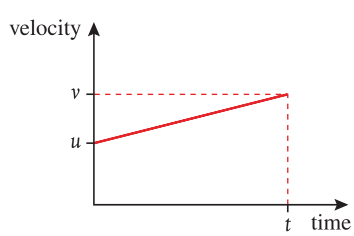


A.1 Figure 23 Constant acceleration.

Velocity–time

Figure 24 is a straight line with a positive gradient showing that the acceleration is constant. The line does not start from the origin since the initial velocity is *u*.

The gradient of this line is $\frac{(v-u)}{t}$, which we know from the *suvat* equations is acceleration.

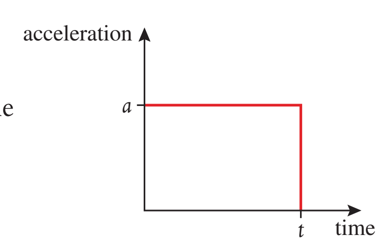


A.1 Figure 24 Constant acceleration.

The area under the line makes the shape of a trapezium. The area of this trapezium is $\frac{1}{2}(v+u)t$. This is the *suvat* equation for *s*.

Acceleration–time

The acceleration is constant so the acceleration–time graph is a horizontal line as shown in Figure 25. The area under this line is $a \times t$, which we know from the *suvat* equations equals $(v-u)$.

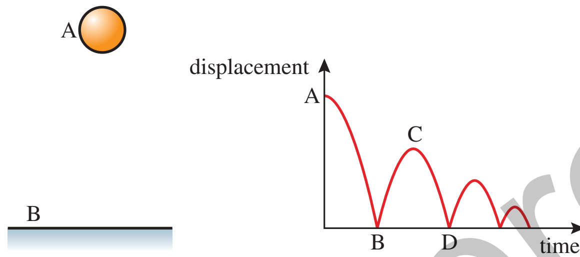


A.1 Figure 25 Constant acceleration.

Example 2: The bouncing ball

Consider a rubber ball dropped from position A above the ground onto hard surface B. The ball bounces up and down several times. Figure 26 shows the displacement–time graph for four bounces. From the graph, we see that the ball starts above the ground then falls with increasing velocity (as shown by the increasing negative gradient). When the ball bounces at B, the velocity suddenly changes from negative to positive as the ball begins to travel back up. As the ball goes up, its velocity decreases until it stops at C and begins to fall again.

A.1 Figure 26 Vertical displacement vs time.



Why is the height reached by a bouncing ball less than the height of release? (A.3)

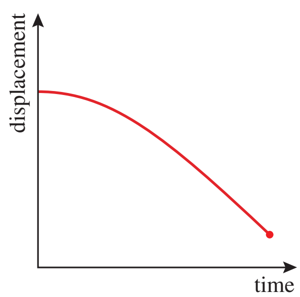
Exercise

Q15. By considering the gradient of the displacement–time graph in Figure 26, plot the velocity–time graph for the motion of the bouncing ball.

Example 3: A ball falling with air resistance

Figure 27 shows the motion of a ball that is dropped several hundred meters through the air. It starts from rest and accelerates for some time. As the ball accelerates, the air resistance increases, which stops the ball from getting any faster. At this point, the ball continues with constant velocity.

A.1 Figure 27 Vertical displacement vs time



Exercise

Q16. By considering the gradient of the displacement–time graph, plot the velocity–time graph for the motion of the falling ball in Figure 27.

Projectile motion

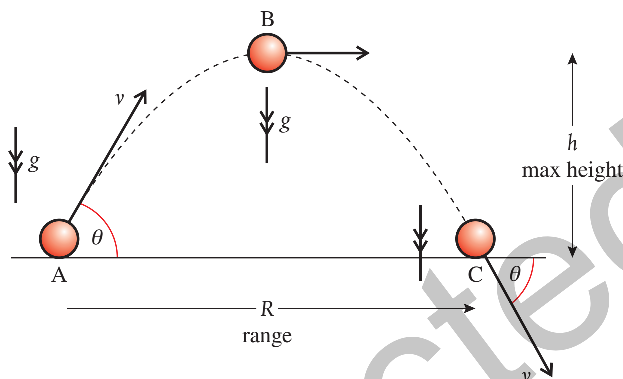
We all know what happens when a ball is thrown. It follows a curved path like the one in the photo. We can see from this photo that the path is parabolic and later we will show why that is the case.

A stroboscopic photograph of a projected ball.

When can problems on projectile motion be solved by applying conservation of energy instead of kinematic equations? (A.3)

Modeling projectile motion

All examples of motion up to this point have been in one dimension but projectile motion is two-dimensional. However, if we take components of all the vectors vertically and horizontally, we can simplify this into two simultaneous one-dimensional problems. The important thing to realize is that the vertical and horizontal components are independent of each other. You can test this by dropping an eraser off your desk and flicking one forward at the same time – they both hit the floor together. The downward motion is not changed by the fact that one stone is also moving forward.



A.1 Figure 28 A projectile launched at an angle θ.

Consider a ball that is projected at an angle θ to the horizontal, as shown in Figure 28. We can split the motion into three parts, beginning, middle and end, and analyze the vectors representing displacement, velocity and time at each stage. Notice that the path is symmetrical, so the motion on the way down is the same as on the way up.

Horizontal components

At A (time = 0)	At B (time = $\frac{t}{2}$)	At C (time = t)
displacement = zero	displacement = $\frac{R}{2}$	displacement = R
velocity = $v \cos \theta$	velocity = $v \cos \theta$	velocity = $v \cos \theta$
acceleration = 0	acceleration = 0	acceleration = 0

Vertical components

At A	At B	At C
displacement = zero	displacement = h	displacement = zero
velocity = $v \sin \theta$	velocity = zero	velocity = $-v \sin \theta$
acceleration = $-g$	acceleration = $-g$	acceleration = $-g$

Note that, at C, we are using the magnitude of θ (which is unchanged from position A). Therefore the negative sign is in place to provide the correct velocity direction; the projectile is moving downward.

We can see that the vertical motion is constant acceleration and the horizontal motion is constant velocity. We can therefore use the *suvat* equations.

Since the horizontal displacement is proportional to t, the path has the same shape as a graph of vertical displacement plotted against time. This is parabolic since the vertical displacement is proportional to t^2.

suvat for horizontal motion

Since acceleration is zero, there is only one equation needed to define the motion.

suvat	A to C
$v = \frac{s}{t}$	$R = v\cos\theta t$

A.1 Table 4

suvat for vertical motion

When considering the vertical motion, it is worth splitting the motion into two parts.

suvat	At B	At C
$s = \frac{1}{2}(u+v)t$	$h = \frac{1}{2}(v\sin\theta)\frac{t}{2}$	$0 = \frac{1}{2}(v\sin\theta - v\sin\theta)t$
$v^2 = u^2 + 2as$	$0 = v^2\sin^2\theta - 2gh$	$(-v\sin\theta)^2 = (v\sin\theta)^2 - 0$
$s = ut + \frac{1}{2}at^2$	$h = v\sin\theta t - \frac{1}{2}g\left(\frac{t}{2}\right)^2$	$0 = v\sin\theta t - \frac{1}{2}gt^2$
$a = \frac{v-u}{t}$	$g = \frac{v\sin\theta - 0}{\frac{t}{2}}$	$g = \frac{v\sin\theta - (-v\sin\theta)}{t}$

For a given value of v, the maximum range is when $v\cos\theta t$ is a maximum value.

$t = \frac{2v\sin\theta}{g}$

If we substitute this for t we get:

$R = \frac{2v^2\cos\theta\sin\theta}{g}$

Now, $2\sin\theta\cos\theta = \sin^2\theta$ (a trigonometric identity)

So, $R = \frac{v^2\sin^2\theta}{g}$

This is maximum when $\sin^2\theta$ is a maximum ($\sin^2\theta = 1$), which is when $\theta = 45°$.

Some of these equations are not very useful since they simply state that 0 = 0. However, we do end up with three useful ones (highlighted):

$$R = v\cos\theta t \quad (7)$$

$$0 = v^2\sin^2\theta - 2gh \quad \text{or} \quad h = \frac{v^2\sin^2\theta}{2g} \quad (8)$$

$$0 = v\sin\theta t - \frac{1}{2}gt^2 \quad \text{or} \quad t = \frac{2v\sin\theta}{g} \quad (9)$$

Solving problems

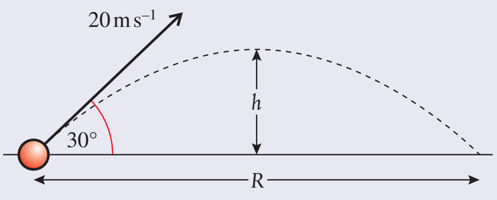
In a typical problem, you will be given the magnitude and direction of the initial velocity and asked to find either the maximum height or range. To calculate h, you can use equation (8), but to calculate R, you need to find the time of flight so must use (9) first. (You could also substitute for t into equation (6) to give another equation but we have enough equations already.)

How does the motion of a mass in a gravitational field compare to the motion of a charged particle in an electric field? (D.2)

You do not have to remember a lot of equations to solve a projectile problem. If you understand how to apply the *suvat* equations to the two components of the projectile motion, you only have to remember the *suvat* equations (and they are in the data booklet).

Worked example

A ball is thrown at an angle of 30° to the horizontal at a speed of 20 m s^{-1}. Calculate its range and the maximum height reached.



Solution

First, draw a diagram, including labels defining all the quantities known and unknown.

Now we need to find the time of flight. If we apply $s = ut + \frac{1}{2}at^2$ to the whole flight we get:

$$t = \frac{2v\sin\theta}{g} = \frac{(2 \times 20 \times \sin 30°)}{10} = 2\,\text{s}$$

We can now apply $s = vt$ to the whole flight to find the range:

$$R = v\cos\theta t = 20 \times \cos 30° \times 2 = 34.6\,\text{m}$$

Finally, to find the height, we apply $s = ut + \frac{1}{2}at^2$ to the vertical motion, but remember that this is only half the complete flight so the time is 1 *s*.

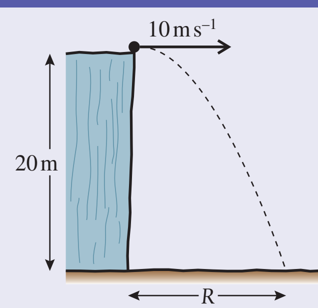
$$h = v\sin\theta t - \frac{1}{2}gt^2 = 20 \times \sin 30° \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5\,\text{m}$$

When a bullet is fired at a distant target, it will travel in a curved path due to the action of gravity. Precision marksmen adjust their sights to compensate for this. The angle of this adjustment could be based on calculation or experiment (trial and error).

If you have ever played golf, you will know that it is not true that the maximum range is achieved with an angle of 45°. The angle is actually much less. This is because the ball is held up by the air like a plane is. In this photo, Alan Shepard is playing golf on the Moon. Here, the maximum range will be at 45°.

Worked example

A ball is thrown horizontally from a cliff top with a horizontal speed of $10\,\text{m}\,\text{s}^{-1}$.
If the cliff is 20 m high, what is the range of the ball?



Solution

This is an easy one since there are no angles to deal with. The initial vertical component of the velocity is zero and the horizontal component is $10\,\text{m}\,\text{s}^{-1}$.
To calculate the time of flight, we apply $s = ut + \frac{1}{2}at^2$ to the vertical component.
Knowing that the final displacement is −20 m, this gives:

$$-20\,\text{m} = 0 - \frac{1}{2}gt^2 \text{ so } t = \sqrt{\frac{(2 \times 20)}{10}} = 2\,\text{s}$$

We can now use this value to find the range by applying the equation $s = vt$ to the horizontal component: $R = 10 \times 2 = 20\,\text{m}$

Exercise

Q17. Calculate the range of a projectile thrown at an angle of 60° to the horizontal with a velocity of 30 m s^{-1}.

Q18. You throw a ball at a speed of 20 m s^{-1}.

(**a**) At what angle must you throw the ball so that it will just get over a wall that is 5 m high?

(**b**) How far away from the wall must you be standing?

Q19. A gun is aimed so that it points directly at the center of a target 200 m away. If the bullet travels at 200 m s^{-1}, how far below the center of the target will the bullet hit?

Q20. If you can throw a ball at 20 m s^{-1}, what is the maximum distance you can throw it?

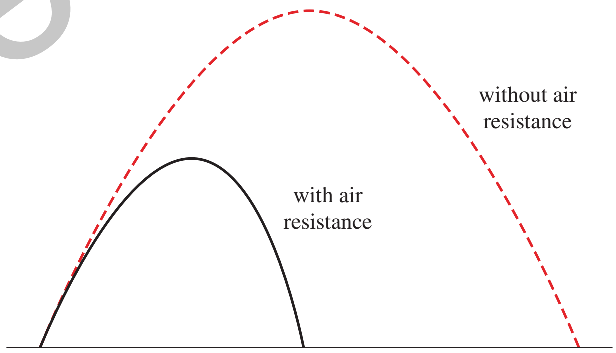
How does gravitational force allow for orbital motion? (A.2)

Challenge yourself

1. A projectile is launched perpendicular to a 30° slope at 20 m s^{-1}. Calculate the distance between the launching position and landing position.

Projectile motion with air resistance

In all the examples above, we have ignored the fact that the air will resist the motion of the ball. Air resistance opposes motion and increases with the speed of the moving object. The actual path of a ball including air resistance is likely to be as shown in Figure 29.



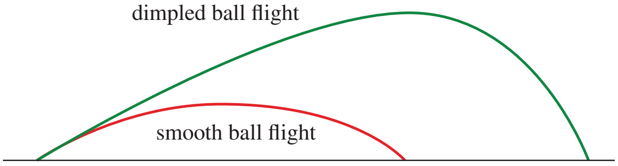
A.1 Figure 29 When air resistance is present, the projectile's motion is asymmetric

Notice that both the maximum height and the range are less. The path is also no longer a parabola – the way down is steeper than the way up.

The equation for this motion is complex. Horizontally, there is negative acceleration and so the horizontal component of velocity decreases. Vertically, there is increased magnitude of acceleration on the way up and a decreased magnitude of acceleration on the way down. None of these accelerations are constant so the *suvat* equations cannot be used. Luckily, all you need to know is the shape of the trajectory and the qualitative effects on range and time of flight.

Alternative air effects

The air does not always reduce the range of a projectile. A golf ball travels further than a ball projected in a vacuum. This is because the air holds the ball up, in the same way that it holds up a plane, due to the dimples in the ball and its spin.



A.1 Figure 30 The path of a smooth ball and a dimpled golf ball.

Guiding Questions revisited

How can the motion of a body be described quantitatively and qualitatively?

How can the position of a body in space and time be predicted?

How can the analysis of motion in one and two dimensions be used to solve real-life problems?

In this chapter, we have considered real-life examples to show that:

- Displacement is the straight-line distance between the start and end points of a body's motion and it has a direction.
- Velocity is the rate of change of displacement (and the vector equivalent of speed).
- Acceleration is the rate of change of velocity (and can therefore be treated as a vector).
- Motion graphs of displacement and velocity (or acceleration) against time enable qualitative changes in these quantities to be described and calculations of other quantities to be performed.
- The *suvat* equations of uniformly accelerated motion can be used to predict how position and velocity change with time (or one another) when a body experiences a constant acceleration.
- Vector quantities can be split into perpendicular components that can be treated independently, making it possible to solve problems in two dimensions using the *suvat* equations twice, for example, vertically and then horizontally for a projectile.
- Air resistance changes the acceleration in both perpendicular components, which means that the *suvat* equations cannot be used.

Practice questions

1. Police car P is stationary by the side of a road. Car S passes car P at a constant speed of 18 m s^{-1}. Car P sets off to catch car S just as car S passes car P. Car P accelerates at 4.5 m s^{-2} for 6.0 s and then continues at a constant speed. Car P takes *t* seconds to draw level with car S.

(**a**) State an expression, in terms of *t*, for the distance car S travels in *t* seconds. (1)

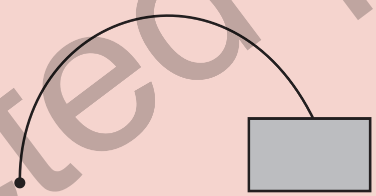
(**b**) Calculate the distance traveled by car P during the first 6.0 s of its motion. (1)

(**c**) Calculate the speed of car P after it has completed its acceleration. (1)

(**d**) State an expression, in terms of *t*, for the distance traveled by car P during the time that it is traveling at constant speed. (1)

(**e**) Using your answers to (**a**) to (**d**), determine the total time *t* taken by car P to draw level with car S. (2)

2. A ball is kicked with a speed of 14 m s^{-1} at 60° to the horizontal and lands on the roof of a 4 m high building.



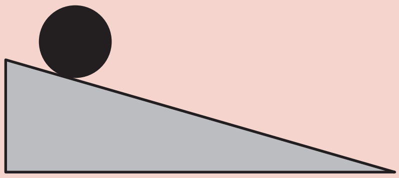
(**a**) (**i**) State the final vertical displacement of the ball. (1)

(**ii**) Calculate the time of flight. (3)

(**iii**) Calculate the horizontal displacement between the start point and the landing point on the roof. (2)

(**b**) The ball is kicked vertically upward. Explain the difference between the time to reach the highest point and the time from the highest point back to the ground. (3)

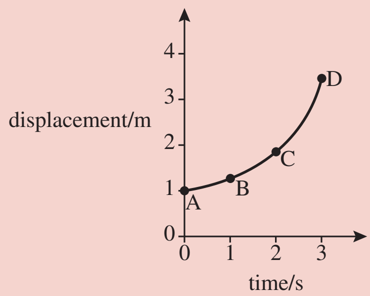
3. Two boys kick a football up and down a hill that is at an angle of 30° to the horizontal. One boy stands at the top of the hill and one boy stands at the bottom of the hill.



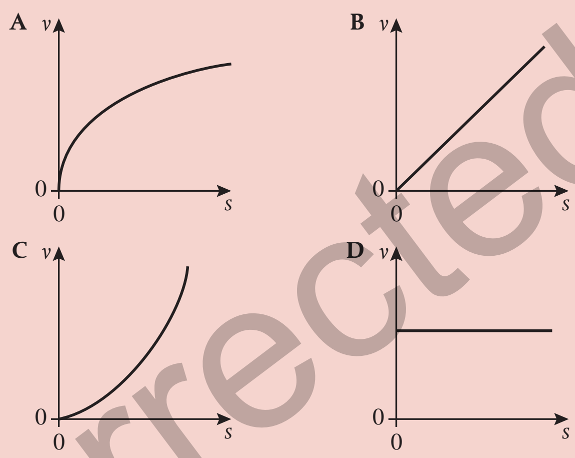
(**a**) Assuming that each boy kicks the ball perfectly to the other boy (without spin or bouncing), sketch a single path that the ball could take in either direction. (2)

(**b**) Compare the velocities with which each boy must strike the ball to achieve this path. (2)

4. The graph shows how the displacement of an object varies with time. At which point (A, B, C or D) does the instantaneous speed of the object equal its average speed over the interval from 0 to 3 s? (1)

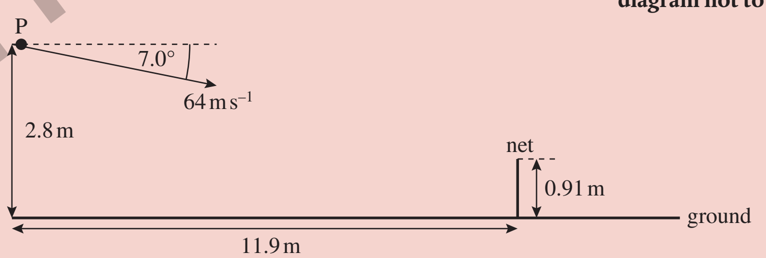


5. A runner starts from rest and accelerates at a constant rate. Which graph (A, B, C or D) shows the variation of the speed v of the runner with the distance traveled s? (1)



6. A student hits a tennis ball at point P, which is 2.8 m above the ground. The tennis ball travels at an initial speed of 64 m s^{-1} at an angle of 7.0° to the horizontal. The student is 11.9 m from the net and the net has a height of 0.91 m.

diagram not to scale



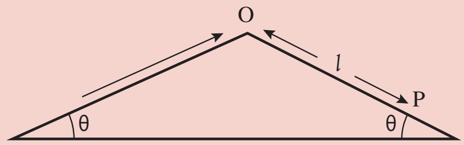
(**a**) Calculate the time it takes the tennis ball to reach the net. (2)

(**b**) Show that the tennis ball passes over the net. (3)

(**c**) Determine the speed of the tennis ball as it hits the ground. (2)

7. Estimate from what height, under free-fall conditions, a heavy stone would need to be dropped if it were to reach the surface of the Earth at the speed of sound ($330\,m\,s^{-1}$). (2)

8. A motorbike is ridden up the left side of a symmetrical ramp. The bike reaches the top of the ramp at speed *u*, becomes airborne and falls to a point P on the other side of the ramp.



In terms of *u*, *l* and *g*, obtain expressions for:

(**a**) the time *t* for which the motorbike is in the air (2)

(**b**) the distance OP (= *l*) along the right side of the ramp. (3)

A.2 Forces and momentum

The launch of the James Webb Space Telescope took place on 25 December 2021. When an object ejects a gas in a downward direction, the gas exerts an equal and upward force on the object. This is an example of a Newton's third law pair.

Guiding Questions

How can we use our knowledge and understanding of the forces acting on a system to predict changes in translational motion?

How can Newton's laws be modeled mathematically?

How can the conservation of momentum be used to predict the behavior of interacting objects?

The motion of a body traveling with constant acceleration can be modeled using the *suvat* equations of uniformly accelerated motion. But what causes the acceleration?

First we must introduce a second body, which interacts with the original body. When two bodies interact, we say that they exert forces on one another in accordance with Newton's three laws of motion. Forces can take many forms, but the presence of a force is required for one body to change the speed or course of another (an acceleration).

The effect of the force depends upon its direction, so we use an arrow to represent the size and direction of a given force. By adding all the arrows together as vectors, we can calculate the overall size and direction of the resultant force. Using this direction of force and, therefore, acceleration in combination with the *suvat* equations, we can predict the new position and velocity of the original body.

Momentum is the product of mass and velocity, two quantities that we met in the previous chapter. What makes it worth defining in its own right? Momentum is always conserved in any collision provided there are no external forces. This conservation is a direct consequence of Newton's three laws and can be used as a quick way to apply them.

Nature of Science

Newton's three laws of motion are a set of statements, based on observation and experiment, that can be used to predict the motion of a point object from the forces acting on it.

Students should understand:

Newton's three laws of motion
forces as interactions between bodies
forces acting on a body can be represented in a free-body diagram
free-body diagrams can be analyzed to find the resultant force on a system

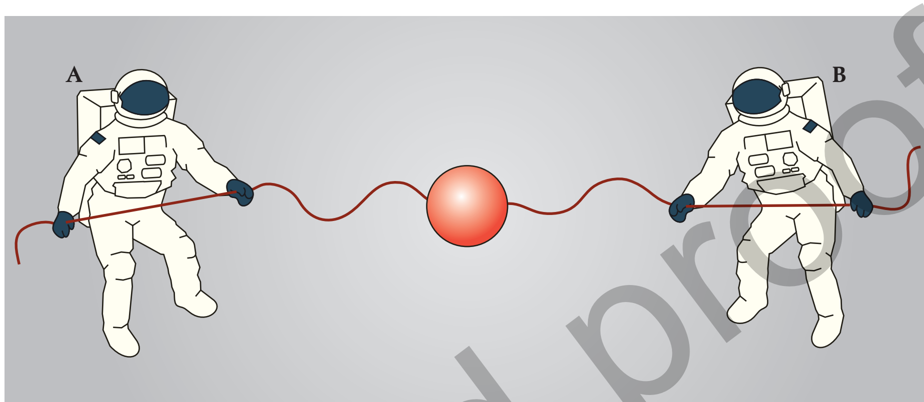
the nature and use of the following contact forces: • normal force F_N is the component of the contact force acting perpendicular to the surface that counteracts the body • surface frictional force F_f acting in a direction parallel to the plane of contact between a body and a surface, on a stationary body as given by $F_f \leq \mu_s F_N$ or a body in motion as given by $F_f = \mu_d F_N$ where μ_s and μ_d are the coefficients of static and dynamic friction respectively • elastic restoring force F_H following Hooke's law as given by $F_H = -kx$ where k is the spring constant • viscous drag force F_d acting on a small sphere opposing its motion through a fluid as given by $F_d = 6\pi\eta rv$ where η is the fluid viscosity, r is the radius of the sphere, and v is the velocity of the sphere through the fluid • buoyancy F_b acting on a body due to the displacement of the fluid as given by $F_b = \rho Vg$ where V is the volume of fluid displaced
the nature and use of the following field forces: • gravitational force F_g as the weight of the body and calculated as given by $F_g = mg$ • electric force F_e • magnetic force F_m
linear momentum as given by $p = mv$ remains constant unless the system is acted upon by a resultant external force
a resultant external force applied to a system constitutes an impulse J as given by $J = F\Delta t$ where F is the average resultant force and Δt is the time of contact
the applied external impulse equals the change in momentum of the system
Newton's second law in the form $F = ma$ assumes mass is constant whereas $F = \frac{\Delta p}{\Delta t}$ allows for situations where mass is changing
the elastic and inelastic collisions of two bodies
explosions
energy considerations in elastic collisions, inelastic collisions, and explosions
bodies moving along a circular trajectory at a constant speed experience an acceleration that is directed radially toward the center of the circle – known as a centripetal acceleration as given by $a = \frac{v^2}{r} = \omega^2 r = \frac{4\pi^2 r}{T^2}$
circular motion is caused by a centripetal force acting perpendicular to the velocity
a centripetal force causes the body to change direction even if its magnitude of velocity may remain constant
the motion along a circular trajectory can be described in terms of the angular velocity ω which is related to the linear speed v by the equation as given by $v = \frac{2\pi r}{T} = \omega r$.

