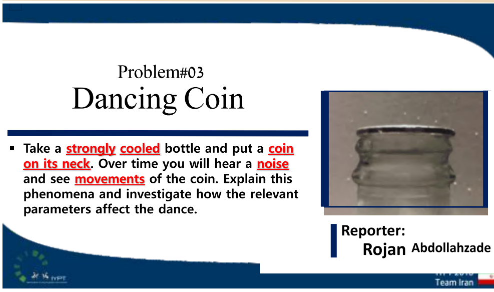


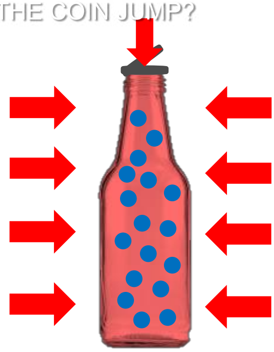
OUTLINE

Approach

- What makes the coin jump?
- Set up
- Heat transfer
- Pressure VS Time
- Pressure VS Temperature
- Conclusion

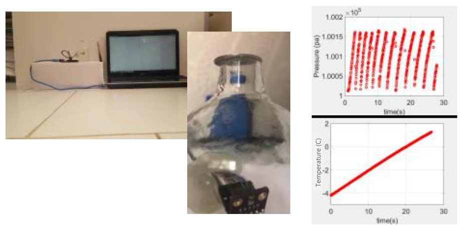
WHAT MAKES THE COIN JUMP?

$$F_C = S\Delta P$$

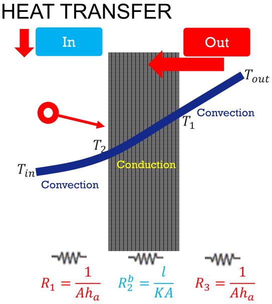


SET UP

SET UP



HEAT TRANSFER



- $\dot{Q} = h_a A(T_{out} - T_1)$ Convection
- $\dot{Q} = \frac{KA}{l}(T_1 - T_2)$ Conduction
- $Q = h_a A(T_{out} - T_m)$ Convection

With no energy radiation and absorption

Bottle : $R_T^b = R_1 + R_2^b + R_3$

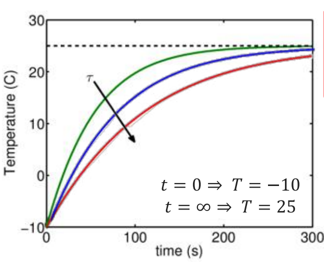
Coin : $R_T^m = R_1 + R_2 + R_3$

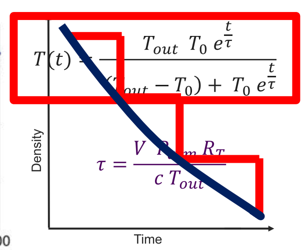
$$R_T = \frac{R_T^b \times R_T^m}{R_T^b + R_T^m}$$

$$\dot{Q} = \frac{R_T}{\frac{1}{h_a A} + \frac{l}{KA} + \frac{1}{h_a A}}$$

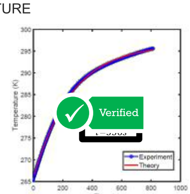
$$\dot{Q} = \frac{T_{out} - T_{in}}{T_{out} - T_{in}}$$

HEAT PROFILE AND TEMPERATURE



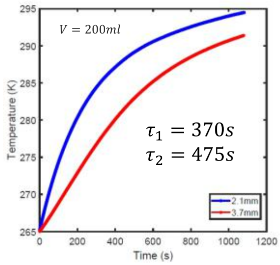


TEMPERATURE



ART AN AMAZING FACT IN SCIENCE

THE EFFECT OF BOTTLES THICKNESS

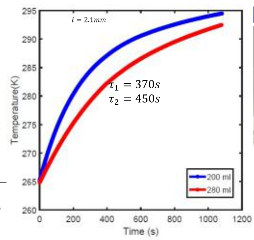


$$\tau = \frac{V\ P_{atm}\,R_T}{c\,T_{out}}$$

$$\frac{200\times 10^{-6}\times 10^5\times R_T}{2\,39\times 301} = 133$$

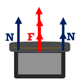
$$\frac{200\times 10^{-6}\times 10^5\times R_T}{2\,39\times 301} = 170\,85$$

THE EFFECT OF VOLUME

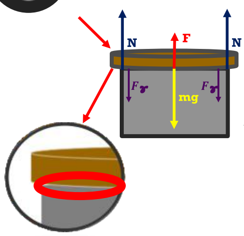


$$N_t - F_{\gamma} - mg = 0$$

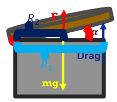
$$N_t + F - F_{\gamma} - mg = 0$$



$$F_C = mg - N + F_{\gamma}$$



WHAT MAKES THE COIN MOVING UPWARD?



$$_dR_1 + F_P R_2 + mgR_2 = I\ddot{\alpha}$$

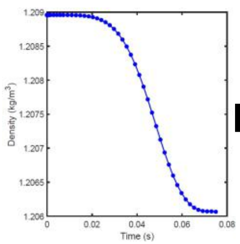
$F_d = Cv^2$ $F_P = S\,\Delta P$

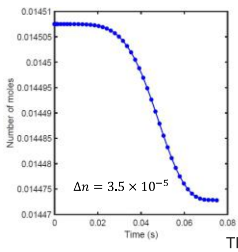
$* \; P = \rho R_{sp} T$ $* \; \Delta P = \frac{1}{2}\rho v^2$

$$2CR_{sp}\left(\frac{\rho T - \rho_0 T_{out}}{\rho}\right)R_1 + SR_{sp}(\rho T - \rho_0 T_{out})R_2 - mgR_2 = I\ddot{\alpha}$$

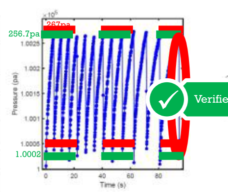
$$\frac{d\rho}{dt} = -\rho\frac{\alpha^2 R^2}{V}\sqrt{2R_{sp}\left(\frac{\rho T - \rho_0 T_{out}}{\rho}\right)}$$

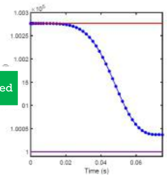
WHAT MAKES THE COIN MOVING UPWARD?