Hooke's law and the elastic restoring force is discussed in A.3. The definitions of elastic and inelastic collisions can also be found in A.3. Information about electric and magnetic forces can be found in D.2.

Force

A force is a push or a pull. The unit of force is the newton.

We can now model the motion of a constantly accelerating body but what makes it accelerate? From experience, we know that to make something move we must push or pull it. We call this **applying a force**. One simple way of applying a force to a body is to attach a string and pull it. Imagine a sphere floating in space with two strings attached. The sphere will not start to move unless one of the astronauts pulls the string as in Figure 1.



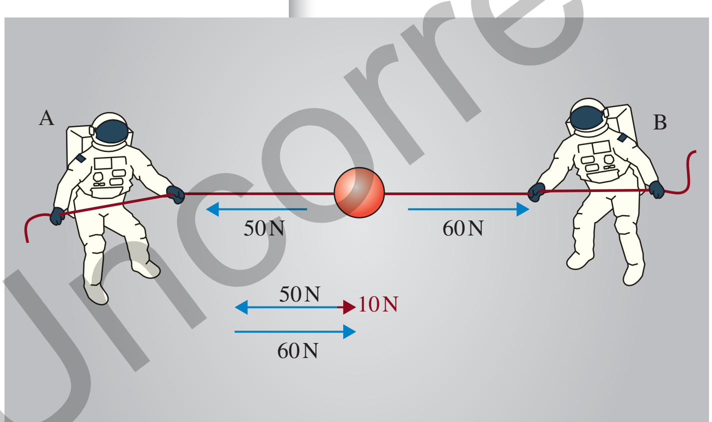
A.2 Figure 1 Two astronauts and a red ball.

If you hold an object of mass 100 g in your hand, then you will be exerting an upward force of about one newton (1 N).

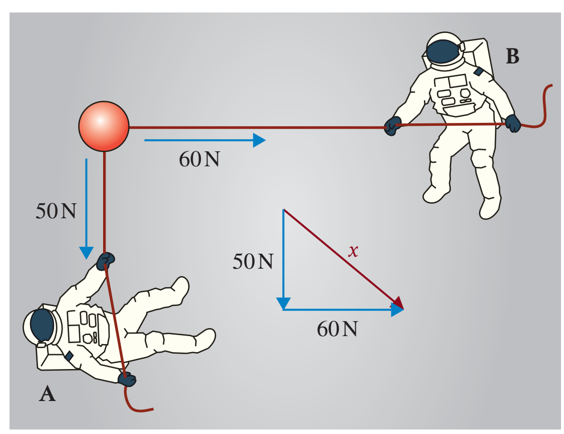
If A pulls the string, then the body will move to the left, and if B pulls the string, it will move to the right. We can see that force is a **vector** quantity since it has **direction**.

Addition of forces

Since force is a vector, we must add forces vectorially, so if A applies a force of 50 N and B applies a force of 60 N, the resultant force will be 10 N toward B, as can be seen in Figure 2.



A.2 Figure 2 Astronaut B pulls harder than A.



A.2 Figure 3 Astronauts pulling at right angles.

Astronauts in space are considered here so that no other forces (except for very low gravity) are present. This makes things simpler.

Or, in two dimensions, we can use trigonometry as in Figure 3.

In this case, because the addition of forces forms a right-angled triangle, we can use Pythagoras to find x:

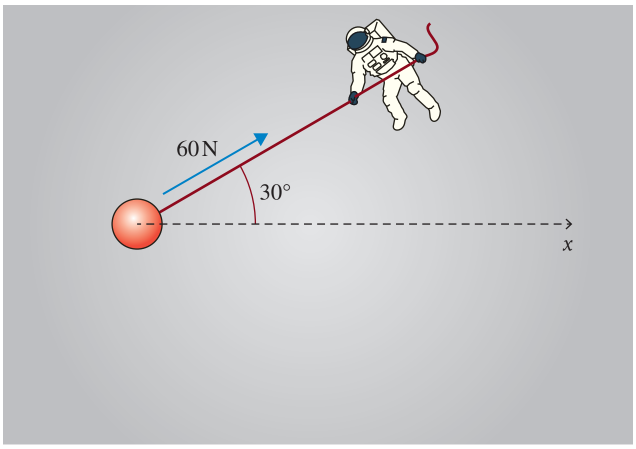
$$x = \sqrt{50^2 + 60^2} = 78\,\text{N}$$

Taking components

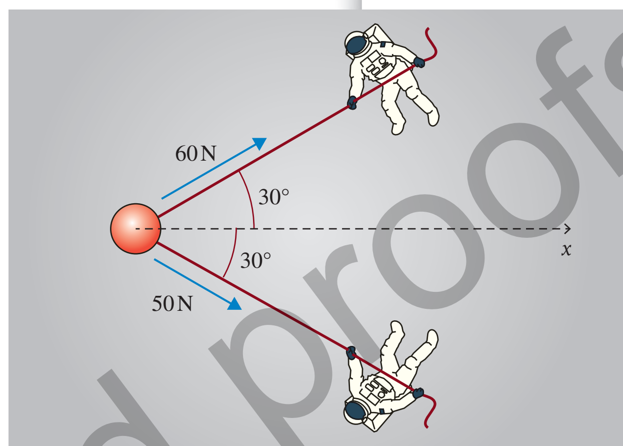
As with other vector quantities, we can calculate components of forces. For example, we might want to know the resultant force in a particular direction.

In Figure 4, the component of the force in the *x*-direction is: $F_x = 60 \times \cos 30° = 52\,N$

This is particularly useful when we have several forces.



▲ **A.2 Figure 4** Pulling at an angle.



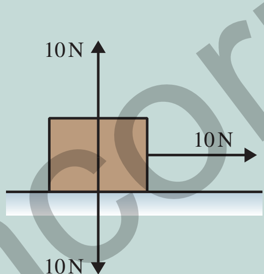
▲ **A.2 Figure 5** Astronauts not pulling in line.

In the example shown in Figure 5, we can use components to calculate the resultant force in the *x*-direction: $60 \times \cos 30° + 50 \times \cos 30° = 52 + 43 = 95\,N$

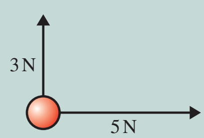
Exercise

Q1. Find the resultant force in the following examples:

(a)

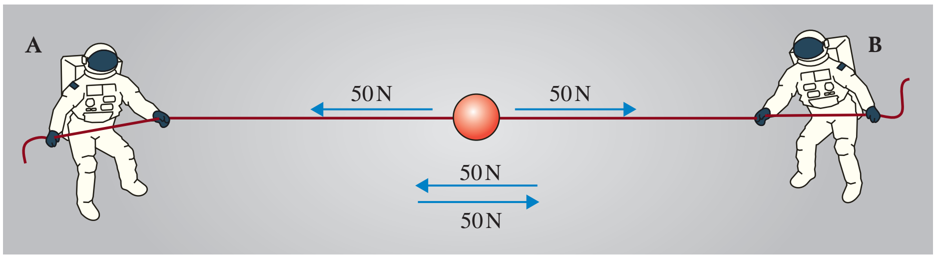


(b)



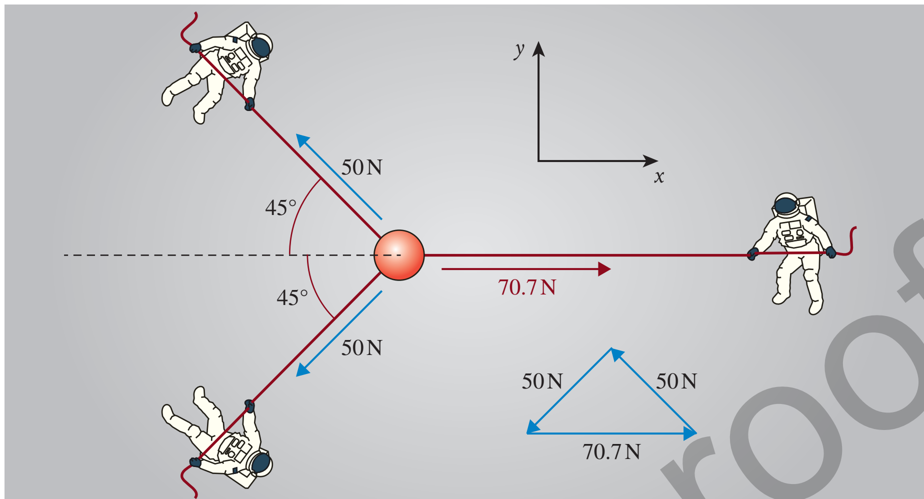
Equilibrium

If the resultant force on a body is zero, as in Figure 6, then we say the forces are **balanced** or the body is in **equilibrium**.



◀ **A.2 Figure 6** Balanced forces.

Or with three forces as in Figure 7.



A.2 Figure 7 Three balanced forces.

In this example, the two blue forces are perpendicular, making the trigonometry easy. Adding all three forces gives a right-angled triangle. We can also see that if we take components in any direction, then the forces must be balanced.

Taking components in the *x*-direction:

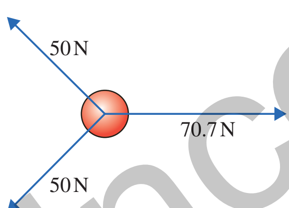
$-50 \times \cos 45° - 50 \times \cos 45° + 70.7 = -35.35 - 35.35 + 70.7 = 0$

Taking components in the *y*-direction:

$50 \times \sin 45° - 50 \times \sin 45° = 0$

Free-body diagrams

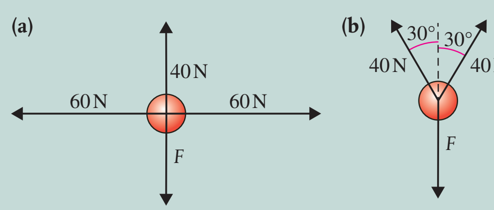
Problems often involve more than one body. For example, the previous problem involved four bodies, three astronauts, and one red ball. All of these bodies will experience forces, but if we draw them all on the diagram, it would be very confusing. For that reason, we only draw forces on the body we are interested in; in this case, the red ball. This is called a free-body diagram, as shown in Figure 8. Note that we treat the red ball as a point object by drawing the forces acting on the center. Not all forces actually act on the center, but when adding forces, it can be convenient to draw them as if they do.



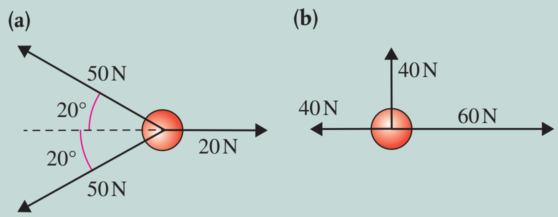
A.2 Figure 8 A free-body diagram of the forces in Figure 7.

Exercise

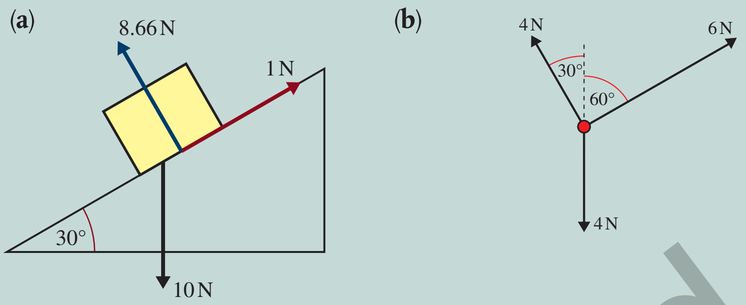
Q2. In the following examples, calculate the force *F* required to balance the forces.



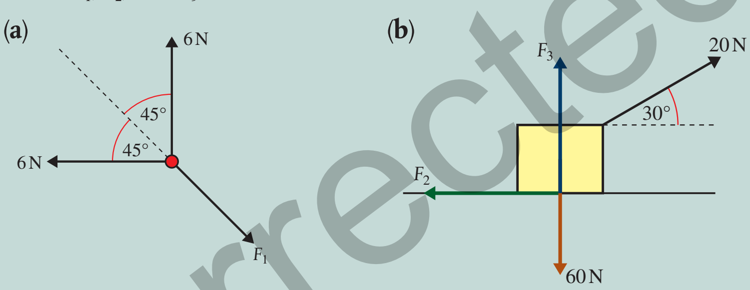
Q3. Calculate the resultant force for the following.



Q4. By resolving the vectors into components, calculate if the following bodies are in translational equilibrium or not. If not, calculate the resultant force.



Q5. If the following two examples are in equilibrium, calculate the unknown forces F_1, F_2, and F_3.



Newton's first law of motion

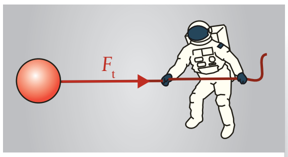
From observation, we can conclude that to make a body move we need to apply an unbalanced force to it. What is not so obvious is that once moving it will continue to move with a constant velocity unless acted upon by another unbalanced force. Newton's first law of motion is a formal statement of this:

A body will remain at rest or moving with constant velocity unless acted upon by an unbalanced force.

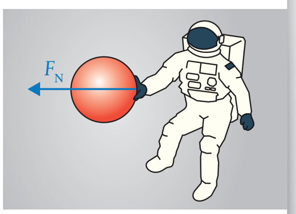
The reason that this is not obvious to us on Earth is that we do not tend to observe bodies traveling with constant velocity with no forces acting on them; in space, it would be more obvious. Newton's first law can be used in two ways. If the forces on a body are balanced, then we can use Newton's first law to predict that it will be at rest or moving with constant velocity. If the forces are unbalanced, then the body will not be at rest or moving with constant velocity. This means its velocity changes – in other words, it accelerates. Using the law the other way round, if a body accelerates, then Newton's first law predicts that the forces acting on the body are unbalanced. To apply this law in real situations, we need to know a bit more about the different types of force.

TOK

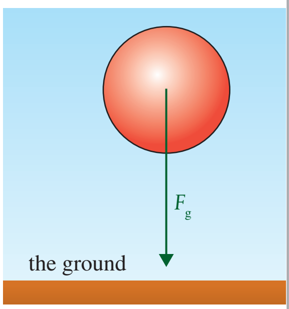
A law in physics is a very useful tool. If applied properly, it enables us to make a very strong argument that what we say is true. If asked 'will a box move?' you can say that you think it will and someone else could say it will not. You both have your opinions and you would then argue as to who is right. However, if you say that Newton's law says it will move, then you have a much stronger argument (assuming you have applied the law correctly).



▲ **A.2 Figure 9** Exerting tension with a string.



▲ **A.2 Figure 10** A normal reaction force is exerted when a hand is in contact with a ball.



▲ **A.2 Figure 11** A ball in free fall.

Types of force

Tension

Tension is the name of the force exerted by the astronauts on the red ball. If you attach a string to a body and pull it, then you are exerting tension, as in Figure 9.

Normal reaction

Whenever two surfaces are in contact with (touching) each other, there will be a force between them. This force is perpendicular to the surface so it is called the **normal reaction force**. If the astronaut pushes the ball with his hand as in Figure 10, then there will be a normal reaction between the hand and the ball.

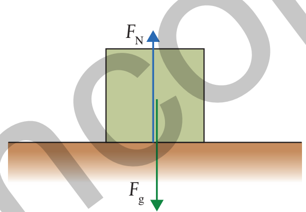
Note that the force acts on both surfaces so the astronaut will also experience a normal force. However, since we are interested in the ball, not the astronaut, we take the ball as our 'free body' so only draw the forces acting on it.

Gravitational force (weight)

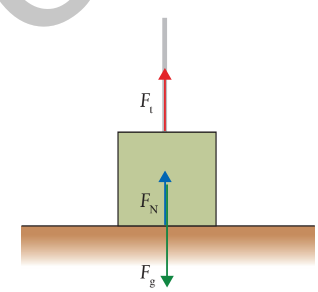
Back on Earth, if a body is released above the ground as in Figure 11, it accelerates downward. According to Newton's first law, there must be an unbalanced force causing this motion. This force is called the **weight**. The weight of a body is directly proportional to its mass: $F_g = mg$ where gravitational field strength, $g = 9.81\,\text{N kg}^{-1}$ close to the surface of the Earth. Note that this is the same as the acceleration of free fall. You will find out why later on.

Note that the weight acts at the **center** of the body.

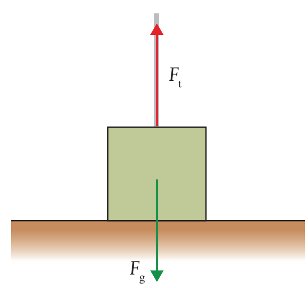
If a block is at rest on the floor, then Newton's first law implies that the forces are balanced. The forces involved are weight (because the block has mass and is on the Earth) and normal force (because the block is in contact with the ground). Figure 12 shows the forces.



▲ **A.2 Figure 12** A free-body diagram of a box resting on the ground.



▲ **A.2 Figure 13** A string applies an upward force on the box.



▲ **A.2 Figure 14** The block is lifted as the tension is bigger than its weight.

These forces are balanced so: $-F_g + F_N = 0$ or $F_g = F_N$

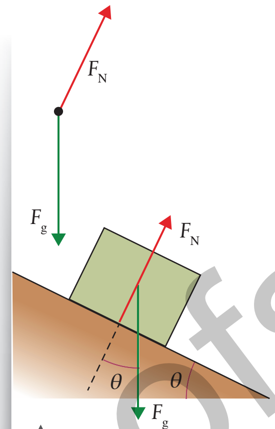
If the mass of the block is increased, then the normal reaction will also increase.

If a string is added to the block, then we can exert tension on the block as in Figure 13.

The forces are still balanced since $F_t + F_N = F_g$. Notice how F_g has remained the same but F_N has got smaller. If we pull with more force,we can lift the block as in Figure 14. At this point, the normal reaction F_N will be zero. The block is no longer in contact with the ground; now $F_t = F_g$.

The block in Figure 15 is on an inclined plane (slope) so the weight still acts downward. In this case, it might be convenient to split the weight into components, one acting parallel to the slope and one acting perpendicular to the slope.

The component of weight perpendicular to the slope is $F_g \cos\theta$. Since there is no movement in this direction, the force is balanced by F_N. The component of weight parallel to the slope is $F_g \sin\theta$. This force is unbalanced, causing the block to accelerate parallel to the slope. If the angle of the slope is increased, then $\sin\theta$ will also increase, resulting in a greater force down the slope.



▲ **A.2 Figure 15** Free-body diagram for a block on a slope.

Electric and magnetic forces

Electric forces act on charged particles. Magnetic forces act on moving charged particles and magnetic materials.

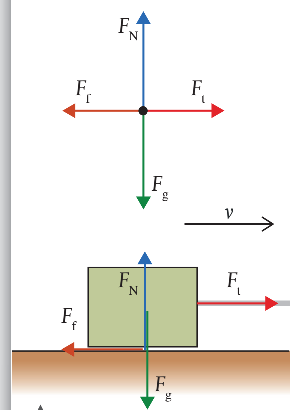
Like weight (i.e. gravitational forces), electric and magnetic forces act at a distance. Unlike weight, they can be attractive and repulsive.

What assumptions (NOS) about the forces between molecules of gas allow for ideal gas behavior? (B.3)

Friction

There are two types of friction: **static friction**, which is the force that stops the relative motion between two touching surfaces, and **dynamic friction**, which opposes the relative motion between two touching surfaces. In both cases, the force is related to both the normal force and the nature of the surfaces, so pushing two surfaces together increases the friction between them.

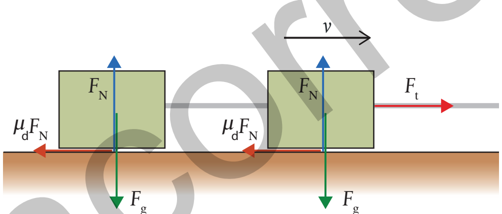
$F_f = \mu F_N$ where μ is the coefficient of friction (static or dynamic).



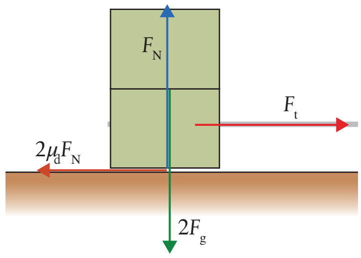
▲ **A.2 Figure 16** The force experienced by a block pulled along a table.

Dynamic friction

In Figure 21, a block is being pulled along a table at a constant velocity.



▲ **A.2 Figure 17** Two blocks joined with a rope.



▲ **A.2 Figure 18** Two blocks on top of each other.

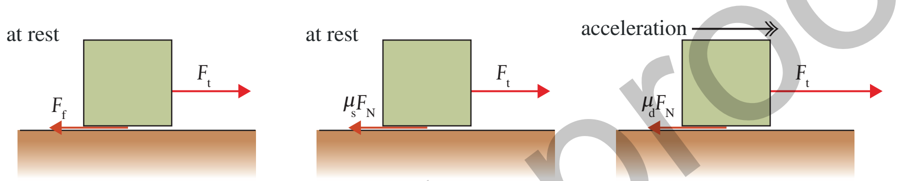
Since the velocity is constant, Newton's first law implies that the forces are balanced so $F_t = F_f$ and $F_g = F_N$. Notice that friction does not depend on the area of contact. We can show this by considering two identical blocks sliding at constant velocity across a table top joined together by a rope as in Figure 17. The friction under each cube is $\mu_d F_N$ so the total friction would be $2\mu_d F_N$.

If one cube is now placed on top of the other as in Figure 18, the normal force under the bottom cube will be twice as much so the friction is now $2\mu_d F_N$. It does not matter if the blocks are side by side (large area of contact) or on top of each other (small area of contact); the friction is the same.

If this is the case, then why do racing cars have wide tires with no tread pattern (slicks)? There are several reasons for this but one is to increase the friction between the tires and the road. This is strange because friction is not supposed to depend on area of contact. In practice, friction is not so simple. When one of the surfaces is sticky like the tires of a racing car, the force *does* depend upon the surface area. The type of surfaces we are concerned with here are quite smooth, non-sticky surfaces like wood and metal.

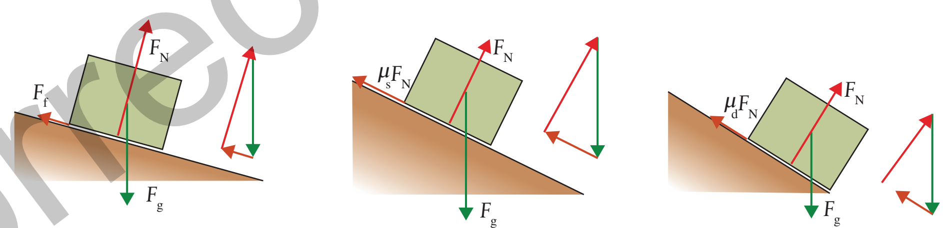
Static friction

If a very small force is applied to a block at rest on the ground, it will not move. This means that the forces on the block are **balanced** (Newton's first law): the applied force is balanced by the static friction.



A.2 Figure 19 μF_N is the maximum size of friction.

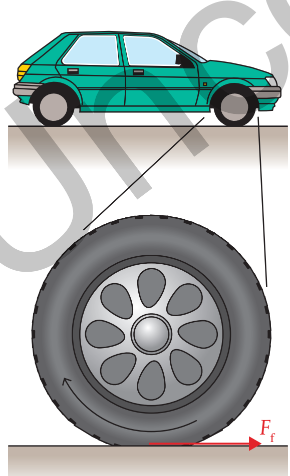
In this case, the friction simply equals the applied force: $F_f = F_t$. As the applied force is increased, the friction will also increase. However, there will be a point when the friction cannot be any bigger. If the applied force is increased past that point, the block will start to move; the forces have become **unbalanced** as illustrated in Figure 24. The maximum value that friction can have is $\mu_s F_N$ where μ_s is the coefficient of static friction. The value of static friction is always *greater* than dynamic friction. This can easily be demonstrated with a block on an inclined plane as shown in Figure 20.



A.2 Figure 20 A block rests on a slope until the forces become unbalanced.

In the first example, the friction is balancing the component of weight down the plane, which equals $F_g \sin\theta$, where θ is the angle of the slope. As the angle of the slope is increased, the point is reached where the static friction = $\mu_s F_N$. The forces are still balanced but the friction cannot get any bigger, so if the angle is increased further, the forces become unbalanced and the block will start to move. Once the block moves, the friction becomes dynamic friction. Dynamic friction is less than static friction, so this results in a bigger resultant force down the slope, causing the block to accelerate.

Friction does not just slow things down; it is also the force that makes things move. Consider the tire of a car as it starts to drive away from the traffic lights. The rubber of the tire is trying to move relative to the road. In fact, if there was no friction, the wheel would spin as the tire slipped backward on the road. The force of friction that opposes the motion of the tire slipping backward on the road is therefore in the **forward** direction.

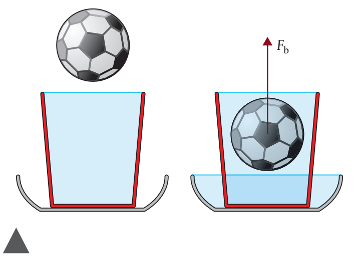


A.2 Figure 21 Friction pushes the car forwards.

If the static friction between the tire and the road is not big enough, the tire will slip. Once this happens, the friction becomes dynamic friction, which is less than static friction, so once tires start to slip, they tend to continue slipping.

Buoyancy

Buoyancy is the name of the force experienced by a body totally or partially immersed in a fluid (a fluid is a liquid or gas). The size of this force is equal to the weight of fluid displaced. It is this force that enables a boat to float and a helium balloon to rise in the air. Let us consider a football and a bucket full of water.



A.2 Figure 22 A football immersed in a bucket of water.

If you take the football and push it under the water, then water will flow out of the bucket (luckily a big bowl was placed there to catch it). The weight of this displaced water is equal to the upward force on the ball. To keep the ball under water, you would therefore have to balance that force by pushing the ball down.

The forces on a floating object are balanced so the weight must equal the buoyant force. This means that the ball must have displaced its own weight of water as in Figure 28.

$$F_g = \rho V_g$$

where V is the volume of fluid displaced and ρ is the density of the fluid.

Air resistance

Air resistance is the force that opposes the motion of a body through the air. More broadly, this is known as fluid resistance or **drag**. The size of this force depends on the speed, size, and shape of the body. At low speeds, the drag force experienced by a sphere is given by Stokes' law:

$$F_d = 6\pi\eta v r$$

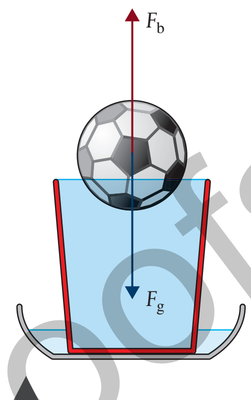
where η = viscosity (a constant)

v = velocity

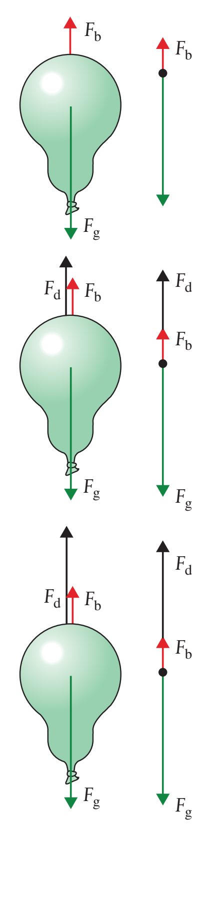
r = radius

When a balloon is dropped, it accelerates downward due to the force of gravity. As it falls through the air, it experiences a drag force opposing its motion. As the balloon's velocity gets bigger so does the drag force, until the drag force balances its weight, at which point its velocity will remain constant (Figure 24). This maximum velocity is called its **terminal velocity**.

The same thing happens when a parachutist jumps out of a plane. The terminal velocity in this case is around 54 m s^{-1} (195 km h^{-1}). Opening the parachute increases the drag force, which slows the parachutist down to a safer 10 m s^{-1} for landing.

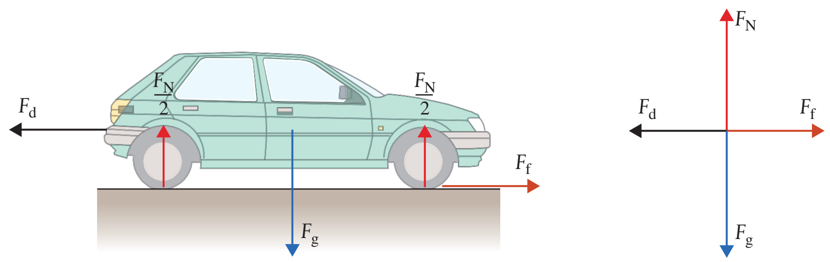


A.2 Figure 23 A football floats in a bucket of water.

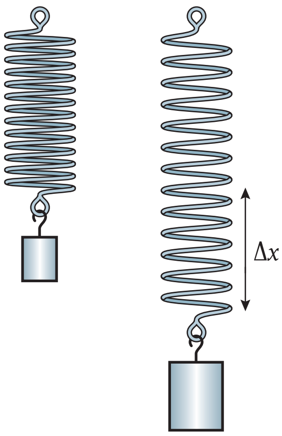


A.2 Figure 24 A balloon reaches terminal velocity as the forces become balanced. Notice the buoyant force is also present.

As it is mainly the air resistance that limits the top speed of a car, a lot of time and money is spent by car designers to try to reduce this force. This is particularly important at high speeds when the drag force is related to the square of the speed.



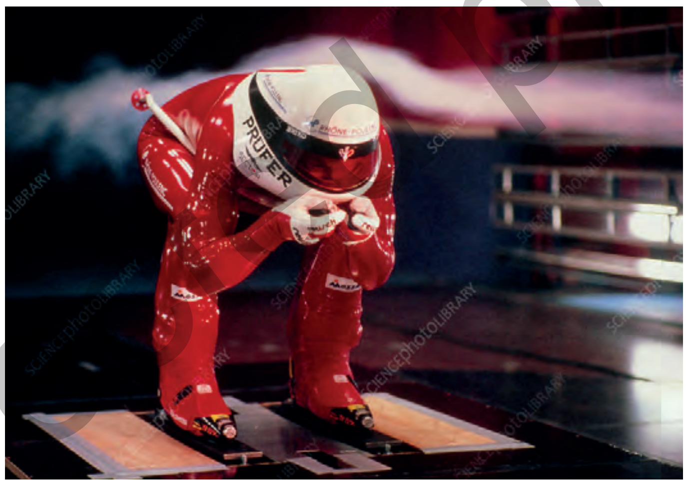
A.2 Figure 25 The forces acting on a car traveling at constant velocity.



A.2 Figure 26 Stretching a spring.

Elastic restoring force

An elastic restoring force, F_H, acts when the shape of an object is changed. An object is stretched when in tension and squashed when in compression. The size of the elastic restoring force increases with the extension (or compression) from the original length.



Speed skiers wear special clothes and squat down like this to reduce air resistance.

How does the application of a restoring force acting on a particle result in simple harmonic motion? (C.1)

Exercise

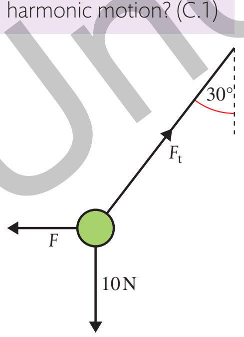
Q6. A ball of weight 10 N is suspended on a string and pulled to one side by another horizontal string as shown in Figure 27. If the forces are balanced:

(**a**) write an equation for the horizontal components of the forces acting on the ball

(**b**) write an equation for the vertical components of the forces acting on the ball

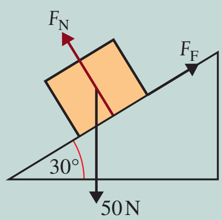
(**c**) use the second equation to calculate the tension in the upper string, F_t

(**d**) use your answer to (c) plus the first equation to find the horizontal force F.



A.2 Figure 27 A ball on a string that is pulled to the side

Q7. The condition for the forces to be balanced is that the sum of components of the forces in any two perpendicular components is zero. In the 'box on a ramp' example, the vertical and horizontal components were taken. However, it is sometimes more convenient to consider components parallel and perpendicular to the ramp.



Consider the situation in the figure. If the forces on this box are balanced:

(a) write an equation for the components of the forces parallel to the ramp

(b) write an equation for the forces perpendicular to the ramp

(c) use your answers to find the friction (F_f) and normal force (F_N).

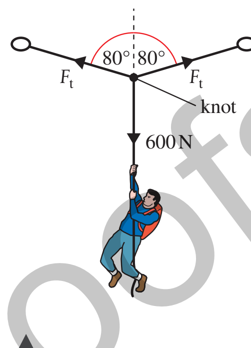
Q8. A rock climber is hanging from a rope attached to the cliff by two bolts as shown in Figure 28. If the forces are balanced:

(a) write an equation for the vertical component of the forces on the knot

(b) write an equation for the horizontal forces exerted on the knot

(c) calculate the tension F_t in the ropes joined to the bolts.

The result of this calculation shows why ropes should not be connected in this way.



A.2 Figure 28 The rope is attached at two bolts

The relationship between force and acceleration

Newton's first law states that a body will accelerate if an unbalanced force is applied to it. Newton's second law tells us how big the acceleration will be and in which direction. Before we look in detail at Newton's second law, we should look at the factors that affect the acceleration of a body when an unbalanced force is applied. Let us consider the example of catching a ball. When we catch the ball, we change its velocity, Newton's first law tells us that we must therefore apply an unbalanced force to the ball. The size of that force depends upon two things: the mass and the velocity. A heavy ball is more difficult to stop than a light one traveling at the same speed, and a fast one is harder to stop than a slow one. Rather than having to concern ourselves with two quantities, we will introduce a new quantity that incorporates both mass and velocity: **momentum**.

Nature of Science

The principle of conservation of momentum is a consequence of Newton's laws of motion applied to the collision between two bodies. If this applies to two isolated bodies, we can generalize that it applies to any number of isolated bodies. Here we will consider colliding balls but it also applies to collisions between microscopic particles such as atoms.

Momentum (p)

Momentum is defined as the product of mass and velocity: $\boldsymbol{p} = m\boldsymbol{v}$

The unit of momentum is kg m s^{-1}. Momentum is a vector quantity.

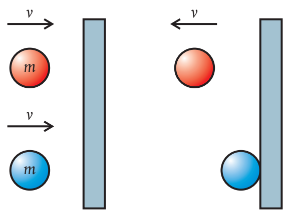
Impulse

When you get hit by a ball, the effect it has on you is greater if the ball bounces off you than if you catch it. This is because the change of momentum, Δp, is greater when the ball bounces, as shown in Figure 35.

The unit of impulse is kg m s^{-1}.

Impulse, J, is the change in momentum and is equal to the product of force and the time over which the force is acting. It is a vector.

$$J = \Delta p = F\Delta t$$



A.2 Figure 29 The change of momentum of the red ball is greater.

Red ball

momentum before = mv

momentum after = $-mv$ (remember momentum is a vector)

change in momentum, $J = -mv - mv = -2mv$

Blue ball

momentum before = mv

momentum after = 0

change in momentum, $J = 0 - mv = -mv$

Exercise

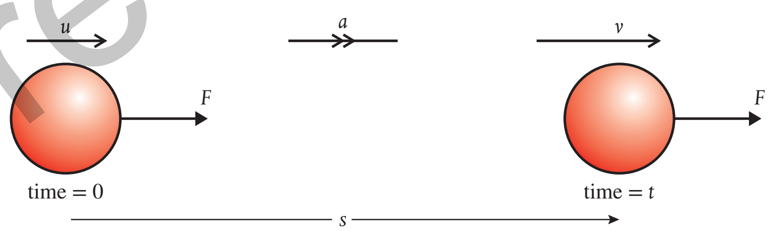
Q9. A ball of mass 200 g traveling at 10 m s^{-1} bounces off a wall as in Figure 29. If after hitting the wall it travels at 5 m s^{-1}, what is the impulse?

Q10. Calculate the impulse on a tennis racket that hits a ball of mass 67 g traveling at 10 m s^{-1} so that it comes off the racket at a velocity of 50 m s^{-1}.

Newton's second law of motion

The rate of change of momentum of a body is directly proportional to the unbalanced force acting on that body and takes place in same direction.

Let us once again consider a ball with a constant force acting on it as in Figure 30.



A.2 Figure 30 A ball gains momentum

If $F = \frac{\text{change in momentum}}{\text{time}}$ then momentum = force × time. So the unit of momentum is N s. This is the same as kg m s^{-1}.

Newton's first law tells us that there must be an unbalanced force acting on the ball since it is accelerating.

Newton's second law tells us that the size of the unbalanced force is directly proportional to the rate of change of momentum. We know that the force is constant so the rate of change of momentum is also constant, which, since the mass is also constant, implies that the acceleration is uniform so the *suvat* equations apply.

If the ball has mass, m we can calculate the change of momentum of the ball.

initial momentum = mu

final momentum = mv

change in momentum = $mv - mu$

The time taken is t so the rate of change of momentum = $\frac{mv - mu}{t}$

This is the same as $\frac{m(v-u)}{t} = ma$

Newton's second law states that the rate of change of momentum is proportional to the force, so $F \propto ma$.

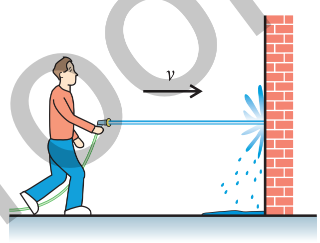
To make things simple, the newton is defined so that the constant of proportionality is equal to 1 so:

$$F = ma$$

So when a force is applied to a body in this way, Newton's second law can be simplified to:

The acceleration of a body is proportional to the force applied and inversely proportional to its mass.

Not all examples are so simple. Consider a jet of water hitting a wall as in Figure 31. The water hits the wall and loses its momentum, ending up in a puddle on the floor.



A.2 Figure 31 A jet becomes a puddle

Newton's first law tells us that since the velocity of the water is changing, there must be a force on the water,

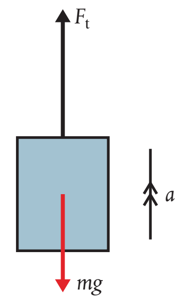
Newton's second law tells us that the size of the force is equal to the rate of change of momentum. The rate of change of momentum in this case is equal to the amount of water hitting the wall per second multiplied by the change in velocity. This is not the same as *ma*. For this reason, it is best to use the first, more general, statement of Newton's second law, since this can always be applied.

However, in this course, most of the examples will be of the $F = ma$ type.

Example 1: Elevator accelerating upward

An elevator has an upward acceleration of $1\,\mathrm{m\,s^{-2}}$. If the mass of the elevator is 500 kg, what is the tension in the cables pulling it up?

First draw a free-body diagram as in Figure 32. Now we can see what forces are acting. Newton's first law tells us that the forces must be unbalanced. Newton's second law tells us that the unbalanced force must be in the direction of the acceleration (upward). This means that F_t is bigger than mg.



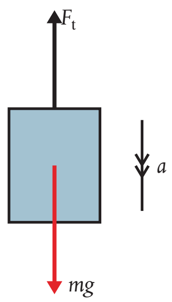
A.2 Figure 32 An elevator accelerating upward. This could either be going up getting *faster* or going down getting *slower*.

Newton's second law also tells us that the size of the unbalanced force equals *ma* so we get the equation:

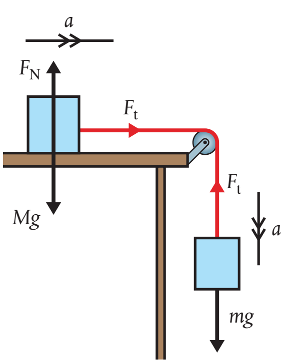
$$F_t - mg = ma$$

Rearranging gives:

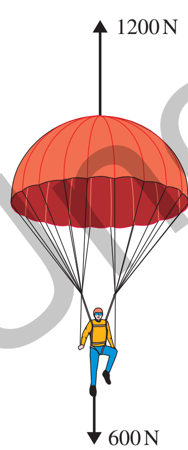
$$\begin{aligned} F_t &= mg + ma \\ &= 500 \times 10 + 500 \times 1 \\ &= 5500\,\mathrm{N} \end{aligned}$$



▲ **A.2 Figure 33** The elevator with downward acceleration.



▲ **A.2 Figure 34**



▲ **A.2 Figure 35** The parachutist just after opening the parachute.

Example 2: Elevator accelerating downward

The same elevator as in Example 1 now has a downward acceleration of 1 m s^{-2} as in Figure 33.

This time, Newton's laws tell us that the weight is bigger than the tension so: $mg - F_t = ma$

Rearranging gives:

$$\begin{aligned} F_t &= mg - ma \\ &= 500 \times 10 - 500 \times 1 \\ &= 4500\,\text{N} \end{aligned}$$

Example 3: Joined masses

Two masses are joined by a rope. One of the masses sits on a frictionless table, while the other hangs off the edge as in Figure 34.

M is being dragged to the edge of the table by m.

Both are connected to the same rope so F_t is the same for both masses. This also means that the acceleration a is the same.

We do not need to consider F_N and Mg for the mass on the table because these forces are balanced. However, the horizontally unbalanced force is F_t.

Applying Newton's laws to the mass on the table gives:

$$F_t = Ma$$

The hanging mass is accelerating down so mg is bigger than F_t. Newton's second law implies that: $mg - F_t = ma$

Substituting for F_t gives: $mg - Ma = ma$ so $a = \dfrac{mg}{M + m}$

Example 4: The free fall parachutist

After falling freely for some time, a free fall parachutist, whose weight is 60 kg, opens his parachute. Suddenly, the force due to air resistance increases to 1200 N. What happens?

Looking at the free-body diagram in Figure 35, we can see that the forces are unbalanced and that, according to Newton's second law, the acceleration, a, will be upward.

The size of the acceleration is given by:

$$ma = 1200 - 600 = 60 \times a$$

So: $$a = 10\,\text{m s}^{-2}$$

The acceleration is in the opposite direction to the motion. This will cause the parachutist to slow down. As he slows down, the air resistance gets less until the forces are balanced. He will then continue down with a constant velocity.

Even before opening their parachutes, base jumpers reach terminal velocity.

Exercise

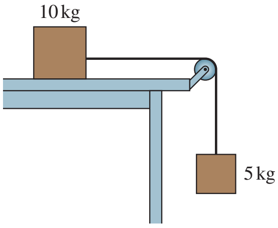
Q11. The helium in a balloon causes an upthrust of 0.1 N. If the mass of the balloon and helium is 6 g, calculate the acceleration of the balloon.

Q12. A rope is used to pull a felled tree (mass 50 kg) along the ground. A tension of 1000 N causes the tree to move from rest to a velocity of 0.1 m s^{-1} in 2 s. Calculate the force due to friction acting on the tree.

Q13. Two masses are arranged on a frictionless table as shown in the figure on the right. Calculate:

(**a**) the acceleration of the masses

(**b**) the tension in the string.



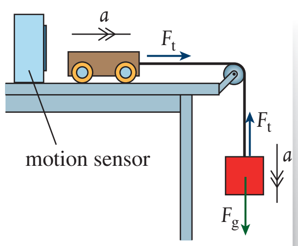
Q14. A helicopter is lifting a load of mass 1000 kg with a rope. The rope is strong enough to hold a force of 12 kN. What is the maximum upward acceleration of the helicopter?

Q15. A person of mass 65 kg is standing in an elevator that is accelerating upwards at 0.5 m s^{-2}.
What is the normal force between the floor and the person?

Q16. A plastic ball is held under the water by a child in a swimming pool. The volume of the ball is 4000 cm^3.

(**a**) If the density of water is 1000 kg m^{-3}, calculate the buoyant force on the ball (buoyant force = weight of fluid displaced).

(**b**) If the mass of the ball is 250 g, calculate the theoretical acceleration of the ball when it is released. Why will the ball not accelerate this quickly in a real situation?



A.2 Figure 36 Apparatus for finding the relationship between force and acceleration.

SKILLS

It is not easy to apply a constant known force to a moving body: just try pulling a cart along the table with a force meter and you will see. One way this is often done in the laboratory is by hanging a mass over the edge of the table as shown in Figure 36.

If we ignore any friction in the pulley or in the wheels of the trolley, then the unbalanced force on the trolley = F_t. Since the mass is accelerating down, then the weight is bigger than F_t so $F_g - F_t = ma$ where m is the mass hanging on the string. The tension is therefore given by: $F_t = mg - ma = m(g - a)$

There are several ways to measure the acceleration of the trolley; one is to use a motion sensor. This senses the position of the trolley by reflecting an ultrasonic pulse off it. Knowing the speed of the pulse, the software can calculate the distance between the trolley and sensor. As the trolley moves away from the sensor, the time taken for the pulse to return increases; the software calculates the velocity from these changing times. Using this apparatus, the acceleration of the trolley for different masses was measured, and the results are given in the Table 1.

A.2 Table 1

Mass/kg ±0.0001	Acceleration/ms^{-2} ±0.03	Tension ($F_t = mg - ma$)/N	Max F_t/N	Min F_t/N	ΔF_t/N
0.0100	0.10	0.097	0.098	0.096	0.001
0.0500	0.74	0.454	0.453	0.451	0.001
0.0600	0.92	0.533	0.532	0.531	0.001
0.1000	1.49	0.832	0.830	0.828	0.001
0.1500	2.12	1.154	1.150	1.148	0.001

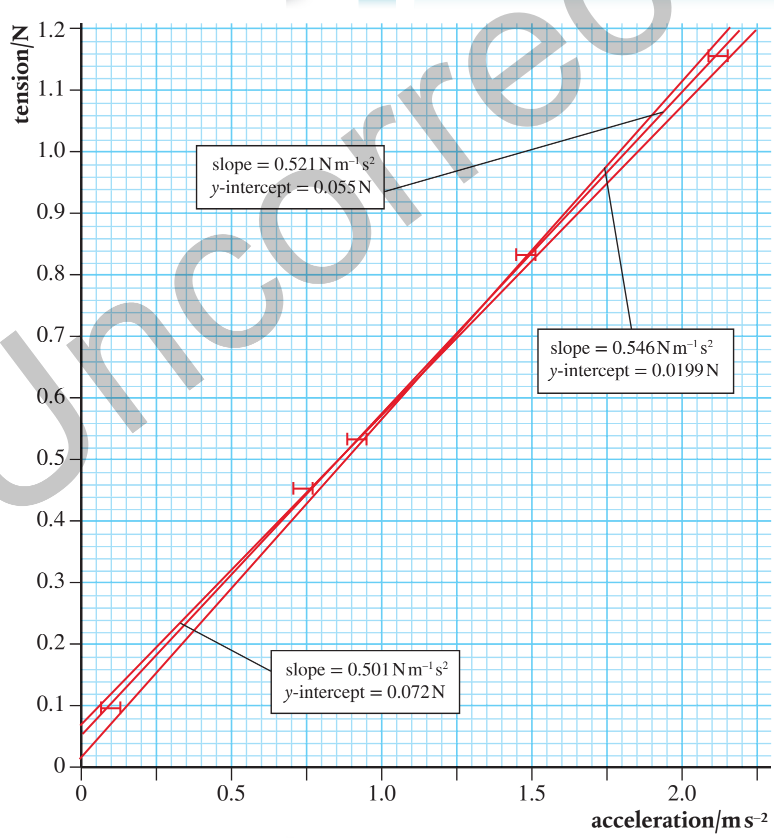
The uncertainty in mass is given by the last decimal place in the scale, and the uncertainty in acceleration by repeating one run several times. To calculate the uncertainty in tension, the maximum and minimum values have been calculated by adding and subtracting the uncertainties.

These results are shown in Figure 37.

Applying Newton's second law to the trolley, the relationship between F_t and a should be $F_t = Ma$ where M is the mass of the trolley. This implies that the gradient of the line should be M. From the graph, we can see that the gradient is 0.52 ± 0.02 kg, which is quite close to the 0.5 kg mass of the trolley.

According to theory, the intercept should be (0, 0) but we can see that there is a positive intercept of 0.05 N. It appears that each value is 0.05 N too big. The reason for this could be friction. If there was friction, then the actual unbalanced force acting on the trolley would be tension – friction. If this is the case, then the results would imply that friction is about 0.05 N.

A.2 Figure 37 Graph of tension against acceleration.



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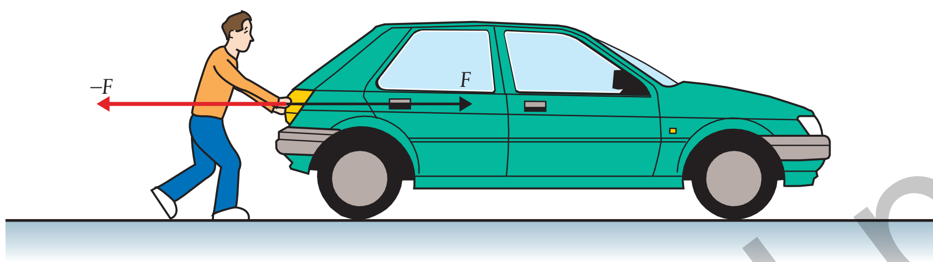
Newton's third law of motion

When dealing with Newton's first and second laws, we are careful to consider only the body that is *experiencing* the forces, not the body that is *exerting* the forces. Newton's third law relates these forces.

If body A exerts a force on body B, then body B will exert an equal and opposite force on body A.

So if someone is pushing a car with a force F as shown in Figure 38, the car will push back on the person with a force $-F$. In this case, both of these forces are normal to the car's surface.



A.2 Figure 38 The man pushes the car and the car pushes the man.