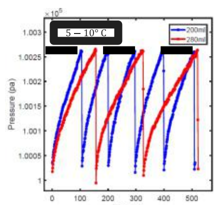


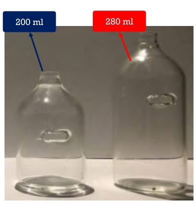
PRESSURE



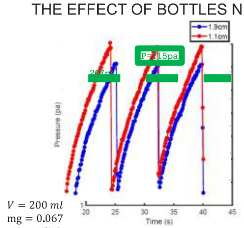


THE EFFECT OF VOLUME

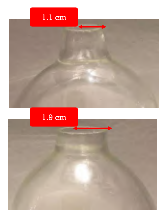




THE EFFECT OF BOTTLES NECK DIAMETER



V = 200 ml
mg = 0.067
10 − 15° C



THE EFFECT OF BOTTLES NECK DIAMETER

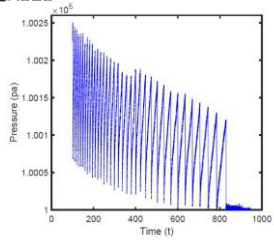
$$F_C = mg - N + F_{\gamma}$$

$$\Delta P \times S = mg - N + F_{\gamma}$$

$$250 \times 9 \times 10^{-5} = 0.067 + 0.0045 - N$$

$$N = 0.15N$$

ΔP DECREASES



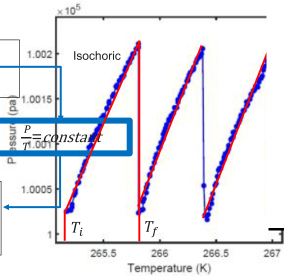
ART AN AMAZING FACT IN SCIENCE

PRESSURE VS TEMPERATURE

$$P = P_{in} + \frac{F_c}{S}$$

Ideal gas law $\frac{P}{T}=constant$

$$T_f = T_i\left(1+\frac{F_c}{S\,P_{in}}\right)$$



PRESSURE VS TEMPERATURE

$T_1 = T_{in}(1+\alpha)$ $\quad \alpha = \frac{F_c}{S\,P_{in}}$

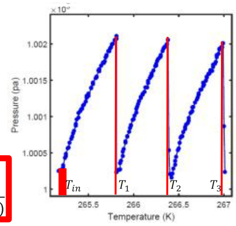
$T_2 = T_{in}(1+\alpha)^2$

⋮

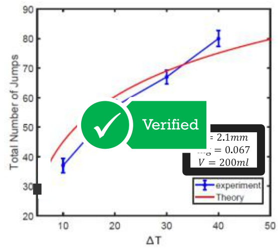
$T_n = T_{in}(1+\alpha)^n$

$\frac{T_{out}}{T_{in}} = (1+\alpha)^n$

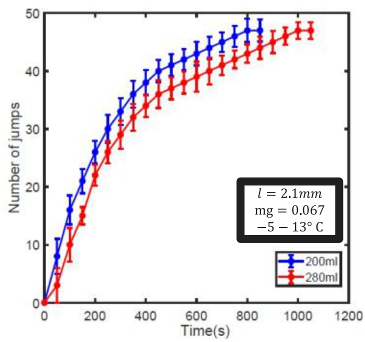
$$n = \frac{\ln\left(\frac{T_{out}}{T_{in}}\right)}{\ln(1+\alpha)}$$



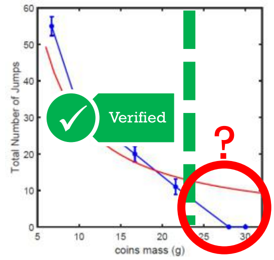
THE EFFECT OF ΔT



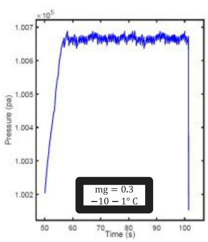
THE EFFECT OF VOLUME



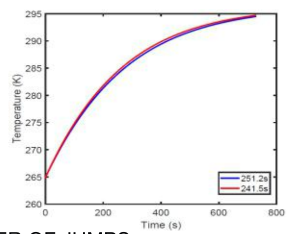
THE EFFECT OF COINS WEIGHT



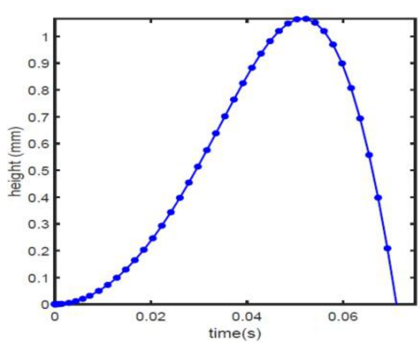
NO JUMPS AT ALL!



THE EFFECT OF SENSOR



THE HEIGHT OF COIN



TOTAL NUMBER OF JUMPS

$$P = P_{atm} + \frac{F_c}{S}$$

$$* \frac{P}{T} = constant$$

$$\frac{P_i}{T_i} = \frac{P_f}{T_f} \Rightarrow T_f = T_i\frac{P_f}{P_i}$$

$$T_f = T_i\left(1+\frac{F_c}{S\,P_{atm}}\right)$$

ART AN AMAZING FACT IN SCIENCE

EFFECTIVE PARAMETERS

$$n = \frac{\ln\left(\frac{T_{out}}{T_{in}}\right)}{\ln(1+\alpha)} \qquad \alpha = \frac{F_c}{S\,P_{atm}}$$

$T_{out} \uparrow \; n \uparrow$
$T_{in} \uparrow \; n \downarrow$
$\alpha \uparrow \; n \downarrow$

$$T = \frac{T_{out}\; T_{in}\; e^{\frac{t}{\tau}}}{(T_{out} - T_{in}) + T_{in}\, e^{\frac{t}{\tau}}}$$

$T_{out} \uparrow \; T \uparrow$
$T_{in} \uparrow \; T \uparrow$
$\tau \uparrow \; T \downarrow$

$$P = P_{atm} + \frac{F_c}{S}$$

$P_{atm} \uparrow P \uparrow$
$F_c \uparrow \; P \uparrow$
$S \uparrow \; P \downarrow$