If experimental measurements contain uncertainties, how can laws be developed based on experimental evidence? (NOS)

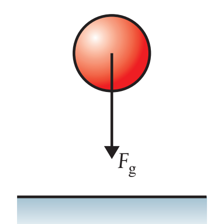
It is very important to realise that Newton's third law is about two bodies. Avoid statements of this law that do not mention anything about there being two bodies.

You might think that, since these forces are equal and opposite, they will be balanced, and, in that case, how does the person get the car moving? This is wrong. The forces act on different bodies so cannot balance each other.

In summary, a Newton's third law pair is made up of two forces of the same type and magnitude acting in opposite directions on different bodies.

Example 1: A falling body

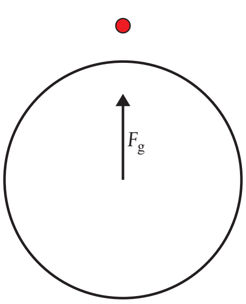
A body falls freely toward the ground as in Figure 39. If we ignore air resistance, there is only one force acting on the body – the force due to the gravitational attraction of the Earth, which we call weight.



A.2 Figure 39 A falling body pulled down by gravity.

Applying Newton's third law

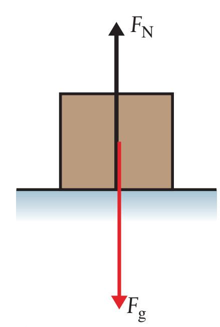
If the Earth pulls the body down, then the body must pull the Earth up with an equal and opposite force. We have seen that the gravitational force always acts on the center of the body, so Newton's third law implies that there must be a force equal to F_g acting upward on the center of the Earth as in Figure 40.



A.2 Figure 40 The Earth pulled up by gravity.

Example 2: A box rests on the floor

A box sits on the floor as shown in Figure 41. Let us apply Newton's third law to this situation.



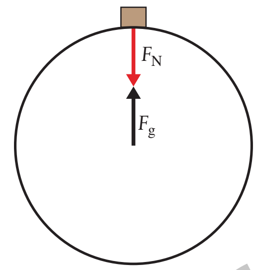
A.2 Figure 41 Forces acting on a box resting on the floor.

There are two forces acting on the box.

Normal force: The floor is pushing up on the box with a force F_N. According to Newton's third law, the box must therefore push down on the floor with a force of magnitude F_N.

Students often think that Newton's third law implies that the normal force = −weight, but *both* of these forces act on the box. If the box is at *rest*, these forces are indeed equal and opposite but this is due to Newton's *first* law.

Weight: The Earth is pulling the box down with a force F_g. According to Newton's third law, the box must be pulling the Earth up with a force of magnitude F_g as shown in Figure 42.



A.2 Figure 42 Forces acting on the Earth according to Newton's third law.

Example 3: Recoil of a gun

When a gun is fired, the velocity of the bullet changes. Newton's first law implies that there must be an unbalanced force on the bullet. This force must come from the gun. Newton's third law says that if the gun exerts a force on the bullet, the bullet must exert an equal and opposite force on the gun. This is the force that makes the gun recoil or 'kick back'.

Example 4: The water cannon

When water is sprayed at a wall from a hosepipe, it hits the wall and stops. Newton's first law says that if the velocity of the water changes, there must be an unbalanced force on the water. This force comes from the wall. Newton's third law says that if the wall exerts a force on the water, then the water will exert a force on the wall. This is the force that makes a water cannon so effective at dispersing demonstrators.

A boat tests its water cannons.

Exercise

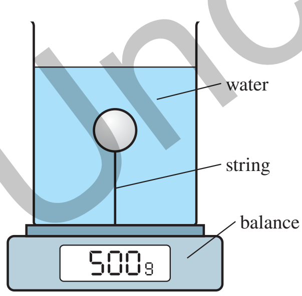
Q17. Use Newton's first and third laws to explain the following:

(**a**) When burning gas is forced downward out of a rocket motor, the rocket accelerates up.

(**b**) When the water cannons on the boat in the photo are operating, the boat accelerates forward.

(**c**) When you step forwards off a skateboard, the skateboard accelerates backward.

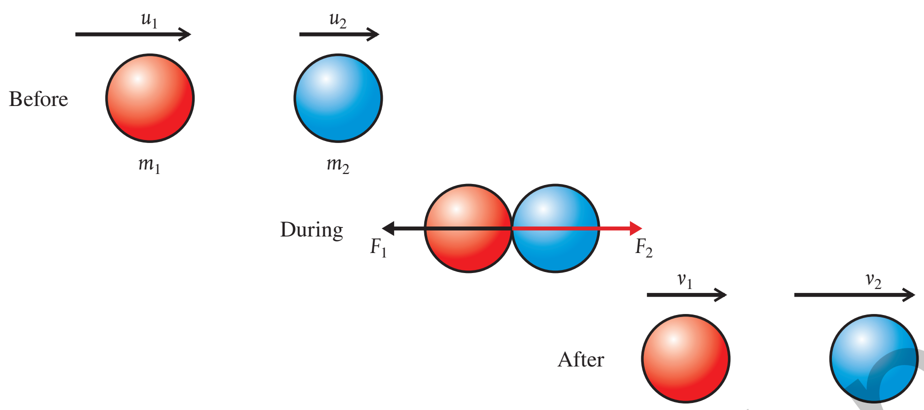
(**d**) A table tennis ball is immersed in a fluid and held down by a string as shown in Figure 43. The container is placed on a balance. What will happen to the reading of the balance if the string breaks?



A.2 Figure 43 A table tennis ball attached to a mass balance

Collisions

In this section, we have been dealing with the interaction between two bodies (gun–bullet, skater–skateboard, hose–water). To develop our understanding of the interaction between bodies, let us consider a simple collision between two balls as illustrated in Figure 44.



A.2 Figure 44 Collision between two balls.

> An isolated system is one in which no external forces are acting. When a ball hits a wall, the momentum of the ball is not conserved because the ball and wall is not an isolated system, since the wall is attached to the ground. If the ball and wall were floating in space, then momentum would be conserved.

Let us apply Newton's three laws to this problem.

Newton's first law

In the collision, the red ball slows down and the blue ball speeds up. Newton's first law tells us that this means there is a force acting to the left on the red ball (F_1) and to the right on the blue ball (F_2).

Newton's second law

This law tells us that the force will be equal to the rate of change of momentum of the balls so if the balls are touching each other for a time Δt:

$$F_1 = \frac{m_1v_1 - m_1u_1}{\Delta t}$$

$$F_2 = \frac{m_2v_2 - m_2u_2}{\Delta t}$$

Newton's third law

According to the third law, if the red ball exerts a force on the blue ball, then the blue ball will exert an equal and opposite force on the red ball.

$$F_1 = -F_2$$

$$\frac{m_1v_1 - m_1u_1}{\Delta t} = \frac{-(m_2v_2 - m_2u_2)}{\Delta t}$$

Rearranging gives: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

In other words, the momentum at the start equals the momentum at the end. We find that this applies not only to this example but to all interactions.

The law of the conservation of momentum

For a system of isolated bodies, the total momentum is always the same.

How are concepts of equilibrium and conservation applied to understand matter and motion from the smallest atom to the whole Universe? (B.3, D.1)

In which way is conservation of momentum relevant to the workings of a nuclear power station? (E.4)

This is not a new law since it is really just a combination of Newton's laws. However, it provides a useful short cut when solving problems.

In many examples, we will have to pretend everything is in space isolated from the rest of the Universe, otherwise they are not isolated and the law of conservation of momentum will not apply.

You may have noticed that some collisions enable the bodies to bounce off one another, while there are other collisions where the bodies stick together. We'll discuss this further in A.3 Work, energy and power.

Nature of Science

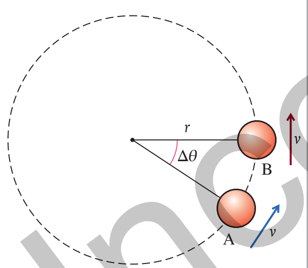
By applying what we know about motion in a straight line, we can develop a model for motion in a circle. This is a common way that models are developed in physics: start simple and add complexity later.

Circular motion

When dealing with circular motion in physics, we always measure the angle in radians.

If a car travels around a bend at 30 km h^{-1}, it is obviously traveling at a constant speed, since the speedometer registers 30 km h^{-1} all the way round. However, it is not traveling at constant velocity. This is because velocity is a vector quantity, and for a vector quantity to be constant, both magnitude and direction must remain the same. Bends in a road can be many different shapes, but, to simplify things, we will only consider circular bends taken at constant speed.

Quantities of circular motion



A.2 Figure 45

Consider the body in Figure 45 traveling in a circle radius r, with constant speed v. In time Δt, the body moves from A to B. As it does this, the radius sweeps out an angle $\Delta\theta$.

When describing motion in a circle, we often use quantities referring to the angular motion rather than the linear motion. These quantities are:

Time period (T)

The time period is the time taken to complete one circle.

The unit of the time period is the second.

Angular displacement (θ)

The angular displacement is the angle swept out by a line joining the body to the center.

The unit of angular displacement is the radian.

Angular velocity (ω)

The angular velocity is the angle swept out by per unit time.

The unit of angular displacement is the radian s^{-1}.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

The angle swept out when the body completes a circle is 2π and the time taken is by definition the time period T so this equation can also be written:

$$\omega = \frac{2\pi}{T}$$

Frequency (f)

The frequency is the number of complete revolutions per unit time.

$$f = \frac{1}{T}$$

So:

$$\omega = 2\pi f$$

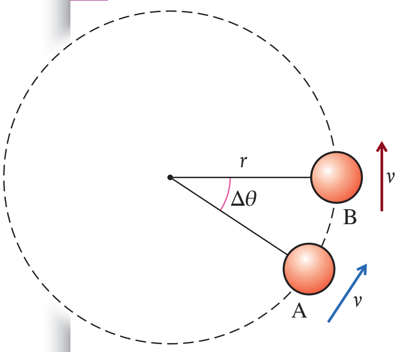
Angular velocity and speed

In a time T, the body in Figure 46 completes one full circle so it travels a distance $2\pi r$, the circumference of the circle. Speed is defined as the $\frac{\text{distance traveled}}{\text{time taken}}$ so $v = \frac{2\pi r}{T}$. In this time, a line joining the body to the center will sweep out an angle of 2π radians so the angular velocity, $\omega = \frac{2\pi}{T}$. Substituting into the equation for v we get:

$$v = \omega r$$

Although the speed is constant, when a body moves in a circle, its direction and velocity are always changing. At any moment in time, the magnitude of the instantaneous velocity is equal to the speed and the direction is perpendicular to the radius of the circle.

Why is no work done on a body moving along a circular trajectory at constant speed? (A.3)

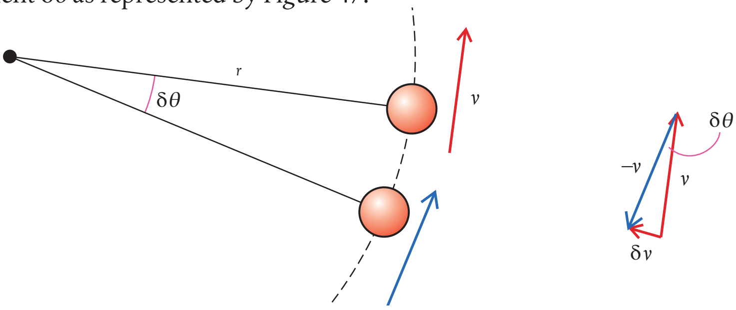


A.2 Figure 46 A body travels at constant speed around a circle of radius *r*.

Centripetal acceleration

From the definition of acceleration, we know that if the velocity of a body changes, it must be accelerating, and that the direction of acceleration is in the direction of the change in velocity. Let us consider a body moving in a circle with a constant speed v. Figure 46 shows two positions of the body separated by a short time.

To derive the equation for this acceleration, let us consider a very small angular displacement $\delta\theta$ as represented by Figure 47.

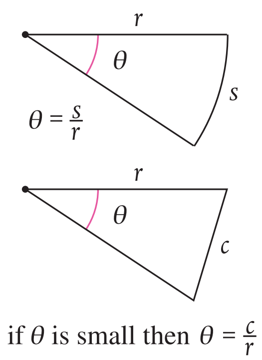


A.2 Figure 47 Angular displacement.

If this small angular displacement has taken place in a short time δt, then the angular velocity, $\omega = \frac{\delta\theta}{\delta t}$.

From the definition of acceleration: $a = \frac{\text{change of velocity}}{\text{time}}$

If we took only the magnitude of velocity, then the change of velocity would be zero. However, velocity is a vector so change in velocity is found by taking the final velocity vector − initial velocity vector as in the vector addition in Figure 48. This triangle is not a right-angled triangle so cannot be solved using Pythagoras. However, since the angle $\delta\theta$ is small, we can say that the angle $\delta\theta$ in radians is approximately equal to $\frac{\delta v}{v}$.



A.2 **Figure 48** The small angle approximation.

Rearranging gives: $\delta v = v\delta\theta$

$$\text{acceleration} = \frac{\delta v}{\delta t} = \frac{v\delta\theta}{\delta t}$$

$$a = v\omega$$

But we know that $v = \omega r$ so we can substitute for v and get $a = \omega^2 r$

Or substituting for: $\omega = \frac{v}{r}$

$$a = \frac{v^2}{r}$$

The direction of this acceleration is in the direction of δv. Now, as the angle $\delta\theta$ is small, the angle between δv and v is approximately 90°, which implies that the acceleration is perpendicular to the velocity. This makes it directed *toward* the center of the circle; hence the name **centripetal acceleration**.

You will not be asked to reproduce this derivation in the exam.

Exercise

Q18. A body travels with constant speed of $2\,\text{m}\,\text{s}^{-1}$ around a circle of radius 5 m. Calculate:

(**a**) the distance moved in one revolution
(**b**) the displacement in one revolution
(**c**) the time taken for one revolution
(**d**) the frequency of the motion
(**e**) the angular velocity
(**f**) the centripetal acceleration.

Centripetal force

From Newton's first law, we know that if a body accelerates, there must be an unbalanced force acting on it. The second law tells us that this force is in the direction of the acceleration. This implies that there must be a resultant force acting toward the center. This force is called the **centripetal force**.

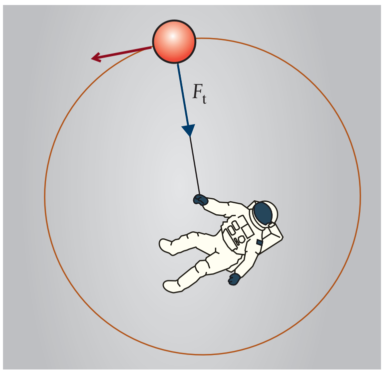
From Newton's second law, we can also deduce that $F = ma$ so $F = \frac{mv^2}{r} = m\omega^2 r$.

$$F = m\omega^2 r$$

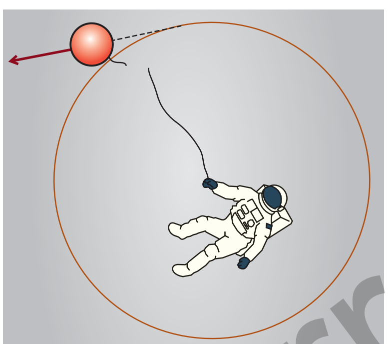
Examples of circular motion

All bodies moving in a circle must be acted upon by a force toward the center of the circle. However, this can be provided by many different forces.

Mass on a string in space



A.2 Figure 49 An astronaut playing with a mass on a string.



A.2 Figure 50 The string breaks.

If you take a mass on the end of a string, you can easily make it move in a circle, but the presence of gravity makes the motion difficult to analyze. It will be simpler if we start by considering what this would be like if performed by an astronaut in deep space: much more difficult to do but easier to analyze. Figure 49 shows an astronaut making a mass move in a circle on the end of a string. The only force acting on the mass is the tension in the string.

In this case, it is obvious that the centripetal force is provided by the tension so:

$$F_t = \frac{mv^2}{r}$$

From this, we can predict that the force required to keep the mass in its circular motion will increase if the speed increases. This will be felt by the astronaut who, according to Newton's third law, must be experiencing an equal and opposite force on the other end of the string. If the string were to break, the ball would have no forces acting on it so would travel at a constant velocity in the same direction as it was moving when the break occurred. This would be at some tangent to the circle as in Figure 50.

How can knowledge of electric and magnetic forces allow the prediction of changes to the motion of charged particles? (D.2)

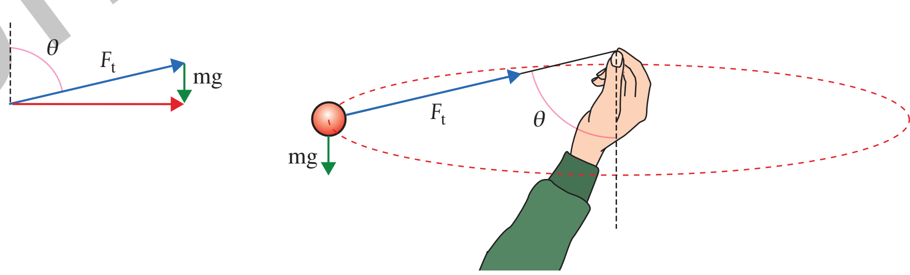
In this example, the astronaut has a much larger mass than the ball. If this was not the case, the astronaut would be pulled out of position by the equal and opposite force acting on the other end of the string.

TOK

People often think that the mass will fly outward if the string breaks. This is because they feel themselves being forced outward so think that if the string breaks, the mass will move in this direction. Applying Newton's laws, we know that this is not the case. This is an example of a case where intuition gives the wrong answer.

Mass on a string on the Earth (horizontal)

When playing with a mass on a string on the Earth, there will be gravity acting as well as tension. We will first consider how this changes the motion when the mass is made to travel in a horizontal circle as in Figure 51.



A.2 Figure 51 A mass swung in a horizontal circle.

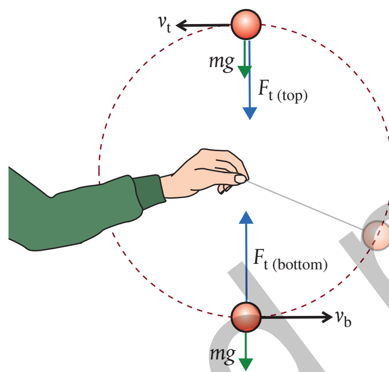
For the motion to be horizontal, there will be no vertical acceleration so the weight must be balanced by the vertical component of tension ($F_t \cos \theta = mg$). This means that the string cannot be horizontal but will always be at an angle, as shown in Figure 51. The centripetal force is provided by the horizontal component of tension ($F_t \sin \theta = \frac{mv^2}{r}$), which is equal to the vector sum of the two forces.

Mass on a string on the Earth (vertical)

As a mass moves in a vertical circle, the force of gravity sometimes acts in the same direction as its motion and sometimes against it. For this reason, it is not possible to keep it moving at a constant speed, so here we will just consider it when it is at the top and the bottom of the circle as shown in Figure 52.

At the top, the centripetal force $\frac{mv_t^2}{r} = F_{t\,(top)} + mg$ so $F_{t\,(top)} = \frac{mv_t^2}{r} - mg$

At the bottom, $\frac{mv_b^2}{r} = F_{t\,(bottom)} - mg$ so $F_{t\,(bottom)} = \frac{mv_b^2}{r} + mg$



A.2 Figure 52 A mass swung in a vertical circle.

When the mass approaches the top of the circle, its kinetic energy is transferred to potential energy, resulting in a loss of speed. If it were to stop at the top, then it would fall straight down. The minimum speed necessary for a complete circle is when the weight of the ball is enough to provide the centripetal force without any tension. So if $F_{t\,(top)} = 0$, then $\frac{mv_t^2}{r} = mg$.

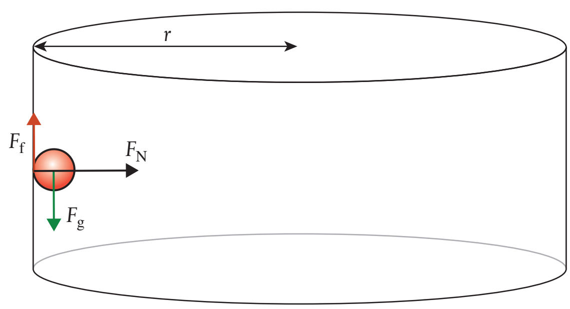
When you rotate a mass in a vertical circle, you definitely feel the change in the tension as it decreases toward the top and increases toward the bottom.

The wall of death

In the wall of death, motorbikes and cars travel around the inside of a cylinder with vertical walls.

▲ Wall of death.

In the wall of death shown in Figure 53, the centripetal force is provided by the normal reaction, F_N. The weight is balanced by the friction between the ball and wall, which is dependent on the normal reaction $F_f = \mu F_N$. If the velocity is too slow, the normal force will be small, which means the friction will not be large enough to support the weight.

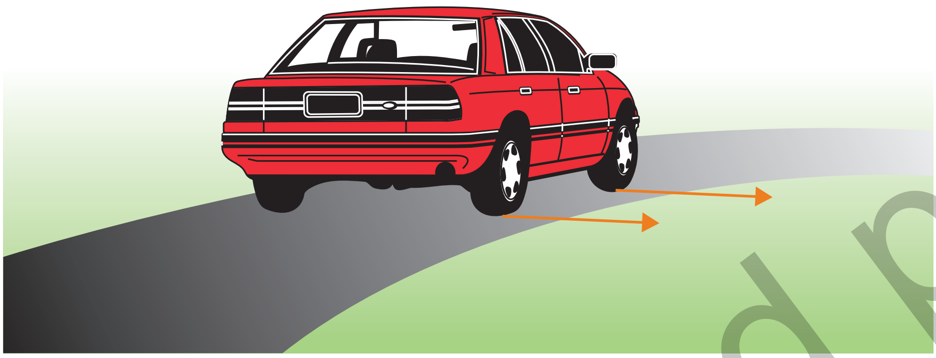


A.2 Figure 53 The wall of death with a ball rather than bike.

Car on a circular track

When a car travels around a circular track, the centripetal force is provided by the friction between the tires and the road. The faster you go, the more friction you need. The problem is that friction has a maximum value given by $F_f = \mu F_N$, so if the centripetal force required is greater than this, the car will not be able to maintain a circular path. Without friction, for example, on an icy road, the car would travel in a straight line. This means that you would hit the kerb on the outside of the circle, giving the impression that you have been thrown outward. This is of course not the case since there is no force acting outward.

Dynamic friction is less than static friction so once a car starts to skid on a corner it will continue. This is also why it is not a good idea to spin the wheels of a car while going round a corner.



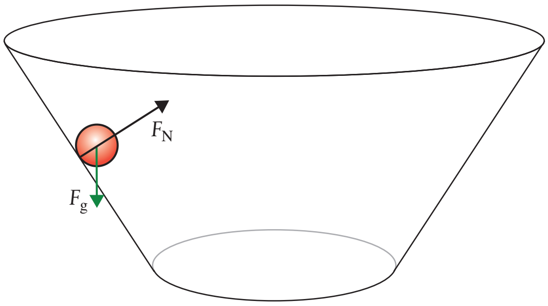
A.2 Figure 54 A car rounding a bend.

Cyclist on a banked track

A banked track is a track that is angled to make it possible to go faster around the bends. These are used in indoor cycle racing. In the case shown in Figure 55, where the bike is represented by a ball, the centripetal force is provided by the horizontal component of the normal reaction force, so even without friction, the ball can travel around the track. If the track was angled the other way, then it would have the opposite effect. This is called an adverse camber and bends like this should be taken slowly.

A racing cyclist on a banked track in a velodrome.

Remember when solving circular motion problems, centripetal force is not an extra force – it is one of the existing forces. Your task is to find which force (or a component of it) points toward the center.



A.2 Figure 55 A ball on a banked track.

Exercise

Q19. Calculate the centripetal force for a 1000 kg car traveling around a circular track of radius 50 m at 30 km h^{-1}.

Q20. A 200 g ball is made to travel in a circle of radius 1 m on the end of a string. If the maximum force that the string can withstand before breaking is 50 N, what is the maximum speed of the ball?

Q21. A rollercoaster is designed with a 5 m radius vertical loop. Calculate the minimum speed necessary to get around the loop without falling down.

Q22. A 200 g ball moves in a vertical circle on the end of a 50 cm long string. If its speed at the bottom is 10 m s^{-1}, calculate:

(**a**) the velocity at the top of the circle

(**b**) the tension at the top of the circle.

Challenge yourself

1. A car of mass 1000 kg is driving around a circular track with radius 50 m. If the coefficient of friction between the tires and road is 0.8, calculate the maximum speed of the car before it starts to slip. What would the maximum speed be if the track was banked at 45°?

Guiding Questions revisited

How can we use our knowledge and understanding of the forces acting on a system to predict changes in translational motion?

How can Newton's laws be modeled mathematically?

How can the conservation of momentum be used to predict the behavior of interacting objects?