HEAT PROFILE AND TEMPERATURE

$$f = \frac{dQ}{dt} = \frac{T_{out} - T_{in}}{R_T}$$

$$Q = m\, c_v\, \Delta T$$

$$dQ = \frac{m\, c_v\, dT}{dt} = \frac{T_{out} - T_{in}}{R_T}$$

$$\frac{VPc_v}{R_{sp}\, T}\frac{dT}{dt} = \frac{T_{out} - T}{R_T}$$

$$\int_{T_{in}}^{T_{out}} \frac{dT}{T(T_{out}-T)} = \int_0^t \frac{dt\, c}{VPR_T}$$

$$PV = nRT \Rightarrow P = \frac{M}{V} \times \frac{R_{sp}}{m} \times T$$

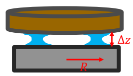
$$P = \rho\, R_{sp}\, \mathrm{T}$$

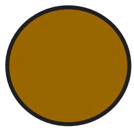
$$\rho = \frac{P}{R_{sp}\, T}$$

ESTIMATING THE SURFACE TENSION FORCE

$$F_C = mg - N + F_\gamma$$

$$\Delta E_\gamma = \gamma A = 4\pi r \Delta z \gamma$$





$M = 6.7 \times 10^{-3}\, kg$

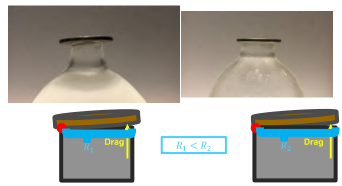
$\gamma_w = 73 \times 10^{-3}\, N/m$

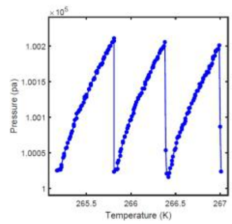
$r = 5 \times 10^{-3}\, m$

$F_\gamma = 4.5 \times 10^{-3}\, N$

$$\frac{6.7 \times 10^{-2}\mathrm{N}}{4.5 \times 10^{-3}\mathrm{N}} \approx 8$$

THE EFFECT OF DRAG FORCE ON JUMP





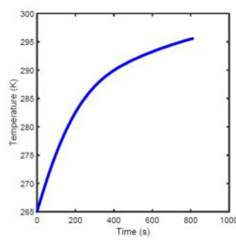
CONCLUSION

- Temperature VS Time
- Pressure VS Time
- Pressure VS Temp

$$T = \frac{T_{out}\; T_{in}\; e^{\frac{t}{\tau}}}{(T_{out} - T_{in}) + T_{in}\, e^{\frac{t}{\tau}}}$$

$$P = P_{atm} + \frac{F_c}{S}$$

$$n = \frac{\ln\left(\frac{T_{out}}{T_{in}}\right)}{\ln(1+\alpha)}$$



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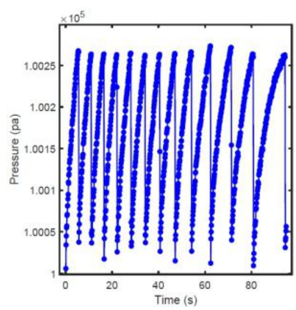
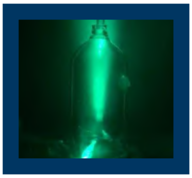
The Dancing Penny (umanitoba.ca), www.umanitoba.ca/outreach/crystal/resources%20for%20teachers/The%20Dancing%20Coin.doc
[2] Robert T. Bailey & Wayne L. Elban
Heat Transfer Eng. 29, 7, 643-650 (2008)
[3] Dancing Penny Experiment (youtube, kentchemistry.com, Jan 31, 2010), https://youtu.be/RU0B5cl8qo4
[4] Jumping Coin (youtube, Amar Chitra Katha Pvt Ltd, Aug 19, 2013), https://youtu.be/20yD8PLCJ98
[5] Jumping Coin Experiment by Manman (youtube, Manman Isaac, Mar 20, 2014)
May 7, 2013), https://youtu.be/3TjcbvmjqIA
[6] Vibrating Coin experiment (youtube, Geraldine6824, Oct 5, 2014), https://youtu.be/E4bQ14_CQZA
[7]Dancing Penny Experiment (youtube, Joe Tarlizzo



Problem#5

Filling Up a Bottle

When a vertical water jet enters a bottle, sound may be produced, and, as the bottle is filled up, the properties of the sound may change. Investigate how relevant parameters such as speed and dimensions of the jet, size and shape of the bottle or water temperature affect the sound.