In this chapter, we have considered new quantities and accepted laws to describe how:

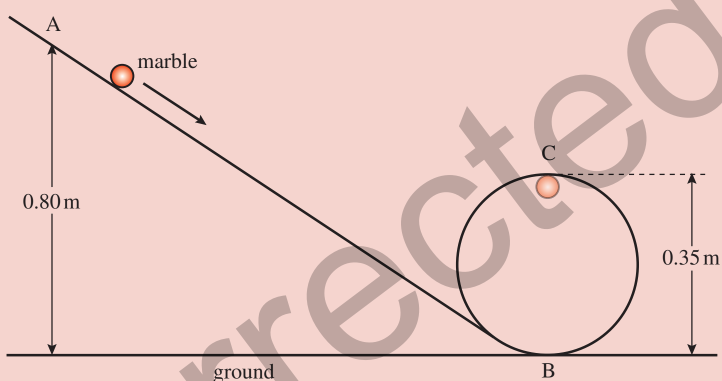
- Different types of force can be distinguished from one another.
- Forces are vector quantities that can be represented by arrows of appropriate length and direction on free-body diagrams (in which the forces acting on, rather than exerted by, a given body are shown).
- Newton's first law states that an object will remain at constant velocity unless acted upon by a resultant force.
- Newton's second law states that the rate of change of momentum is proportional to and in the same direction as the resultant force.
- A resultant force acting at an angle to a body's velocity leads to a change in direction and, when the force and velocity are perpendicular, circular motion.
- Linear acceleration and centripetal acceleration are both defined as the rate of change of velocity, but are calculated using different equations.
- Newton's third law states that when body A exerts a force on body B, body B exerts an equal and opposite force (of the same type) on body A.

- Momentum, the product of mass and velocity, is conserved in the absence of external forces, which means that the initial momentum and final momentum of a system can be equated.
- When objects interact, they exert forces on one another, which means that momentum is exchanged between them.
- Newton's second law can be rephrased mathematically as impulse, the product of force acting and the time of the interaction, which is equal to change in momentum.

Practice questions

1. **(a)** A car goes round a curve in a road at constant speed. Explain why, although its speed is constant, it is accelerating. (2)

In the diagram, a marble (small glass sphere) rolls down a track, the bottom part of which has been bent into a loop. The end A of the track, from which the marble is released, is at a height of 0.80 m above the ground. Point B is the lowest point and point C the highest point of the loop. The diameter of the loop is 0.35 m.



The mass of the marble is 0.050 kg. Friction forces and any gain in kinetic energy due to the rotating of the marble can be ignored. The acceleration due to gravity, $g = 10\,\text{m}\,\text{s}^{-2}$.

Consider the marble when it is at point C.

(b) **(i)** Copy the diagram and on the diagram, draw an arrow to show the direction of the resultant force acting on the marble. (1)

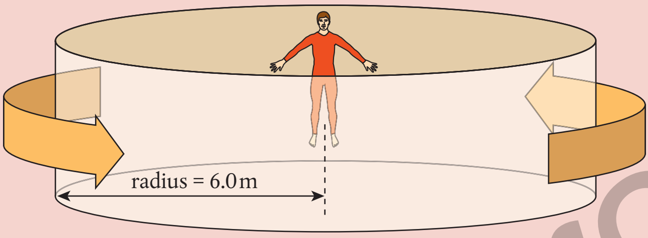
(ii) State the names of the **two** forces acting on the marble. (2)

(iii) Deduce that the speed of the marble is $3.0\,\text{m}\,\text{s}^{-1}$. (3)

(iv) Determine the resultant force acting on the marble and hence determine the reaction force of the track on the marble. (4)

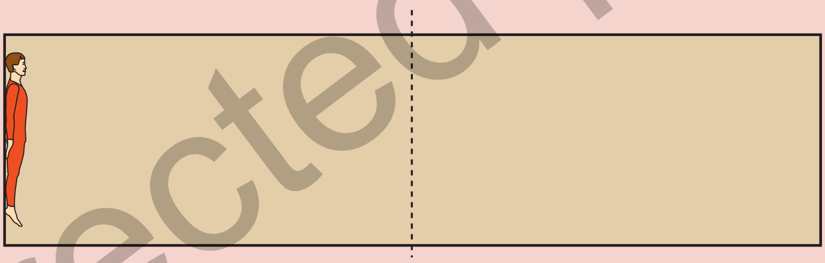
2. **(a)** Define what is meant by **coefficient of friction**. (1)

The diagram shows a particular ride at a funfair (sometimes called 'the fly') that involves a spinning circular room. When it is spinning fast enough, a person in the room feels 'stuck' to the wall. The floor is lowered and they remain held in place on the wall. Friction prevents the person from falling.



(b) **(i)** Explain whether the friction acting on the person is static, dynamic, **or** a combination of both. (2)

The diagram below shows a cross section of the ride when the floor has been lowered.



(ii) Copy the diagram and, on your diagram, draw labeled arrows to represent the forces acting on the person. (3)

(c) Use the data given below:

mass of person = 80 kg

coefficient of friction between the person and the wall = 0.40

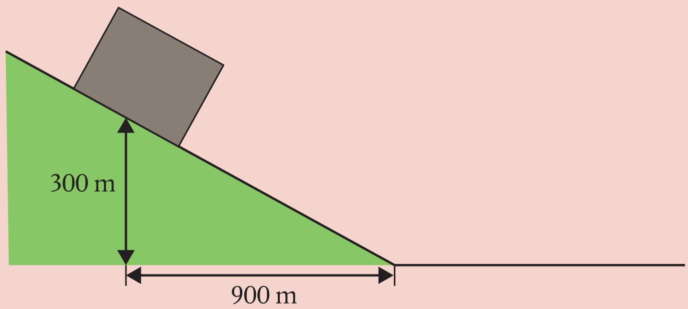
radius of circular room = 6.0 m

Calculate each of the following:

(i) the magnitude of the minimum resultant horizontal force on the person (2)

(ii) the minimum speed of the wall for a person to be 'stuck' to it. (2)

3. A 10 000 kg cubic rock (a boulder) rests on the side of a mountain as shown in the diagram below.



(a) Calculate the frictional force acting on the rock. (3)

(b) After a prolonged period of rain, the rock starts to slide down the slope. Show that this will happen when the coefficient of friction between the rock and the slope is equal to 0.33. (3)

(c) Once the block starts to slide, the coefficient of friction is reduced to 0.2. Calculate:

(i) the acceleration of the rock (2)

(ii) the speed of the rock when it reaches the bottom of the hill. (2)

(d) The rock then slides along the flat section of ground. Assuming there is no change in the coefficient of friction calculate:

(i) the frictional force on the rock (1)

(ii) the distance traveled by the rock. (2)

4. A student investigates the forces involved in holding a climbing rope by standing on bathroom scales while holding a rope as shown in the diagram.

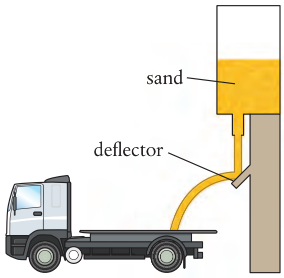
(a) If both students have mass 60 kg, calculate:

(i) the tension in the rope (1)

(ii) the reading on the scales (2)

(iii) the friction between the rope holder's feet and the scales. (2)

(b) Explain why the rope-holding student should lower the hanging student smoothly. (2)



5. Sand flows out of a container at a rate of $5\,\text{kg s}^{-1}$ and falls a vertical distance of 1 m, where it is deflected into the back of a truck by a deflector placed at an angle of 45°.

(**a**) Calculate:

(**i**) the velocity of the sand as it hits the deflector (1)

(**ii**) the horizontal velocity of the sand after hitting the deflector (1)

(**iii**) the force exerted by the sand on the deflector. (3)

(**b**) The sand stops when it hits the back of the truck.

(**i**) Explain why there must be a frictional force between the tires and the road. (3)

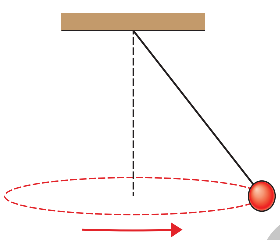
(**ii**) Calculate the force of friction between the truck and the ground. (2)

6. Two forces act on an object in different directions. The magnitudes of the forces are 18 N and 27 N. The mass of the object is 9.0 kg. What is a possible value for the acceleration of the object? (1)

A $0\,\text{m s}^{-2}$ **B** $0.5\,\text{m s}^{-2}$ **C** $2.0\,\text{m s}^{-2}$ **D** $6.0\,\text{m s}^{-2}$

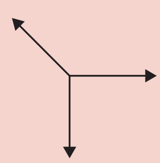
7. An object of mass *m* strikes a vertical wall horizontally at speed *U*. The object rebounds from the wall horizontally at speed *V*. What is the magnitude of the change in the momentum of the object? (1)

A 0 **B** $m(V - U)$ **C** $m(U - V)$ **D** $m(U + V)$

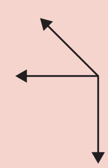


8. A sphere is suspended from the end of a string and rotates in a horizontal circle. Which free-body diagram, to the correct scale, shows the forces acting on the sphere? (1)

A



B



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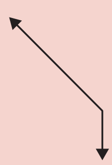

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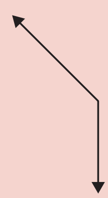
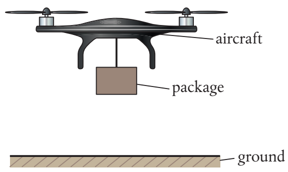


9. A company delivers packages to customers using a small unmanned aircraft. Rotating horizontal blades exert a force on the surrounding air. The air above the aircraft is initially stationary.

The air is propelled vertically downward with speed v. The aircraft hovers motionless above the ground. A package is suspended from the aircraft on a string. The mass of the aircraft is 0.95 kg and the combined mass of the package and string is 0.45 kg. The mass of air pushed downward by the blades in one second is 1.7 kg.



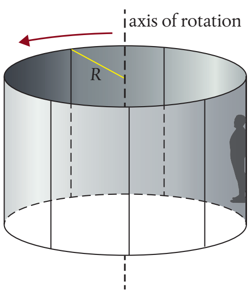
(**a**) State the value of the resultant force on the aircraft when hovering. (1)

(**b**) Outline, by reference to Newton's third law, how the upward lift force on the aircraft is achieved. (2)

(**c**) Determine v. State your answer to an appropriate number of significant figures. (3)

(**d**) The package and string are now released and fall to the ground. The lift force on the aircraft remains unchanged. Calculate the initial acceleration of the aircraft. (2)

10. The Rotor is an amusement park ride that can be modeled as a vertical cylinder of inner radius R rotating about its axis. When the cylinder rotates fast enough, the floor drops out and the passengers stay motionless against the inner surface of the cylinder. The diagram shows a person taking the Rotor ride. The floor of the Rotor has been lowered away from the person.



(**a**) Draw and label the free-body diagram for the person. (2)

(**b**) The person must not slide down the wall. Show that the minimum angular velocity of the cylinder for this situation is:

$$\omega = \sqrt{\frac{g}{\mu R}}$$

where μ is the coefficient of static friction between the person and the cylinder. (2)

(**c**) The coefficient of static friction between the person and the cylinder is 0.40. The radius of the cylinder is 3.5 m. The cylinder makes 28 revolutions per minute. Deduce whether the person will slide down the inner surface of the cylinder. (3)

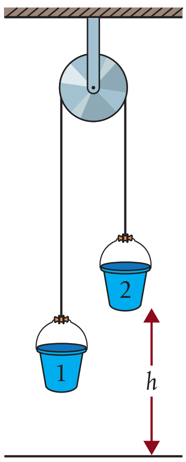
11. A boulder is many times heavier than a pebble; that is, the gravitational force that acts on a boulder is many times that which acts on the pebble. If you drop a boulder and a pebble at the same time, they will fall together with equal accelerations (neglecting air resistance). The principal reason the heavier boulder does not accelerate more than the pebble has to do with what? (1)

A Energy **B** Weight **C** Mass

D Surface area **E** none of these

12. The force exerted on a house by a 120 mph hurricane wind is how many times as strong as the force exerted on the same house by a 60 mph gale wind? (1)

A Equally **B** Two times **C** Three times **D** Four times



13. Two buckets hang from a rope over a frictionless pulley. The bucket on the right has a mass m_2, which is greater than the mass of the bucket on the left m_1. Bucket 2 starts at height h above the ground. If the buckets are released from rest, determine:

(**a**) the speed with which bucket 2 hits the ground in terms of m_1, m_2, h, and the acceleration due to gravity g (2)

(**b**) the further increase in height of bucket 1 after bucket 2 hits the ground and stops. (2)

Ignore resistive effects and assume the rope is long compared to the height above the ground.

A.3 Work, energy and power

◀ A Rube Goldberg machine starts with a small action (like the pressing of a button or the knocking over of a domino tile) that sets off a chain reaction of different events, which continues for an extended time. Objects that are stretched, at a height, or moving, combine – often with entertaining consequences (as seen in films, music videos, advertisements and social media).

Guiding Questions

How are concepts of work, energy and power used to predict changes within a system?

How can a consideration of energetics be used as a method to solve problems in kinematics?

How can transfer of energy be used to do work?

If body A pushes body B, body B may start to move. This movement might cause body B to hit body C, after which, body C moves toward body D (you get the idea). The 'ability to push' seems to be passed on from one object to another. We call this energy, and the fact that it is conserved can lead to a simple way of solving problems that bypasses all the complications of forces and motion.

For example, when calculating the final velocity of a box pushed up a slope by a constant force, we need to find the component of force up the slope, use that to find acceleration, and then apply the *suvat* equations to calculate final velocity. Or, using conservation of energy, we can state that the gain in potential energy and kinetic energy is equal to the work done.

It is the same when we study gases. It is much easier to understand how work done on a gas increases its temperature than to model the motion of each molecule.

Students should understand:

the principle of the conservation of energy
work done by a force is equivalent to a transfer of energy
energy transfers can be represented on a Sankey diagram
work W done on a body by a constant force depends on the component of the force along the line of displacement as given by $W = Fs\cos\theta$
work done by the resultant force on a system is equal to the change in the energy of the system
mechanical energy is the sum of kinetic energy, gravitational potential energy and elastic potential energy
in the absence of frictional, resistive forces, the total mechanical energy of a system is conserved
if mechanical energy is conserved, work is the amount of energy transformed between different forms of mechanical energy in a system, such as: • the kinetic energy of translational motion as given by $E_k = \frac{1}{2}mv^2 = \frac{p^2}{2m}$ • the gravitational potential energy, when close to the surface of the Earth, as given by: $\Delta E_p = mg\Delta h$ • the elastic potential energy as given by $E_H = \frac{1}{2}k\Delta x^2$

power P is the rate of work done, or the rate of energy transfer, as given by $P = \frac{E}{t} = Fv$
efficiency η in terms of energy transfer or power as given by $\eta = \frac{E_{output}}{E_{input}} = \frac{P_{output}}{P_{input}}$
energy density of the fuel sources.

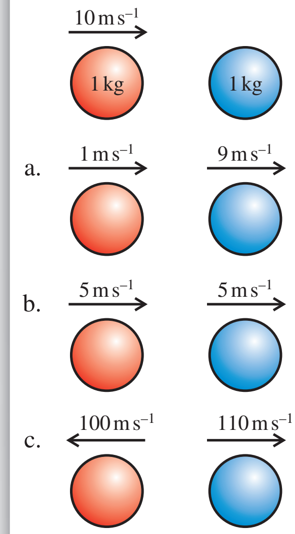
Nature of Science

Scientists should remain sceptical but that does not mean you have to doubt everything you read. The law of conservation of energy is supported by many experiments and is the basis of countless predictions that turn out to be true. If someone now found that energy was not conserved, there would be a lot of explaining to do. Once a law is accepted, it gives us an easy way to make predictions. For example, if you are shown a device that produces energy from nowhere, you know it must be a fake without even finding out how it works because it violates the law of conservation of energy.

In A.1 and A.2, we dealt with the motion of a small red ball and now understand what causes acceleration. We have also investigated the interaction between a red ball and a blue one and have seen that the red one can cause the blue one to move when they collide. But what enables the red one to push the blue one? To answer this question, we need to define some more quantities.

Work

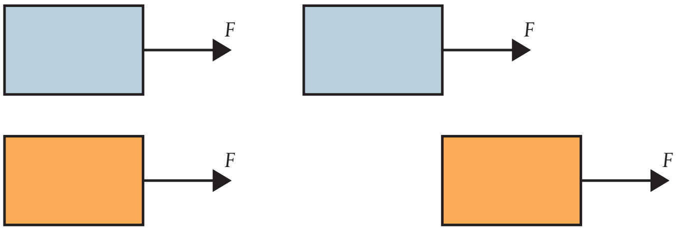
In the introduction to this book, it was stated that by developing models, our aim is to understand the physical world so that we can make predictions. At this point, you should understand certain concepts related to the collision between two balls, but we still cannot predict the outcome. To illustrate this point, let us again consider the red and blue balls. Figure 1 shows three possible outcomes of the collision.



A.3 Figure 1 The red ball hits the blue ball but what happens?

If we apply the law of conservation of momentum, we realize that all three outcomes are possible. The original momentum is 10 N s and the final momentum is 10 N s in all three cases. But which one actually happens? This we cannot say (yet). All we know is that from experience the last option is not possible – but why?

When body A hits body B, body A exerts a force on body B. This force causes B to have an increase in velocity. The amount that the velocity increases depends on how big the force is and over what distance the collision takes place. To make this simpler, consider a constant force acting on two blocks as in Figure 2.



A.3 Figure 2 The force acts on the orange block for a greater distance

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Both blocks start at rest and are pulled by the same force, but the orange block will gain more velocity because the force acts over a longer distance. To quantify this difference, we say that in the case of the orange block, the force has done more work. Work is done when the point of application of a force moves in the direction of the force.

Work is defined in the following way:

$$\textit{work done} = \textit{force} \times \textit{distance moved in the direction of the force}$$

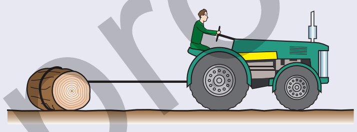
The unit of work is the newton meter (N m), which is the same as the joule (J).

Work is a scalar quantity.

How do traveling waves allow for a transfer of energy without a resultant displacement of matter? (C.2, A.1)

Worked example

A tractor pulls a felled tree along the ground for a distance of 200 m. If the tractor exerts a force of 5000 N, how much work will be done?



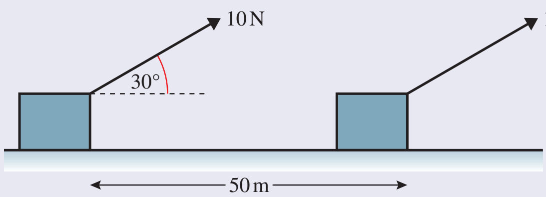
Solution

work done = force × distance moved in direction of force

$$\text{work done} = 5000 \times 200 = 1\ \text{MJ}$$

Worked example

A force of 10 N is applied to a block, pulling it 50 m along the ground as shown. How much work is done by the force?



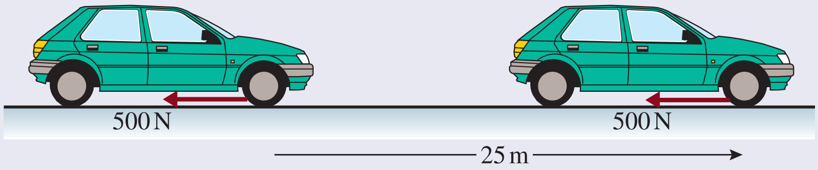
Solution

In this example, the force is not in the same direction as the movement. However, the horizontal component of the force is 10 × cos 30°.

$$\text{work done} = 10 \times \cos 30° \times 50 = 433\ \text{N}$$

Worked example

When a car brakes, it slows down due to the friction force between the tires and the road. This force opposes the motion as shown. If the friction force is a constant 500 N and the car comes to rest in 25 m, how much work is done by the friction force?



Solution

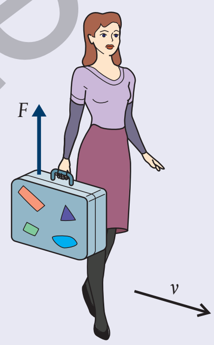
This time, the force is in the opposite direction to the motion.

$$\text{work done} = -500 \times 25 = -12\,500\,\text{J}$$

The negative sign tells us that the car's kinetic energy is decreasing. The internal energy of the brake disks has increased; positive work has been done on them by friction.

Worked example

The woman in the figure walks along with a constant velocity holding a suitcase.
How much work is done by the force holding the case?



Solution

In this example, the force is acting perpendicular to the direction of motion, so there is no movement in the direction of the force.

$$\text{work done} = \text{zero}$$

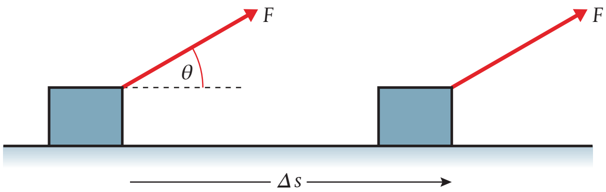
It may seem strange that when you carry a heavy bag you are not doing any work – that is not what it feels like. In reality, lots of work is being done, since to hold the bag you use your muscles. Muscles are made of microscopic fibres, which are continuously contracting and relaxing, so are doing work.

General formula

In general:

$$\text{work} = F\cos\theta \times \Delta s$$

where θ is the angle between the displacement, Δs, and force, F3.

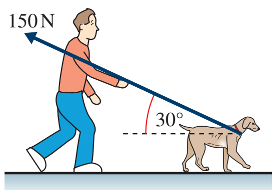


All the previous examples can be solved using this formula.

If $\theta < 90°$, $\cos\theta$ is positive so the work is positive.

If $\theta = 90°$, $\cos\theta = 0$ so the work is zero.

If $\theta > 90°$, $\cos\theta$ is negative so the work is negative.



Exercise

Q1. The figure shows a boy taking a dog for a walk.

(**a**) Calculate the work done by the force shown when the dog moves 10 m forward.

(**b**) Who is doing the work?

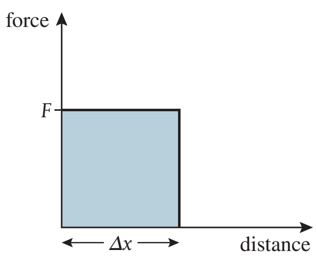
Q2. A bird weighing 200 g sits on a tree branch. How much work does the bird do on the tree?

Q3. As a box slides along the floor, it is slowed down by a constant force due to friction. If this force is 150 N and the box slides for 2 m, how much work is done against the frictional force?

Graphical method for determining work done

Let us consider a constant force acting in the direction of movement pulling a body a distance Δx. The graph of force against distance for this example is as shown in Figure 3. From the definition of work, we know that work done = $F\Delta x$, which in this case is the area under the graph. From this we can deduce that:

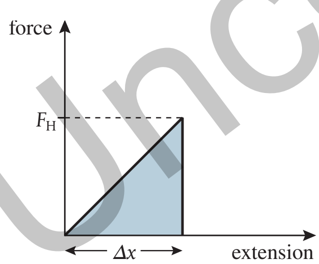
work done = area under force vs distance graph



A.3 Figure 3 Force vs distance for a constant force.

Work done by a varying force

Stretching a spring is a common example of a varying force. When you stretch a spring, it gets more and more difficult the longer it gets. Within certain limits, the force needed to stretch the spring is directly proportional to the extension of the spring: $F_H = kx$. This was first recognized by Robert Hooke in 1676, so is named Hooke's law. If we add different weights to a spring, the more weight we add, the longer it gets. If we draw a graph of force against distance as we stretch a spring, it will look like the graph in Figure 4. The gradient of this line, $\frac{F_H}{\Delta x}$ is called the spring constant, k.



A.3 Figure 4 Force vs extension for a spring.

The work done as the spring is stretched is found by calculating the area under the graph:

$$\text{area} = \tfrac{1}{2}\text{base} \times \text{height} = \tfrac{1}{2}F_H\Delta x$$

So: $$\text{work done} = \tfrac{1}{2}F_H\Delta x$$

But if: $$\frac{F_H}{\Delta x} = k \text{ then } F_H = k\Delta x$$

Substituting for F_H gives:

$$\text{work done} = \frac{1}{2}k\Delta x^2$$

This is equal to the elastic potential energy, E_H

Exercise

Q4. A spring of spring constant $2\,\text{N cm}^{-1}$ and length 6 cm is stretched to a new length of 8 cm.

(**a**) How far has the spring been stretched?

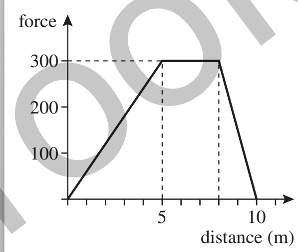
(**b**) What force will be needed to hold the spring at this length?

(**c**) Sketch a graph of force against extension for this spring.

(**d**) Calculate the work done in stretching the spring.

(**e**) The spring is now stretched a further 2 cm. Draw a line on your graph to represent this and calculate how much additional work has been done.

Q5. Calculate the work done by the force represented by the figure on the right.



Energy

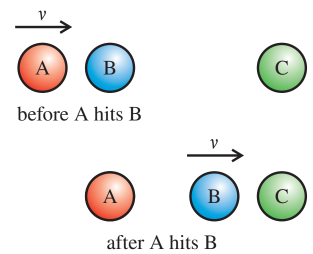
We have seen that it is sometimes possible for body A to do work on body B, but what does A have that enables it to do work on B? To answer this question, we must define a new quantity: energy.

Energy is the quantity that enables body A to do work on body B.

If body A collides with body B as shown in Figure 5, body A has done work on body B. This means that body B can now do work on body C. Energy has been transferred from A to B.

When body A does work on body B, energy is transferred from body A to body B.

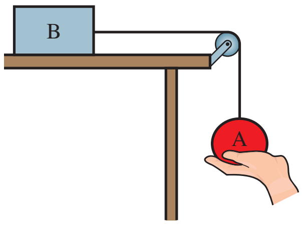
The unit of energy is the joule (J). Energy is a scalar quantity.



A.3 Figure 5 The red ball gives energy to the blue ball.

Different types of energy

If a body can do work, then it has energy. There are two ways that a simple body such as a red ball can do work. In the example above, body A could do work because it was moving – this is called **kinetic energy**. Figure 6 shows an example where A can do work even though it is not moving. In this example, body A is able to do work on body B because of its position above the Earth. If the hand is removed, body A will be pulled downward by the force of gravity, and the string attached to it will then drag body B along the table. If a body is able to do work because of its position, we say it has **potential energy**.



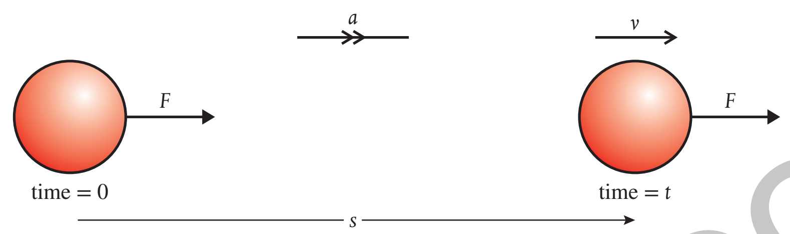
A.3 Figure 6 A has potential energy that could become kinetic energy

TOK

If we say a body has potential energy, it sounds as though it has the potential to do work. This is true, but a body that is moving has the potential to do work too. This can lead to misunderstanding. It might have been better to call it positional energy.

Kinetic energy (E_k)

This is the energy a body has due to its movement. To give a body kinetic energy, work must be done on the body. The amount of work done will be equal to the increase in kinetic energy. If a constant force acts on a red ball of mass m as shown in Figure 7, then the work done is Fs.



A.3 Figure 7 A ball gains kinetic energy

From Newton's second law, we know that $F = ma$, which we can substitute in work = Fs to give work = mas.

We also know that since acceleration is constant, we can use the *suvat* equation $v^2 = u^2 + 2as$, which since $u = 0$ simplifies to $v^2 = 2as$.

Rearranging this gives: $as = \frac{v^2}{2}$ so work = $\frac{1}{2}mv^2$

This work has increased the kinetic energy of the body so we can deduce that:

$$E_k = \frac{1}{2}mv^2$$

In this section, we only deal with examples of potential energy due to a body's position close to the Earth. However, there are other positions that will enable a body to do work (for example, in an electric field). These will be introduced after the concept of fields has been introduced.

Gravitational potential energy (E_p)

This is the energy a body has due to its position above the Earth.

For a body to have potential energy, it must have at some time been lifted to that position. The amount of work done in lifting it equals the potential energy. Taking the example shown in Figure 9, the work done in lifting the mass, m, to a height h is mgh (this assumes that the body is moving at a constant velocity so the lifting force and weight are balanced).

If work is done on the body then energy is transferred so:

$$\text{gain in } E_p = mgh$$

What happens to the energy transferred as we approach the speed of light? (A.5)

Why is the equation for the change in gravitational potential energy only relevant close to the surface of the Earth, and what happens when moving further away from the surface? (D.1)

How are Kirchhoff's and Lenz's laws a consequence of the law of conservation of energy? (B.5, D.4)

The law of conservation of energy

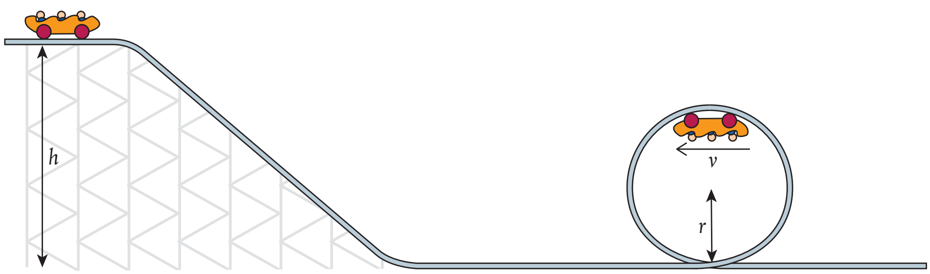
We could not have derived the equations for kinetic energy or potential energy without assuming that the work done is the same as the gain in energy. The law of conservation of energy is a formal statement of this fact.

Energy can neither be created nor destroyed; it can only be transferred from one store to another.

This law is one of the most important laws that we use in physics. If it were not true, you could suddenly find yourself at the top of the stairs without having done any work in climbing them, or a car suddenly has a speed of 200 km h^{-1} without anyone touching the accelerator pedal. These things just do not happen, so the laws we use to describe the physical world should reflect that.

Looping the loop

When looping the loop on a rollercoaster, the situation is very similar to the vertical circle example, except that the tension is replaced by the normal reaction force. This also gives a minimum speed at the top of the loop when $\frac{mv_t^2}{r} = F_g$.



A.3 Figure 8 Looping the loop.

If the ride is propelled by gravity, then the designer must make sure that the car has this minimum speed when it reaches the top.

Applying the law of conservation of energy to the car in Figure 8, if no energy is lost, then the E_p at the top of the hill = $E_p + E_k$ at the top of the loop.

The minimum speed to complete the loop is $mg = \frac{mv^2}{r}$ so at the top of the loop:

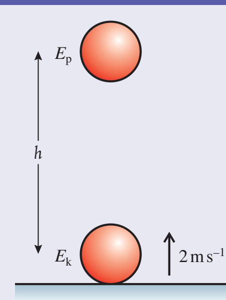
$$\frac{1}{2}mv^2 = \frac{1}{2}mgr$$

The height of the car at the top of the loop is $2r$ so $E_p = mgh = mg2r$.

So E_p at top of slope = $2mgr + \frac{1}{2}mgr = 2.5mgr$, which means the height of the slope = $2.5r$. In any real situation, there will be energy lost due to work done against friction and air resistance so the slope will have to be a bit higher.

Worked example

A ball of mass 200 g is thrown vertically upward with a velocity of 2 m s^{-1} as shown in the figure. Use the law of conservation of energy to calculate its maximum height.



A.3 Figure 9

Solution

At the start of its motion, the body has kinetic energy. This enables the body to do work against gravity as the ball travels upward. When the ball reaches the top, all the kinetic energy has been transferred to potential energy. So applying the law of conservation of energy:

$$\text{loss of } E_k = \text{gain in } E_p$$

$$\frac{1}{2}mv^2 = mgh$$

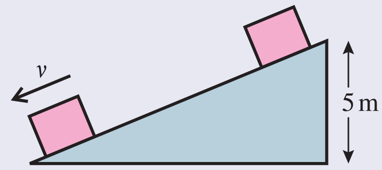
So:

$$h = \frac{v^2}{2g} = \frac{2^2}{2 \times 10} = 0.2\,\text{m}$$

This is exactly the same answer you would get by calculating the acceleration from $F = ma$ and using the *suvat* equations.

Worked example

A block slides down the frictionless ramp shown in the figure. Use the law of conservation of energy to find its speed when it gets to the bottom.



Solution

This time the body loses potential energy and gains kinetic energy so applying the law of conservation of energy:

$$\text{loss of } E_p = \text{gain of } E_k$$

$$mgh = \frac{1}{2}mv^2$$

So: $$v = \sqrt{2gh} = \sqrt{(2 \times 10 \times 5)} = 10\,\text{m}\,\text{s}^{-1}$$

Again, this is a much simpler way of getting the answer than using components of the forces.

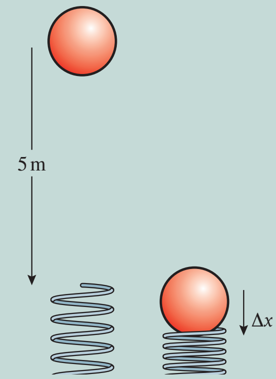
In this example, the spring is compressed not stretched, but Hooke's law still applies.

Exercise

Use the law of conservation of energy to solve the following:

Q6. A stone of mass 500 g is thrown off the top of a cliff with a speed of $5\,\text{m}\,\text{s}^{-1}$. If the cliff is 50 m high, what is its speed just before it hits the ground?

Q7. A ball of mass 250 g is dropped 5 m onto a spring as shown in the figure on the right.



(**a**) How much kinetic energy will the ball have when it hits the spring?

(**b**) How much work will be done as the spring is compressed?

(**c**) If the spring constant is $250\,\text{kN}\,\text{m}^{-1}$, calculate how far the spring will be compressed.

Q8. A ball of mass 100 g is hit vertically upward with a bat. The bat exerts a constant force of 15 N on the ball and is in contact with it for a distance of 5 cm.

(**a**) How much work does the bat do on the ball?

(**b**) How high will the ball go?

Q9. A child pushes a toy car of mass 200 g up a slope. The car has a speed of $2\,\text{m}\,\text{s}^{-1}$ at the bottom of the slope.

(**a**) How high up the slope will the car go?

(**b**) If the speed of the car were doubled, how high would it go now?

Stores of energy

When we are describing the motion of simple red balls, there are only two stores of energy, kinetic energy and potential energy. However, when we start to look at more complicated systems, we discover that we can do work using a variety of different machines, such as petrol engine, electric engine, etc. To do work, these machines must be given energy and this can come from many stores, for example:

- petrol
- solar
- gas
- nuclear
- electricity

All of these (except solar) are related to either the kinetic energy or potential energy of particles.

Where do the laws of conservation apply in other areas of physics? (NOS)

How is the equilibrium state of a system, such as the Earth's atmosphere or a star, determined? (B.2, E.5)

Fuels

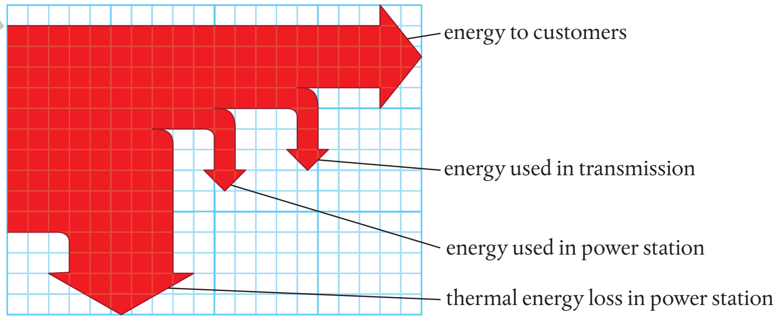
A petrol car gets its energy from petrol, which is mixed with air in the engine and ignited by a spark, causing it to explode. The explosion pushes up a piston that turns a crank, which converts the linear motion of the piston into rotation of the wheels. Petrol is an example of a fuel – a chemical that can be burned to produce heat, which can be used to enable an engine to do work.

Different fuels contain different amounts of energy. Physicists compare the energy contents of fuels in terms of the energy per unit volume (energy density).

Fuel	Energy density/MJ L^{-1}
Crude oil	37
Vegetable oil	30
Diesel	36
Coal	72
Sugar	26
Wood	3

A.3 Table 1

During the process of burning and moving parts of the engine, some energy is lost. The flow of energy can be represented on a Sankey diagram. The width of an arrow represents the amount of that type of energy and, because the total width of all output arrows is equal to the input width, conservation of energy is displayed.

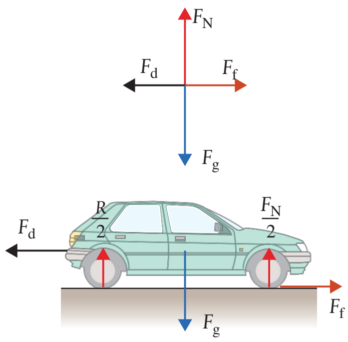


A.3 Figure 10 A Sankey diagram for a coal-fired power station.

Energy transfer

Taking the example of a petrol engine, the energy stored in the petrol is transferred to mechanical energy of the car by the engine. 1 liter of petrol contains 36 MJ of energy. Let us calculate how far a car could travel at a constant 36 km h^{-1} on 1 liter of fuel; that's pretty slow but 36 km h^{-1} is 10 m s^{-1} so it will make the calculation easier.

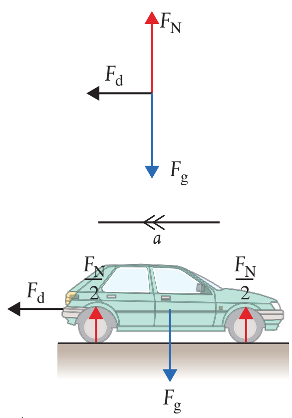
The reason a car needs to use energy when traveling at a constant speed is because of air resistance. If we look at the forces acting on the car, we see that there must be a constant forward force (provided by the friction between tires and road) to balance the air resistance or drag force.



▲ **A.3 Figure 11** The forces acting on a car traveling at constant velocity.

So work is done against the drag force, and the energy to do this work comes from the petrol. The amount of work done = force × distance traveled. So to calculate the work done, we need to know the drag force on a car traveling at 36 km h^{-1}. One way to do this would be to drive along a flat road at a constant 36 km h^{-1} and then take your foot off the accelerator pedal. The car would then slow down because of the unbalanced drag force.

This force will get less as the car slows down but here we will assume it is constant. From Newton's second law, we know that $F = ma$, so if we can measure how fast the car slows down, we can calculate the force. This will depend on the make of car but to reduce the speed by 1 m s^{-1} (about 4 km h^{-1}) would take about 2 s. Now we can do the calculation:



▲ **A.3 Figure 12** The forces on a car traveling at high speed with the foot off the accelerator pedal.

$$\text{acceleration of car} = \frac{(v-u)}{t} = \frac{(9-10)}{2} = -0.5\,\text{m s}^{-2}$$

$$\text{drag force} = ma = 1000 \times -0.5 = -500\,\text{N}$$

So to keep the car moving at a constant velocity, this force would need to be balanced by an equal and opposite force, $F = 500$ N.

Work done = force × distance so the distance traveled by the car = $\frac{\text{work done}}{\text{force}}$.

So if all of the energy in 1 liter of fuel is converted to work, the car will move a distance = $\frac{36 \times 10^6}{500}$ = 72 km. Note that if you reduce the drag force on the car, you increase the distance it can travel on 1 liter of fuel.

If you go over a hump back bridge too quickly, your car might leave the surface of the road. This is because the force needed to keep you moving in a circle is more than the weight of the car.

Circular motion and work

In the previous chapter (A.2), we learned that for an object to move in a circle, a centripetal force (resultant force perpendicular to velocity) was required. An alternative way of deducing that the force acts toward the center is to consider the energy. When a body moves in a circle with constant speed, it will have constant kinetic energy. This means that no work is being done on the mass. But we also know that since the velocity is changing, there must be a force acting on the body. This force cannot be acting in the direction of motion, since if it was, then work would be done and the kinetic energy would increase. We can therefore deduce that the force must be perpendicular to the direction of motion; in other words, toward the center of the circle.

Efficiency

A very efficient road car driven carefully would not be able to drive much further than 20 km on 1 liter of fuel so energy must be lost somewhere. One place where the energy is lost is in doing work against the friction that exists between the moving parts of the engine. Using oil and grease will reduce this but it can never be eliminated. The efficiency of an engine is defined by the equation:

$$\text{efficiency} = \frac{\text{useful work out}}{\text{total energy in}}$$

So if a car travels 20 km at a speed of $10\,\text{m}\,\text{s}^{-1}$, the useful work done by the engine:

$$= \text{force} \times \text{distance} = 500 \times 20000 = 10\,\text{MJ}$$

The total energy put in = 36 MJ so the efficiency $= \frac{10}{36} = 0.28$.

Efficiency is often expressed as a percentage so this would be 28%.

There has been a lot of research into making cars more efficient so that they use less fuel. Is this to save energy or money?

Where does all the energy go?

In this example, we calculated the energy required to move a car along a flat road. The car was traveling at a constant speed so there was no increase in kinetic energy and the road was flat so there was no increase in potential energy. We know that energy cannot be created or destroyed so where has the energy gone? The answer is that it has been transferred to the particles that make up the air and car.

Exercise

Q10. A 45% efficient machine lifts 100 kg through 2 m.

(**a**) How much work is done by the machine?

(**b**) How much energy is used by the machine?

Q11. A 1000 kg car accelerates uniformly from rest to $100\,\text{km}\,\text{h}^{-1}$.

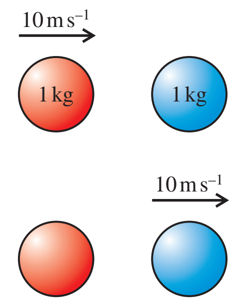
(**a**) Ignoring air resistance and friction, calculate how much work was done by the car's engine.

(**b**) If the car is 60% efficient, how much energy in the form of fuel was given to the engine?

(**c**) If the fuel contains 36 MJ per liter, how many liters of fuel were used?

Energy and collisions

One of the reasons that we brought up the concept of energy was related to the collision between two balls as shown in Figure 13. We now know that if no energy is lost when the balls collide, then the kinetic energy before the collision = kinetic energy after. This enables us to calculate the velocity afterwards and the only solution in this example is quite a simple one. The red ball transfers all its kinetic energy to the blue one, so the red one stops and the blue one continues, with velocity = $10\,\text{m}\,\text{s}^{-1}$. If the balls become squashed, then some work needs to be done to squash them. In this case, not all the kinetic energy is transferred, and we can only calculate the outcome if we know how much energy is used in squashing the balls.



A.3 Figure 13 The red ball gives energy to the blue ball.

Elastic collisions

An elastic collision is a collision in which both momentum and kinetic energy are conserved.

Example: two balls with equal mass

Two balls with equal mass *m* collide as shown in Figure 14. As you can see, the red ball is traveling faster than the blue one before and slower after. If the collision is perfectly elastic, then we can show that the velocities of the balls simply swap so $u_1 = v_2$ and $u_2 = v_1$.

A.3 Figure 14 Collision between two identical balls..

If the collision is elastic, then momentum and kinetic energy are both conserved. If we consider these one at a time we get:

Conservation of momentum:

$$\text{momentum before} = \text{momentum after}$$

$$mu_1 + mu_2 = mv_1 + mv_2$$

$$u_1 + u_2 = v_1 + v_2$$

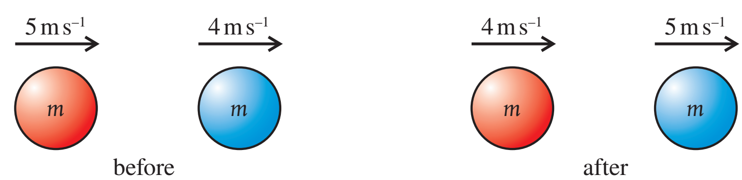
Conservation of kinetic energy:

$$E_k \text{ before} = E_k \text{ after}$$

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$u_1^2 + u_2^2 = v_1^2 + v_2^2$$

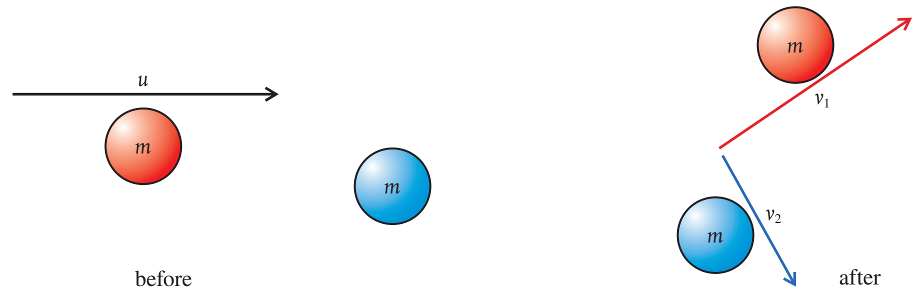
So we can see that the velocities are such that both their sums are equal and the squares of their sums are equal. This is only true if the velocities swap, as in Figure 15.



A.3 Figure 15 A possible elastic collision.

Collision in 2D between two identical balls

Anyone who has ever played pool or snooker will know that balls do not always collide in line; they travel at *angles* to each other. Figure 16 shows a possible collision.



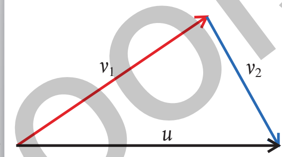
A.3 Figure 16 A 2D collision.

In this case, the blue ball is stationary so applying the conservation laws we get slightly simpler equations:

$$\vec{u} = \vec{v}_1 + \vec{v}_2$$

$$\vec{u}^2 = \vec{v}_1^{\,2} + \vec{v}_2^{\,2}$$

Note the vector notation to remind us that we are dealing with vectors. The first equation means that the sum of the velocity vectors after the collision gives the velocity before. This can be represented by the triangle of vectors in Figure 17.



A.3 Figure 17 Adding the velocity vectors.

The second equation tells us that the sum of the squares of two sides of this triangle = the square of the other side. This is Pythagoras' theorem, which is only true for right-angled triangles. So after an elastic collision between two identical balls, the two balls will always travel away at right angles (unless the collision is perfectly head on). This of course does not apply to balls rolling on a pool table since they are not isolated.

Pool balls may not collide like perfectly elastic isolated spheres but, if the table is included, their motion can be accurately modeled, enabling scientists to calculate the correct direction and speed for the perfect shot. Taking that shot is another matter entirely.

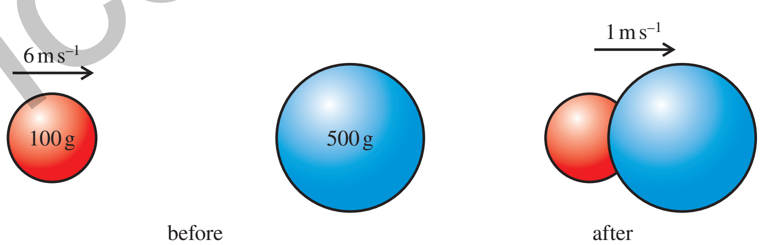
Inelastic collisions

There are many outcomes of an inelastic collision but here we will only consider the case when the two bodies stick together (coalesce). We call this a **totally inelastic collision**.

Why is the internal energy of an ideal gas equal to the sum of the kinetic energies but not the potential energies? (B.3)

Example

When considering the conservation of momentum in collisions, we used the example shown in Figure 18. How much work was done to squash the balls in this example?



A.3 Figure 18 An inelastic collision

How do collisions between charge carriers and the atomic cores of a conductor result in thermal energy transfer? (B.5, B.1)

According to the law of conservation of energy, the work done squashing the balls is equal to the loss in kinetic energy.

$$E_k \text{ loss} = E_k \text{ before} - E_k \text{ after} = \tfrac{1}{2} \times 0.1 \times 6^2 - \tfrac{1}{2} \times 0.6 \times 1^2$$

$$E_k \text{ loss} = 1.8 - 0.3 = 1.5\,\text{J}$$

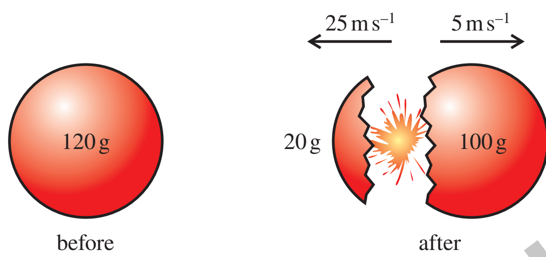
So: work done = 1.5 J

Explosions

Explosions can never be elastic since, without doing work, the parts that fly off after the explosion would not have any kinetic energy and would therefore not be moving. The energy to initiate an explosion often comes from the chemical energy contained in the explosive.

Example

Consider an exploding ball (shown in Figure 18). How much energy was supplied to the ball by the explosive?



A.3 Figure 19 An explosion

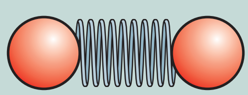
The result of this example is very important; we will use it when dealing with nuclear decay later on. So remember, when a body explodes into two unequal bits, the small bit gets most energy.

According to the law of conservation of energy, the energy from the explosive equals the gain in kinetic energy of the ball.

$$E_k \text{ gain} = E_k \text{ after} - E_k \text{ before}$$

$$E_k \text{ gain} = (\tfrac{1}{2} \times 0.02 \times 25^2 + \tfrac{1}{2} \times 0.1 \times 5^2) - 0 = 6.25 + 1.25 = 7.5\,\text{J}$$

Exercise

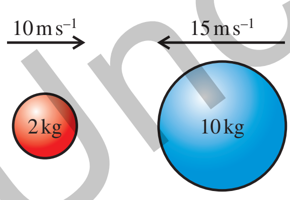


Q12. Two balls are held together by a spring as shown in the figure. The spring has a spring constant of $10\,\text{N cm}^{-1}$ and has been compressed a distance 5 cm.

(**a**) How much work was done to compress the spring?

(**b**) How much kinetic energy will each gain?

(**c**) If each ball has a mass of 10 g, calculate the velocity of each ball.



Q13. Two pieces of modeling clay as shown in the figure collide and stick together.

(**a**) Calculate the velocity of the lump after the collision.