Rojan Abdollahzade

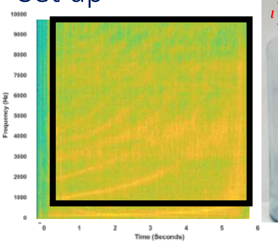
IYPT 2019
Team Iran

Road Map

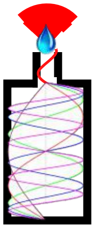
Approach

- The resonance in system
- Set up
- Different phases
- Natural frequency of the system
- The frequency of an one end closed pipe
- The effect of parameters
- Conclusion

Set up



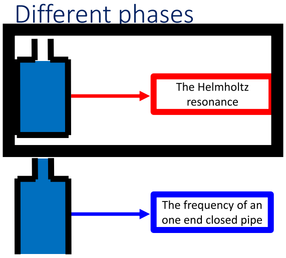
Explanation

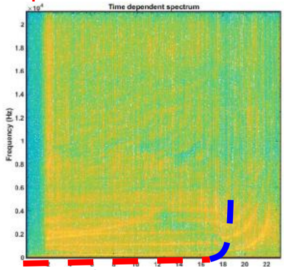


- $f_0 = \frac{1}{2\pi}\sqrt{\frac{k_0}{m_0}}$
- $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

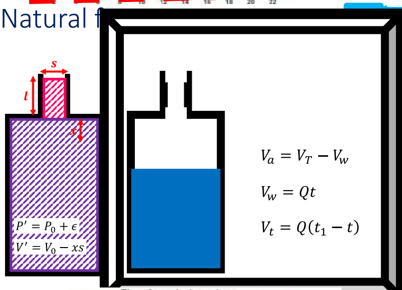
Resonance

Different phases





Natural f



$V_a = V_T - V_w$

$V_w = Qt$

$V_t = Q(t_1 - t)$

$ls\rho\ddot{x} + \frac{\gamma s^2 P_0}{V} x = 0$

k

$f = \frac{1}{2\pi}\sqrt{\frac{\gamma P_0}{\rho}}\sqrt{\frac{s}{LV}}$

$f_t = \frac{1}{2\pi} c \sqrt{\frac{s}{LQ}}\sqrt{\frac{1}{(t_1 - t)}}$

Natural frequency

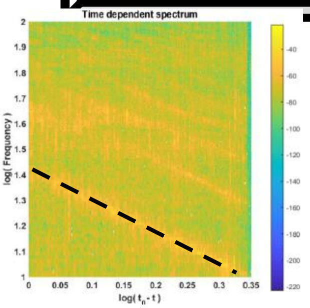
$\frac{1}{2\pi} c \sqrt{\frac{s}{LQ}} = \frac{1}{2\pi} 340 \sqrt{\frac{\pi(0.008)^2}{114\times10^{-6}\times0.029}}$ =422

$f_t = \frac{1}{2\pi} c \sqrt{\frac{s}{LQ}}\sqrt{\frac{1}{(t_1 - t)}}$

$\log f_t = \log c' + \frac{1}{2}\log(t_1 - t)$

constant

$\alpha \approx \frac{1}{2}$



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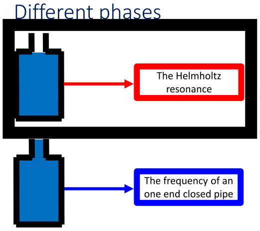
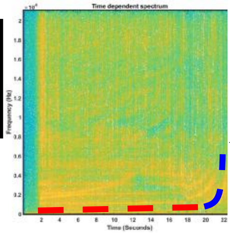
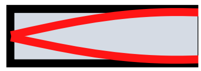
Different phases



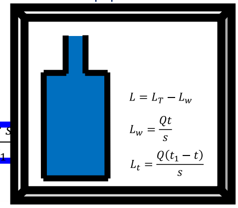


The frequency of an one end closed pipe



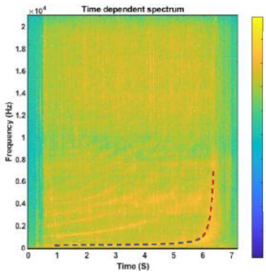
$$f_p = \frac{c}{4L}$$

$$L_t = \frac{Q(t_1 - t)}{s}$$

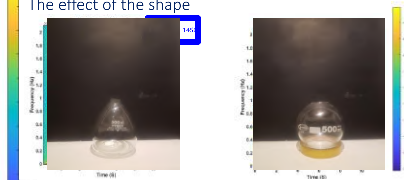


Natural frequency

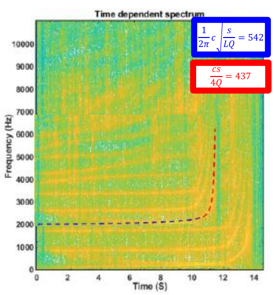
$$\frac{cs}{4Q} = 340\,\frac{\pi(0.008)^2}{4\times114\times10^{-6}} = 149$$

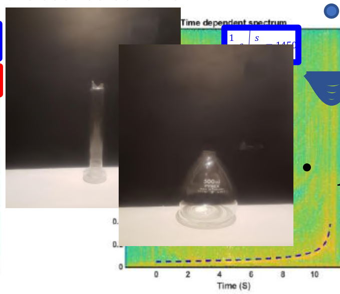


The effect of the shape

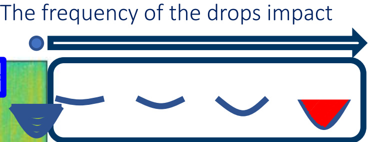


How the opening will change the sounds behavior?





The frequency of the drops impact



$$f = \frac{1}{2\pi r}\left(\frac{2\gamma P_0}{\rho}\right)^{\frac{1}{2}}$$

The effect of the temperature

$$f_t = \frac{1}{2\pi}c\sqrt{\frac{s}{LQ}}\sqrt{\frac{1}{(t_1 - t)}}$$

$$c = \sqrt{\frac{\gamma P_0}{\rho}}$$

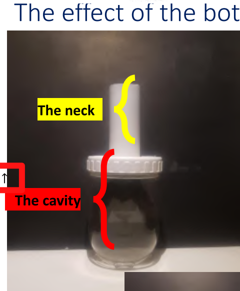
$$PV = nRT$$

$$\rho = \frac{M \times n}{V}$$

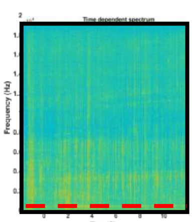
$$c = \sqrt{\frac{\gamma RT}{M}}$$

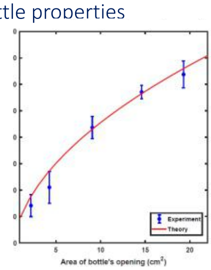
$T \uparrow \longrightarrow c \uparrow$

The effect of the bottle properties



The effect of the bottle properties





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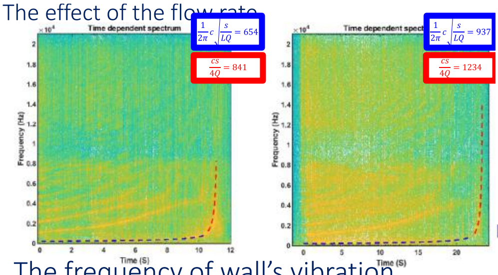
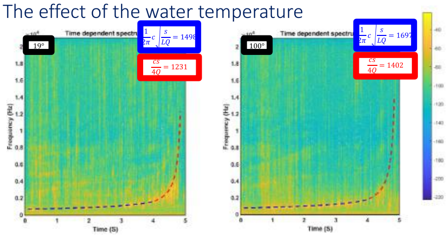
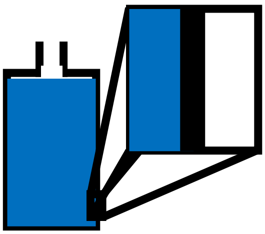
The effect of the flow rate



The effect of the water temperature



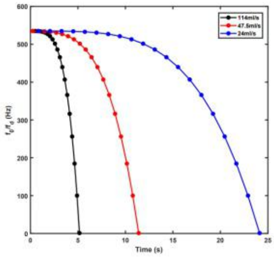
The frequency of wall's vibration



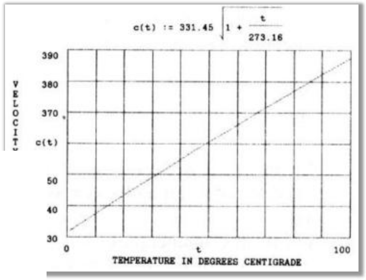
Camilla Shu Yu Yang ," Wine glass acoustics? ",The Journal of the Acoustical Society of America (2011)