(**b**) How much kinetic energy is lost during the collision?

Q14. A red ball traveling at $10\,\text{m s}^{-1}$ to the right collides with a blue ball with the same mass traveling at $15\,\text{m s}^{-1}$ to the left. If the collision is elastic, what are the velocities of the balls after the collision?

Challenge yourself

1. A 200 g red ball traveling at $6\,\text{m s}^{-1}$ collides with a 500 g blue ball at rest, such that after the collision the red ball travels at $4\,\text{m s}^{-1}$ at an angle of 45° to its original direction. Calculate the speed of the blue ball.

Power

We know that to do work requires energy, but work can be done quickly or it can be done slowly. This does not alter the energy transferred but the situations are certainly different. For example, we know that to lift one thousand 1 kg bags of sugar from the floor to the table is not an impossible task – we can simply lift them one by one. It will take a long time but we would manage it in the end. However, if we were asked to do the same task in 5 seconds, we would either have to lift all 1000 kg at the same time or move each bag in 0.005 s; both of which are impossible. Power is the quantity that distinguishes between these two tasks.

Power is defined as:

power = work done per unit time

The unit of power is the J s^{-1} which is the same as the watt (W). Power is a scalar quantity.

Example 1: The powerful car

We often use the term power to describe cars. A powerful car is one that can accelerate from 0 to $100\,\text{km h}^{-1}$ in a very short time. When a car accelerates, energy is being transferred from the chemical energy in the fuel to kinetic energy. To have a big acceleration, the car must gain kinetic energy in a short time; hence be powerful.

Example 2: Power lifter

A power lifter is someone who can lift heavy weights, so should we not say they are strong people rather than powerful? A power lifter certainly is a strong person (if they are good at it) but they are also powerful. This is because they can lift a big weight in a short time.

Worked example

A car of mass 1000 kg accelerates from rest to $100\,\text{km h}^{-1}$ in 5 seconds. What is the average power of the car?

Solution

$$100\,\text{km h}^{-1} = 28\,\text{m s}^{-1}$$

gain in kinetic energy of the car $= \frac{1}{2}mv^2 = \frac{1}{2} \times 1000 \times 28^2 = 392\,\text{kJ}$

If the car does this in 5 s, then:

$$\text{power} = \frac{\text{work done}}{\text{time}} = \frac{392}{5} = 78.4\,\text{kW}$$

If power = $\frac{\text{work done}}{\text{time}}$ then we can also write

$$P = \frac{F\Delta s}{t}$$

So $P = F\frac{\Delta s}{t}$

which is the same as

$$P = Fv$$

where v is the velocity.

This equation is a useful shortcut for calculating the power of a body moving at constant velocity.

Which other quantities in physics involve rates of change? (e.g. A.1, B.5, C.1, E.3)

Horsepower is often used as the unit for power when talking about cars and boats.

746 W = 1 hp

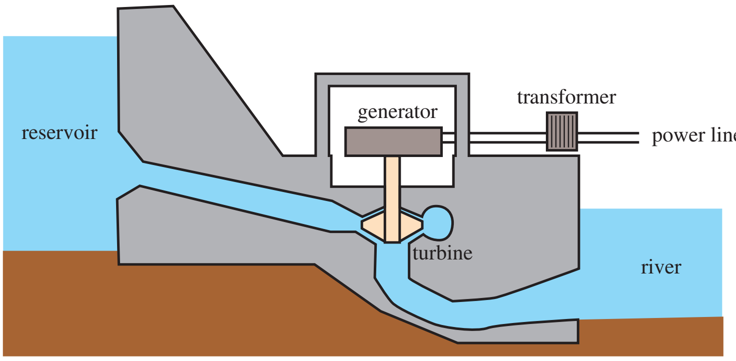
So in the Worked example, the power of the car is 105 hp.

Example 3: Hydroelectric power

It may not be obvious at first, but the energy converted into electrical energy by hydroelectric power stations comes originally from the Sun. Heat from the Sun turns water into water vapor, forming clouds. The clouds are blown over the land and the water vapor turns back into water as rain falls. Rain water falling on high ground has potential energy that can be converted into electricity (see Figure 20). Some countries like Norway have many natural lakes high in the mountains and the energy can be utilized by simply drilling into the bottom of the lake. In other countries rivers have to be dammed.

The Hoover Dam in Colorado can generate 1.5×10^9 watts.

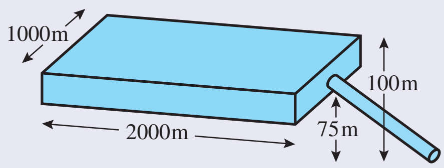
The energy stored in a lake at altitude is gravitational potential energy. This can be calculated from the equation $E_p = mgh$ where h is the height difference between the outlet from the lake and the turbine. Since not all of the water in the lake is the same height, the average height is used (this is assuming the lake is rectangular in cross section).



A.3 Figure 20 The main components in a hydroelectric power station.

Worked example

Calculate the total energy stored and power generated in the figure if water flows from the lake at a rate of $1\ m^3$ per second.



Solution

The average height above the turbine is

$$\frac{(100 + 75)}{2} = 87.5\,m$$

$$\text{Volume of the lake} = 2000 \times 1000 \times 25 = 5 \times 10^7\,m^3$$

$$\text{Mass of the lake} = \text{volume} \times \text{density} = 5 \times 10^7 \times 1000 = 5 \times 10^{10}\,kg$$

$$E_p = mgh = 5 \times 10^{10} \times 9.8 \times 87.5 = 4.29 \times 10^{13}\,J$$

If the water flows at a rate of $1\ m^3$ per second then 1000 kg falls 87.5 m per second

$$\text{So the energy lost by the water} = 1000 \times 9.8 \times 87.5 = 875\,000\,J\,s^{-1}$$

$$\text{Power} = 875\,kW$$

Exercise

Q15. A weightlifter lifts 200 kg 2 m above the ground in 5 s. Calculate the power of the weightlifter in watts.

Q16. In 25 s, a trolley of mass 50 kg runs down a hill. If the difference in height between the top and the bottom of the hill is 50 m, how much power will have been dissipated?

Q17. A car moves along a road at a constant velocity of $20\,m\,s^{-1}$. If the resistance force acting against the car is 1000 N, what is the power developed by the engine?

Efficiency and power

Efficiency is a quantity that gives a sense of the proportion of input energy that is transferred to useful stores. We define efficiency, η, by the equation:

$$\text{efficiency} = \frac{\text{useful work out}}{\text{total work in}}$$

If the work out is done at the same time as the work in, then we can divide the numerator and the denominator by time to give the equivalent equation:

$$\text{efficiency} = \frac{\text{useful power out}}{\text{total power in}}$$

Exercise

Q18. A motor is used to lift a 10 kg mass 2 m above the ground in 4 s. If the power input to the motor is 100 W, what is the efficiency of the motor?

Q19. A motor is 70% efficient. If 60 kJ of energy is put into the engine, how much work is got out?

Q20. The drag force that resists the motion of a car traveling at 80 km h^{-1} is 300 N.

(**a**) What power is required to keep the car traveling at that speed?

(**b**) If the efficiency of the engine is 60%, what is the power of the engine?

Guiding Questions revisited

How are concepts of work, energy and power used to predict changes within a system?

How can a consideration of energetics be used as a method to solve problems in kinematics?

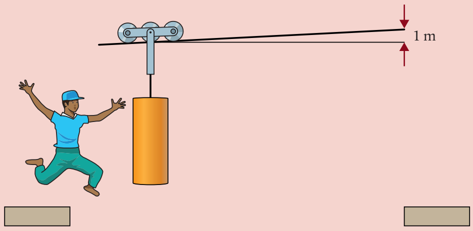
How can transfer of energy be used to do work?

In this chapter, we have provided an alternative model for analyzing the physical changes in a system that requires an understanding of how:

- The work done on a body is the product of the force exerted and the displacement of the body in the direction of the force.
- Gravitational and elastic potential energies are associated with position (relative to positions of zero potential energy that are selected strategically).
- Kinetic energy is associated with momentum (mass and velocity).
- Gravitational potential, elastic potential and kinetic energies are collectively referred to as mechanical energies.
- By considering the types of energy of all bodies at different positions and times, kinematics problems can be solved; unlike the *suvat* equations (which require uniform acceleration), energies can be calculated irrespective of the route taken.
- Energy is a conserved quantity, which means that it can be transferred but not created or destroyed.
- Work is done when energy is transferred and energy is transferred when work is done.
- Power is the rate at which work is done or energy is transferred.
- The upper limit of efficiency, the ratio of the useful energy (or power output) to the total energy (or power input), is 1.
- Sankey diagrams are a visual representation of the input and output energy types and the efficiency of the system.

Practice questions

1. A competition includes an obstacle where the competitor has to jump onto a hanging cylinder, causing it to move along an inclined wire with negligible friction.



(**a**) The cylinder has mass 100 kg and the competitor has mass 80 kg. The competitor is moving at 10 m s^{-1} when they catch the cylinder and leave the ground. Using $g = 10\,m\,s^{-2}$, calculate:

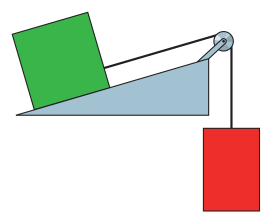
(**i**) the impulse experienced by the cylinder (2)

(**ii**) the impulse experienced by the competitor (1)

(**iii**) the kinetic energy of the cylinder and competitor just after catching the cylinder. (1)

(**b**) Determine whether the competitor will get to the far end of the wire. (2)

2. A 10 kg block is pulled a distance of 4 m along a frictionless ramp inclined at an angle of 20° by an 8 kg hanging mass as shown.



(**a**) Calculate:

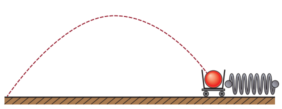
(**i**) the work done by the gravitational force (1)

(**ii**) the increase in potential energy of the sliding mass. (1)

(**iii**) Why are the answers to (**i**) and (**ii**) different? (1)

(**b**) The hanging mass is replaced by an electric motor and winch, which consumes 400 J of energy lifting the block to the same height. If the purpose of the machine is to raise the block, calculate the efficiency of the motor/ramp system. (2)

3. A 0.25 kg ball is launched from the ground with initial speed of 20 m s^{-1} and reaches a maximum height of 10 m.



(**a**) Calculate the speed of the projectile when it reaches maximum height. (2)

(**b**) The projectile lands in a bucket with wheels. Calculate the velocity of the bucket plus ball after the ball has landed in the bucket if the mass of the bucket is 1.25 kg. (2)

(**c**) The bucket and ball are stopped by a buffer, which is made from a spring that becomes compressed. If the change in length of the spring is 10 cm, calculate the spring constant. (2)

4. A stone is falling at a constant velocity vertically down a tube filled with oil. Which of the following statements about the energy transfers of the stone during its motion are correct? (1)

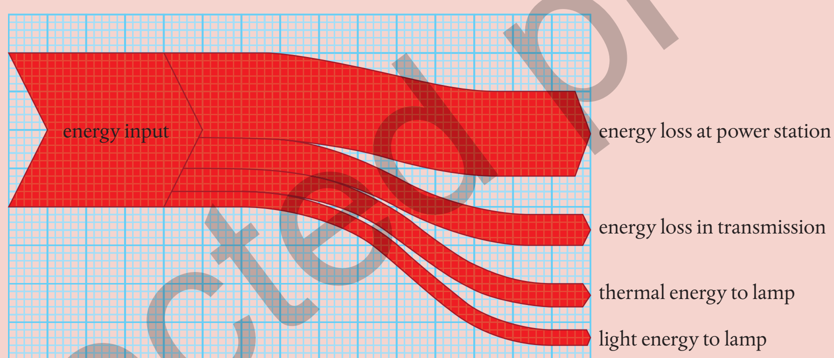
I. The gain in kinetic energy is less than the loss in gravitational potential energy.

II. The sum of kinetic and gravitational potential energy of the stone is constant.

III. The work done by the force of gravity has the same magnitude as the work done by friction.

A I and II only **B** I and III only **C** II and III only **D** I, II and III

5. The Sankey diagram shows the energy input from fuel that is eventually transferred to useful domestic energy in the form of light in a filament lamp. What is true for this Sankey diagram? (1)

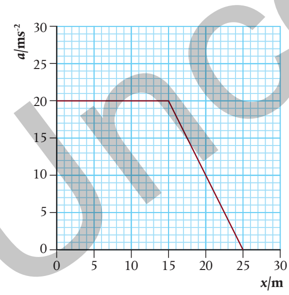


A The overall efficiency of the process is 10%.

B Generation and transmission losses account for 55% of the energy input.

C Useful energy accounts for half of the transmission losses.

D The energy loss in the power station equals the energy that leaves it.



6. The graph shows how the acceleration a of an object varies with distance traveled x. The mass of the object is 3.0 kg. What is the total work done on the object?

A 300 J

B 400 J

C 1200 J

D 1500 J

7. An object of mass m is initially at rest. When an impulse I acts on the object, its final kinetic energy is E_k. What is the final kinetic energy when an impulse of $2I$ acts on an object of mass $2m$ initially at rest? (1)

A $\frac{E_k}{2}$ **B** E_k **C** $2E_k$ **D** $4E_k$

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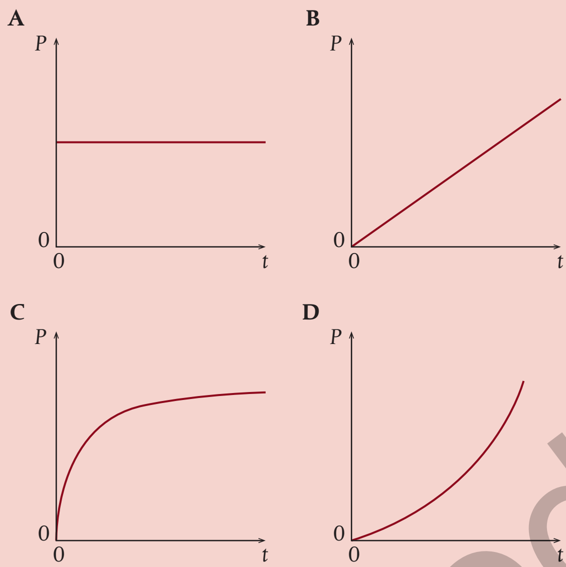
8. A train on a straight horizontal track moves from rest at constant acceleration. The horizontal forces on the train are the engine force and a resistive force that increases with speed. Which graph represents the variation with time *t* of the power *P* developed by the engine? (1)



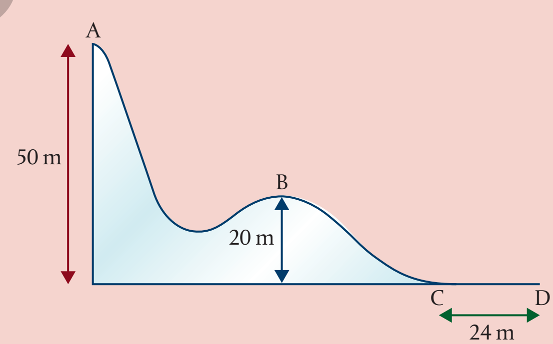
9. A car traveling at a constant velocity covers a distance of 100 m in 5.0 s. The thrust of the engine is 1.5 kN. What is the power of the car? (1)

A 0.75 kW **B** 3.0 kW **C** 7.5 kW **D** 30 kW

10. The energy density of a substance can be calculated by multiplying its specific energy (energy per unit mass) with which quantity? (1)

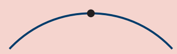
A mass **B** volume **C** $\frac{\text{mass}}{\text{volume}}$ **D** $\frac{\text{volume}}{\text{mass}}$

11. The diagram shows part of a downhill ski course that starts at point A, 50 m above level ground. Point B is 20 m above level ground. A skier of mass 65 kg starts from rest at point A and, during the ski course, some of the gravitational potential energy is transferred to kinetic energy.



(a) From A to B, 24% of the gravitational potential energy is transferred to kinetic energy. Show that the velocity at B is 12 m s^{-1}. (2)

(**b**) The dot on the following diagram represents the skier as she passes point B. Draw and label the vertical forces acting on the skier. (2)



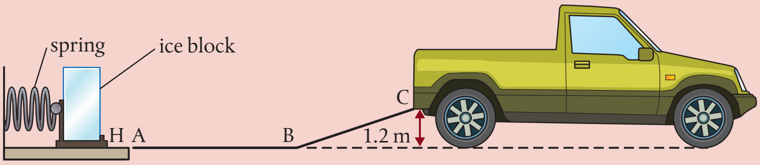
(**c**) The hill at point B has a circular shape with a radius of 20 m. Determine whether the skier will lose contact with the ground at point B. (3)

(**d**) The skier reaches point C with a speed of 8.2 m s^{-1}. She stops after a distance of 24 m at point D. Determine the coefficient of dynamic friction between the base of the skis and the snow. Assume that the frictional force is constant and that air resistance can be neglected. (3)

(**e**) At the side of the course, flexible safety nets are used. Another skier of mass 76 kg falls normally into the safety net with speed 9.6 m s^{-1}. Calculate the impulse required from the net to stop the skier and give an appropriate unit for your answer. (2)

(**f**) Explain, with reference to change in momentum, why a flexible safety net is less likely to harm the skier than a rigid barrier. (2)

12. A company designs a spring system for loading ice blocks onto a truck. The ice block is placed in a holder H in front of the spring, and an electric motor compresses the spring by pushing H to the left. When the spring is released, the ice block is accelerated toward a ramp, ABC. When the spring is fully decompressed, the ice block loses contact with the spring at A. The mass of the ice block is 55 kg. Assume that the surface of the ramp is frictionless and that the masses of the spring and the holder are negligible compared to the mass of the ice block.



(**a**) (**i**) The block arrives at C with a speed of 0.90 m s^{-1}. Show that the elastic energy stored in the spring is 670 J. (2)

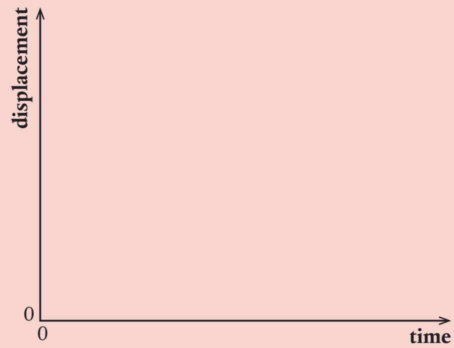
(**ii**) Calculate the speed of the block at A. (2)

(**b**) Describe the motion of the block:

(**i**) from A to B with reference to Newton's first law (1)

(**ii**) from B to C with reference to Newton's second law. (2)

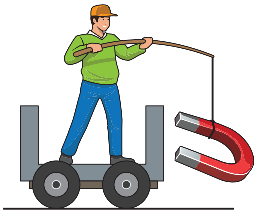
(c) Copy the axes below and sketch a graph to show how the displacement of the block varies with time from A to C. (You do not have to put numbers on the axes.) (2)



(d) The spring decompression takes 0.42 s. Determine the average force that the spring exerts on the block. (2)

(e) The electric motor is connected to an electrical source of power 816 W. The motor takes 1.5 s to compress the spring. Estimate the efficiency of the motor. (2)

13. (a) Will hanging a magnet in front of an iron car, as shown, make the car go? (1)



A Yes, it will go.

B It will move if there is no friction.

C It will not go.

(b) Explain your answer. (1)

14. A neutron moving through heavy water strikes an isolated and stationary deuteron (the nucleus of an isotope of hydrogen) head on in an elastic collision.

(a) Assuming the mass of the neutron is equal to half that of the deuteron, find the ratio of the final speed of the deuteron to the initial speed of the neutron. (2)

(b) What percentage of the initial kinetic energy is transferred to the deuteron? (2)

(c) How many collisions would be needed to slow the neutron down from 10 MeV to 0.01 eV? (2)

eV is a unit of energy and a conversion into J is not required.

A.4 Rigid body mechanics

Japanese breakdancer, Ami Yuasa, is shown mid-spin in Mumbai in 2019. We learn in A.2 that an object in motion continues with constant velocity unless acted upon by a resultant force. The same is true for rotation. Ami will continue to spin with constant angular speed unless acted upon by a resultant torque.

Guiding Questions

How can we use our knowledge and understanding of the torques acting on a system to predict changes in rotational motion?

If no external torque acts on a system, what physical quantity remains constant for a rotating body?

The balls and boxes we have considered in the previous chapters are rigid bodies. However, for simplicity, we have treated them like points by assuming that all the forces act on the center of the body in question. This is fine if the forces do act on or through the center, but what if they do not?

If you lift one end of a plank of wood, it will rotate. To solve mechanics problems properly, we need to understand the relationship between force and rotation. Fortunately for physicists, the relationships between rotational quantities are very similar to the linear ones. We even use the same words but starting with 'angular': angular displacement, angular speed, etc. The letters used are from the Greek alphabet, but the relationships are the same; the *suvat* equations now become the (not so catchy) $\theta\omega_i\omega_f\alpha t$ equations.

Nature of Science

Treating bodies as if they are points is appropriate up to a point, but insufficient for dealing with real-life examples. However, a rigid body is made up of many points so we can use what we know about point bodies. These models can then be applied to practical problems such as the design of buildings and bridges.

The Millau Viaduct in France took three years to build and is higher than the Eiffel Tower.

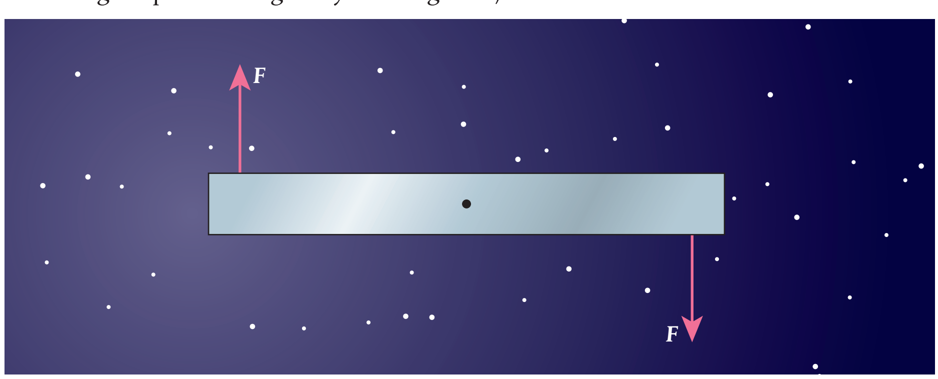
Students should understand:

torque τ of a force about an axis as given by $\tau = Fr\sin\theta$
bodies in rotational equilibrium have a resultant torque of zero
an unbalanced torque applied to an extended, rigid body will cause angular acceleration
the rotation of a body can be described in terms of angular displacement, angular velocity and angular acceleration
equations of motion for uniform angular acceleration can be used to predict the body's angular position θ, angular displacement $\Delta\theta$, angular speed ω and angular acceleration α, as given by: $\Delta\theta = \frac{\omega_f + \omega_i}{2}t$ $\omega_f = \omega_i + \alpha t$ $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
the moment of inertia, I, depends on the distribution of mass of an extended body about an axis of rotation
the moment of inertia for a system of point masses as given by $I = \Sigma mr^2$
Newton's second law for rotation as given by $\tau = I\alpha$ where τ is the average torque
an extended body rotating with an angular speed has an angular momentum L as given by $L = I\omega$
angular momentum remains constant unless the body is acted upon by a resultant torque
the action of a resultant torque constitutes an angular impulse ΔL as given by $\Delta L = \tau\Delta t = \Delta(I\omega)$
the kinetic energy of rotational motion as given by $E_k = \frac{1}{2}I\omega^2 = \frac{L^2}{2I}$

Rotational motion

Up to this point in the course, we have dealt with the motion of a small particle (a red ball), defining quantities related to its motion, deriving relationships relating those quantities, and introducing the concepts of force, momentum and energy to investigate the interaction between bodies. These models were then used to solve problems related to larger bodies, cars, people, etc., by treating them like particles. This works fine provided all the forces act at the center of mass, but what if they do not? Consider the two equal and opposite forces acting on the bar in Figure 1 (notice the bar is floating in space so no gravity is acting on it).

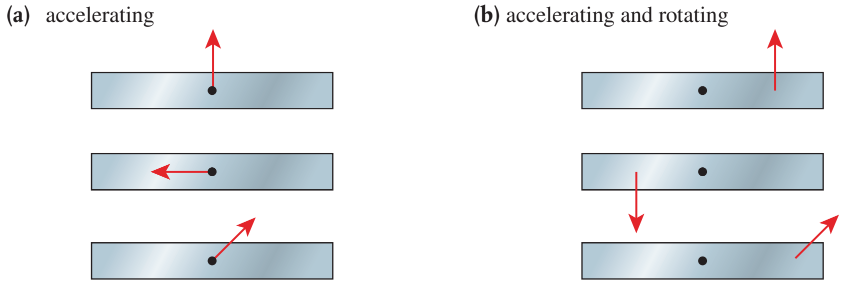
If the bar in Figure 1 was made of rubber, then the problem would be even more complicated as it would also bend. Here we will only consider **rigid** bodies. These are bodies that are made of atoms that do not move relative to one another; in other words, bodies with a fixed shape.



A.4 Figure 1 Forces on a bar

Let us apply Newton's first law to the body. The forces are balanced so the body will be at rest or moving with a constant velocity. However, if we observe what happens, we find that although the center of mass of the body remains stationary, the body rotates. We need to extend our model to include this type of motion.

Torque (τ)

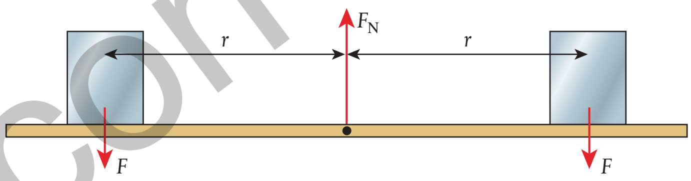


A.4 Figure 2 Forces do not always cause rotation.

If an unbalanced force acts on the center of mass of a rigid body, then it will have linear acceleration but it will not rotate. All the bodies in Figure 2(a) would have the same magnitude of acceleration. However, if the unbalanced force does not act on the center of mass, as in the examples in Figure 2(b), the bodies will rotate as well as accelerate. We can define the center of mass as *the point on a body through which an unbalanced force can act without causing rotation*.

Describing forces acting on bodies floating in space is rather difficult to imagine since it is not something we deal with every day. To make things more meaningful, let us consider something more down to Earth: a seesaw.

A seesaw is a rigid bar with two moveable masses. It only works in a region where the masses are under the influence of gravity, e.g. on the Earth. The forces involved are as shown in Figure 3.

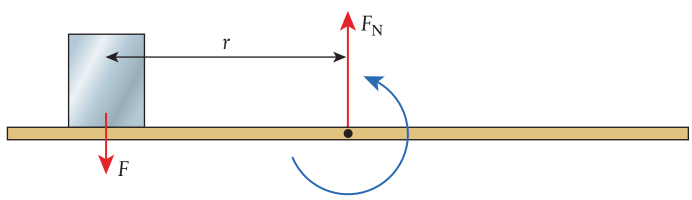


A.4 Figure 3 Balanced seesaw.

A balanced seesaw only moves when you push with your legs.

Here we can see that the forces up = the forces down so there will be no acceleration. There is also no rotation so the turning effect of the two children must be balanced. The normal reaction that holds the bar up does not turn the bar since it acts at the center of mass. If, however, one child was to get off, then the bar would turn.

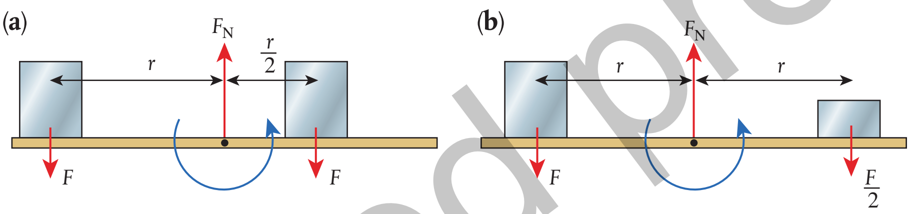
A.4 Figure 4 Seesaw with one child.



The seesaw is held in position by an axle fixed to the center of the bar. This point is called the **pivot**. The axle prevents the bar from accelerating by exerting a force that is equal and opposite to the weight of the children (assuming the bar has negligible weight), but allows it to rotate.

The bar would also turn if one child moved toward the center or was replaced by a child with less weight.

A.4 Figure 5 Unbalanced seesaws.



Balancing the forces when two people lift a heavy object up a flight of stairs, one would expect that each person would exert a force equal to half the weight. But if that is the case, why is it easiest to be at the top? Balancing torques gives the answer.

The turning effect of the force depends upon the force and how far the force is from the pivot. The **torque** gives the turning effect of the force:

torque = force × perpendicular distance from the line of action of the force to a point

So the torque in Figure 4 is $F \times r$. This torque turns the bar in an anticlockwise direction. The torques in Figure 3 are balanced because the clockwise torque = anticlockwise torque, but in Figure 5(a) and (b), the anticlockwise torque ($F \times r$) is greater than the clockwise torque ($F \times \frac{r}{2}$) so the bar will rotate anticlockwise. If we take anticlockwise torques to be positive and clockwise negative, we can say *the bar is balanced when the sum of torques is zero*.

Angular speed and angular acceleration

In this example, the mass of the bar (also called a beam) is negligible but even if it was not, we would not have to consider it since the force at the pivot acts in the same place.

When the bar rotates, we can define the speed of rotation by the **angular speed**. This is the angle swept out by the bar per unit time. If the torques on the bar are unbalanced, then it will begin to rotate. This means there is change in the angular speed (from zero to something); we can say that the bar has **angular acceleration**:

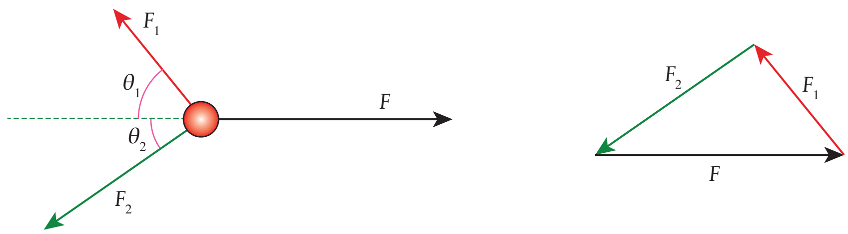
angular speed (ω) is the angle swept out per unit time;
angular acceleration (α) is the rate of change of angular speed.

Equilibrium

When dealing with point masses, we say that a body is in equilibrium when at rest or moving with constant velocity. However, when we define equilibrium for larger, rigid bodies, we should add that there should be no angular acceleration. This means that not only must the forces be balanced but so should the torques.

The sum of all the forces acting on the body is zero

If all the forces acting on a body are added vectorially, the resultant will be zero. With many forces, adding the vectors can lead to some confusing many-sided figures so it is often easier to take components in two perpendicular directions, often vertical and horizontal, then sum these separately. If the total force is zero, then the sum in any two perpendicular directions will also be zero.



A.4 **Figure 6** Summing vectors or taking components.

If the red ball is in equilibrium, the sum of the forces must be zero so the vector sum has a zero resultant as shown by the triangle. This can be solved but it is not a right-angled triangle so is not simple. An easier approach is to take perpendicular components:

vertical: $F_1 \sin \theta_1 - F_2 \sin \theta_2 = 0$

horizontal: $F - F_1 \cos \theta_1 - F_2 \cos \theta_2 = 0$

In other words:

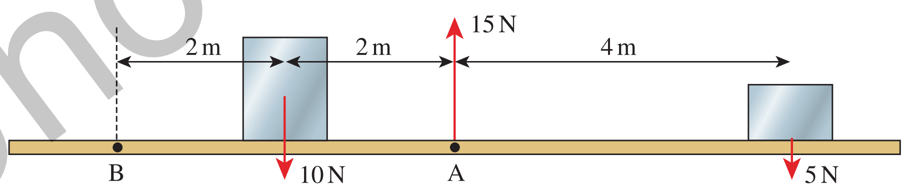
sum of the forces left = sum of the forces right

and

sum of the forces up = sum of the forces down

The sum of all the torques acting on the body is zero when in equilibrium

In the seesaw example, we obviously considered torques about the pivot but if a body is in equilibrium, then the sum of the torques about *any* point will be zero. Take the example in Figure 7.



A.4 **Figure 7** Torques can be calculated about A, B or anywhere else.

Taking torques about A:
clockwise = 5 × 4 = 20 N m
anticlockwise = 2 × 10 = 20 N m

Taking torques about B:
clockwise = 5 × 8 + 10 × 2 = 60 N m (If B was a pivot, both forces would cause a clockwise rotation.)
anticlockwise = 15 × 4 = 60 N m (Here we have taken the normal reaction. If this was the only force and B was a pivot, it would cause the bar to rotate in an anticlockwise direction.)

When solving problems, you can choose the *most convenient* place to take torques about. It does not have to be the pivot.

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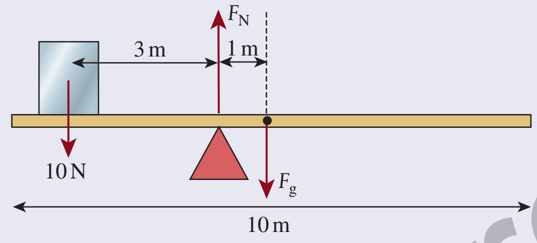
The balanced beam

There are many variations of this problem. In some cases, you can ignore the weight of the beam (as in the seesaw) but in others it must be taken into account.

Worked example

Calculate the weight of the beam balanced as in the figure.

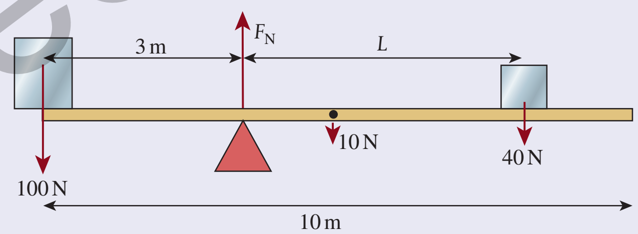


Solution

Taking torques about the pivot we get:
clockwise torques = $F_g \times 1$
anticlockwise torques = 10×3
Since balanced: $F_g = 30\,N$

Worked example

Calculate the length L between the 40 N weight and the pivot needed to balance the beam shown in the figure.



Solution

Taking torques about the pivot:
clockwise torques = $10 \times 2 + 40 \times L = 20 + 40L$
anticlockwise torques = $100 \times 3 = 300$
Since balanced: $300 = 20 + 40L$
$280 = 40L$
$L = 7\,m$

Exercise

Q1. A 1.0 m ruler is balanced on the 30 cm mark by placing a 300 g mass 10 cm from the end. Calculate the mass of the ruler.

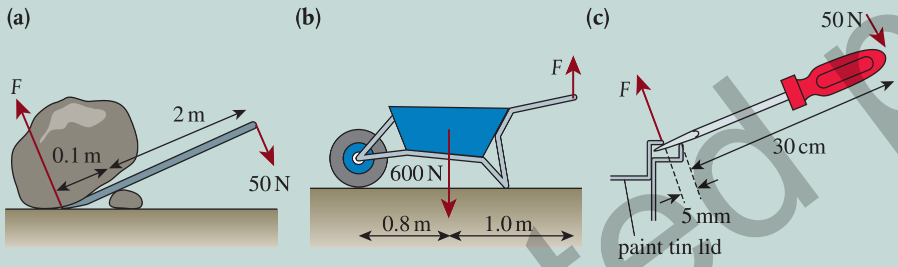
Q2. A 100 g mass is placed at the 10 cm mark on a 20 g ruler. Where must a 350 g mass be placed so that the ruler balances at the 60 cm mark?

Levers

We have seen that the force required to balance the bar depends on how far from the pivot you apply the force. This is the principle of levers and has many applications.

Exercise

Q3. Calculate the unknown force *F* in each of the situations shown below.



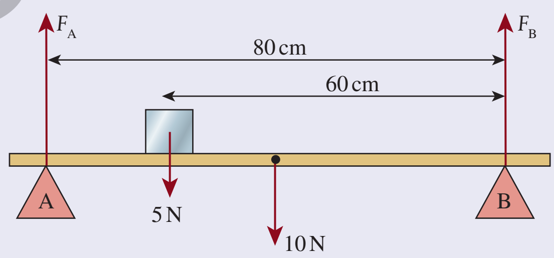
The bridge

A simple bridge consists of a rigid construction spanning the gap between two supports. This may seem nothing to do with rotation, and if built properly, it is not. However, we can use the condition for equilibrium to calculate the forces on the supports.

Advances in engineering have made it possible to construct bridges connecting isolated communities, changing the way people live their lives.

Worked example

A mass of 500 g is placed on the bridge as shown below. If the mass of the bridge is 1.0 kg, calculate the force on each of the supports.



Solution

In this case, if we calculated the torques about the center, we would have two unknowns in the equation so it would be better to find torques about one of the ends. Let us consider end B:

clockwise torques = $F_A \times 0.8$
anticlockwise torques = $5 \times 0.6 + 10 \times 0.4 = 7$ N m

$$F_A = \frac{7}{0.8} = 8.75 \text{ N}$$

To find F_B, we can now use the fact that the vertical forces must also be balanced so:

$$F_A + F_B = 10 + 5$$

$$F_B = 15 - 8.75 = 6.25 \text{ N}$$

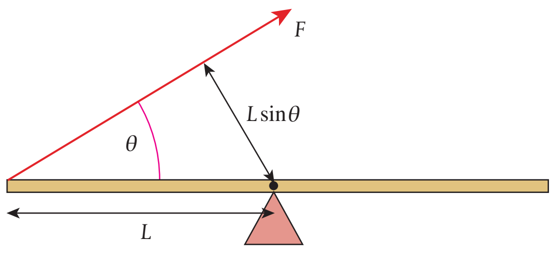
Exercise

Q4. A 5.0 m long ladder is held horizontally between two men. A third man with mass 80 kg sits on the ladder 1.0 m from one end. Calculate the force each man exerts if the mass of the ladder is 10 kg.

Q5. A 1.0 m long ruler of mass 200 g is suspended from two vertical strings tied 10 cm from each end. The force required to break the strings is 6.0 N. An 800 g mass is placed in the middle of the ruler and moved toward one end. How far can the mass move before one of the strings breaks?

Non-perpendicular forces

When a force acts at an angle to the bar as in Figure 8, the perpendicular distance from the line of action to the pivot is reduced so $\tau = F \times L \sin\theta$. This is the same component of the force perpendicular to the bar multiplied by the distance to the pivot. The parallel component does not have a turning effect since the line of action passes through the pivot.

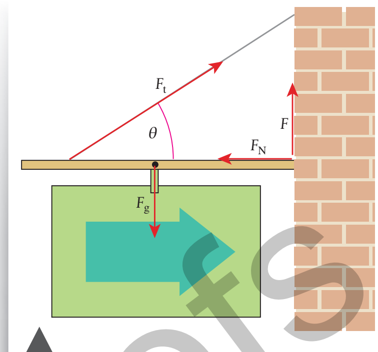


A.4 Figure 8 A non-perpendicular force

The hanging sign

Signs and lights are often hung on brackets fixed to a wall. This can result in a lot of force on the fixings so they are often supported by a wire as shown in Figure 9. Note that in this case the sign hangs from the center of the bar.

Here we can see that, because the wire is attached to the wall, it makes an angle θ with the supporting bar. This must be balanced by an equal and opposite force from the wall; this is the normal reaction F_N. Calculating torques around the point where the wire joins the bar, we see that the bar and sign cause a clockwise torque. This is balanced by the anticlockwise torque caused by the force F at the wall. This force is provided by the fixing plate or by inserting the bar into a hole in the wall.



A.4 Figure 9 A hanging sign.

Exercise

Q6. A sign is hung exactly like the one in Figure 9. The sign has a mass of 50 kg and the bar 10 kg. The bar is 3.0 m in length and the wire is attached 50 cm from the end and makes an angle of 45° with the bar. Calculate:

(**a**) the tension F_t in the wire

(**b**) the normal force F_N

(**c**) the upwards force F.

Q7. Repeat Q6 with the sign hanging from the end of the bar.

How does a torque lead to simple harmonic motion? (C.1)

The leaning ladder

If you have ever used a ladder to paint the wall of a house, you might have wondered what angle the ladder should be: too steep and you might fall backward; not steep enough and it might slip on the ground. By calculating torques, it is possible to find out if the ladder is in equilibrium, but remember the forces change when you start to climb the ladder.

Figure 10 shows a ladder leaning against a frictionless wall in equilibrium. Brick walls are not really frictionless but it makes things easier to assume that this one is. The problem is to find the friction force on the bottom of the ladder.

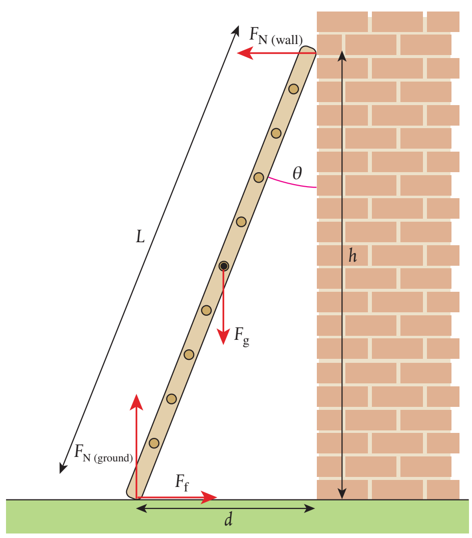
First we can balance the forces: vertical forces: $F_{N\,(ground)} = F_g$

horizontal forces $F_{N\,(wall)} = F_f$

Then, calculating torques about the top of the ladder:

sum of clockwise torques = sum of anticlockwise torques

$$F_{N\,(ground)} \times d = F_f \times h + F_g \times \frac{d}{2}$$



A.4 Figure 10 A leaning ladder.

When a ladder leans against a wall, the friction at the bottom balances the normal force at the top. As you climb the ladder, the normal force increases so the friction must also increase. However, friction cannot be bigger than $\mu F_{N\ (ground)}$. If this is less than the normal force at the top, the ladder will slip. The moral of this tale is that just because the ladder does not slip when you start to climb does not mean it will not slip when you get to the top.

If we were to calculate torques around the bottom of the ladder, we get:

$$F_g \times \frac{d}{2} = F_{N\ (wall)} \times h$$

$$F_{N\ (wall)} = F_g \times \frac{d}{h} \times \frac{1}{2}$$

$$= F_g \times \frac{\tan\theta}{2}$$

But $F_{N\ (wall)} = F_f$:

$$F_f = F_g \times \frac{\tan\theta}{2}$$

So as the angle increases, the friction at the bottom (F_f) increases. This has a maximum value of $\mu F_{N\ (ground)}$ (μ is the coefficient of friction) that limits the maximum angle of the ladder.

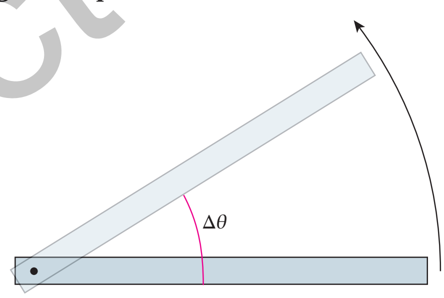
Exercise

Q8. A ladder of length 5 m leans against a wall such that the bottom of the ladder is 3 m from the wall. If the weight of the ladder is 20 kg, calculate the friction between the ground and the bottom of the ladder.

Q9. If the ladder in Q8. is moved a little bit further out and it begins to slip. Calculate the coefficient of static friction between the ground and the ladder.

Constant angular acceleration

Consider a bar pivoted at one end as in Figure 11. As the bar rotates, it sweeps out an angle $\Delta\theta$. This is the **angular displacement** of the bar and is measured in radians.



A.4 Figure 11 An angle is swept out

If the time taken for the bar to sweep out angle $\Delta\theta$ is Δt, then the average **angular speed** of the bar ω is given by the equation:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

An unbalanced torque applied to the bar will cause it to rotate faster. The average rate of change of angular speed is the **angular acceleration**, α.

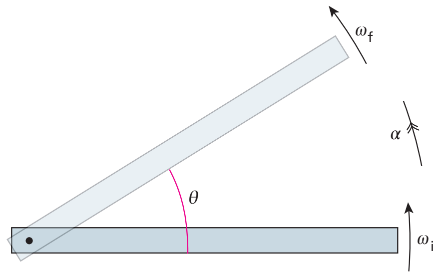
$$\alpha = \frac{\Delta\omega}{\Delta t}$$

These quantities are the rotational equivalents of linear displacement, velocity and acceleration. If the angular acceleration is constant, they are related in the same way, giving angular equivalents of the *suvat* equations (the $\theta\,\omega_i\,\omega_f\,\alpha\,t$ equations!).

Constant angular acceleration equations

A bar rotating at an initial angular speed of ω_i is acted upon by a torque that causes an angular acceleration α, increasing the angular speed to a final value of ω_f in t seconds. During this time, the bar sweeps out an angle θ.

To perform a triple somersault, a gymnast must first initiate the rotation using friction between their feet and the floor. Once the body is rotating, the legs and arms are pulled in to a tucked position, reducing the rotational inertia and resulting in an increase in angular speed. It is also possible to perform a triple somersault with a straight body. In this case, a lot of speed must be built up before take-off to give a high enough angular speed.



A.4 Figure 12 Uniform angular acceleration.

These quantities are related by the equations shown in Table 1.

These angular equations are used to solve problems in exactly the same way as the linear equations.

Angular	Linear
$\omega_f = \omega_i + \alpha t$	$v = u + at$
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	$v^2 = u^2 + 2as$
$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$	$s = ut + \frac{1}{2}at^2$
$\Delta\theta = \frac{\omega_i + \omega_f}{2}t$	$s = \frac{u + v}{2}t$

A.4 Table 1

Worked example

A body rotating at 10 rad s^{-1} accelerates at a uniform rate of 2 rad s^{-2} for 5 seconds. Calculate the final angular speed.

Solution

The data given is:
$\omega_i = 10\,\text{rad s}^{-1}$
$\alpha = 2\,\text{rad s}^{-2}$
$t = 5\,\text{s}$

We wish to find ω_f so the equation to use is $\omega_f = \omega_i + \alpha t$:

$$\omega_f = 10 + 2 \times 5 = 20\,\text{rad s}^{-1}$$

Worked example

Calculate the angle swept out by a body that starts with an angular speed of 2 rad s^{-1} and accelerates for 10 s at a rate of 5 rad s^{-2}.

Solution

The data given is:
$\omega_i = 2\,\text{rad s}^{-1}$
$\alpha = 5\,\text{rad s}^{-2}$
$t = 10\,\text{s}$

We wish to find $\Delta\theta$ so the equation to use is $\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$:

$$\Delta\theta = 2 \times 10 + \frac{1}{2}5 \times 10^2 = 20 + 250 = 270\,\text{rad}$$

This is $\frac{270}{2\pi}$ revolutions.

1 revolution is 2π radians.

Exercise

Q10. A wheel is pushed so that it has a uniform angular acceleration of 2 rad s^{-2} for a time of 5 s. If its initial angular speed was 6 rad s^{-1}, calculate:

(**a**) the final angular speed

(**b**) the number of revolutions made.

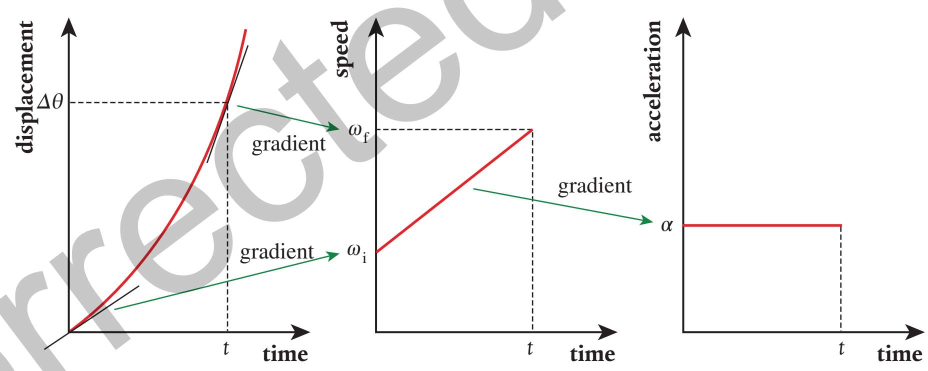
Q11. The frictional force on a spinning wheel slows it down at a constant acceleration until it stops. Initially, the wheel was spinning at 5 revolutions per second. If the wheel was slowed down to stop in one revolution, calculate:

(**a**) the angular acceleration

(**b**) the time taken.

Graphical representation

As with linear motion, angular motion can be represented graphically. In the example considered previously, a bar rotating at an initial angular speed of ω_i is acted on by a torque that causes an angular acceleration α, increasing the angular speed to a final value of ω_f in t seconds. During this time, the bar sweeps out an angle θ. This can be represented by the three graphs shown in Figure 13.

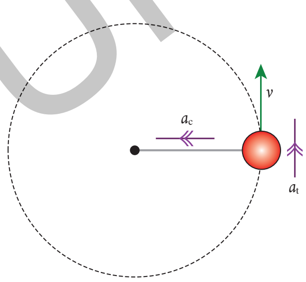


A.4 Figure 13 Rotational motion graphs.

As with the linear equivalents, the gradient of displacement/time ($\frac{\Delta\theta}{\Delta t}$) gives speed and the gradient of speed/time ($\frac{\Delta\omega}{\Delta t}$) gives acceleration. Working the other way around, the area under acceleration/time gives the change of speed and the area under speed/time gives displacement.

Relationship between angular motion and linear motion

Circular motion can be split into two components: one perpendicular to the circumference and one tangential to it. The perpendicular component is dealt with in A.2 (Circular motion) when we considered only bodies moving with constant speed. In this case, there is acceleration toward the center – the centripetal acceleration – but no tangential acceleration. When an unbalanced torque acts, then there will be an increasing centripetal acceleration plus a tangential acceleration in the directions shown in Figure 14.



A.4 Figure 14 Centripetal acceleration is along a radius and tangential acceleration is along a tangent

We know that if $\Delta\theta$ is measured in radians, $\Delta\theta = \frac{\Delta s}{r}$ so $\Delta s = \Delta\theta \times r$.