- $\left(\frac{f_0}{f_d}\right)^2 \approx 1 + \frac{\rho_l R}{5\rho_g a}\left(1 - \frac{d}{H}\right)^4$

f_0	Frequency of empty bottle
f_d	Frequency of partially filled bottle
R	Radius of bottle
ρ_l	Density of liquid
ρ_g	Density of glass
a	Bottle thickness
d	Distance of water from top of bottle
H	Height of glass



The effect of the water temperature

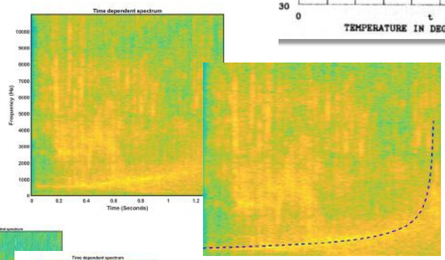


The effect of the bottle properties

$$\frac{1}{2\pi}c\sqrt{\frac{s}{LQ}} = \frac{1}{2\pi}340\sqrt{\frac{\pi(0.015)^2}{114\times10^{-6}\times0.07}} = 509$$

The effect of the bottle properties

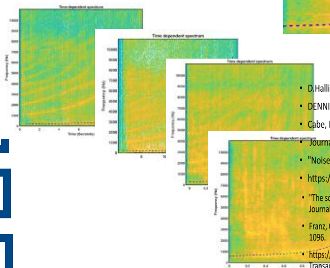
$$\frac{1}{2\pi}c\sqrt{\frac{s}{LQ}} = \frac{1}{2\pi}340\sqrt{\frac{\pi(0.017)^2}{114\times10^{-6}\times0.031}} = 865$$



Conclusion

- The Resonance
- The Natural frequency
- The frequency of an one end closed pipe
- The effect of parameters

$$f_t = \frac{1}{2\pi}c\sqrt{\frac{s}{LQ}}\sqrt{\frac{1}{(t_1 - t)}}$$



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11. Transformers

Hyung Kyu Jun,
Republic of Korea

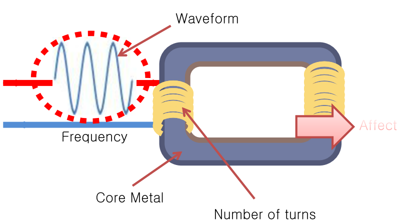
Non ideal behavior?

- Simple Transformer Law

$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$:Deviation from S.T.L

Non ideal behavior

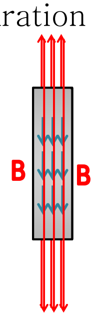
Parameters



$$\frac{V_1}{V_2}$$

Core loss equation

Core loss occurs by magnetic saturation of a core magnet

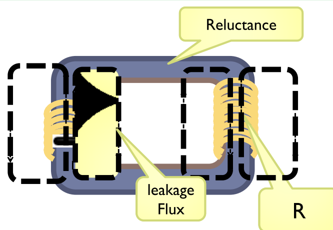


Problem

- Simple Transformer Law $\frac{V_1}{V_2} = \frac{N_1}{N_2}$
- *Investigate*

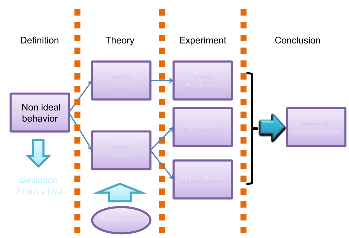
the importance of frequency and other parameters in determining the non- ideal behavior of transformers.

Non Ideal Behavior



Say, M. G. (February, 1984). *Alternating Current Machines 5E*
McLaren. *Elementary Electric Power and Machines*

Flowcharts



Core loss equation

$$coreloss\ P_c = Kf^{\alpha} B^{\beta}$$

K, α, β: constant

W.G. Hurley et al. Optimized Transformer Design: Inclusive of High-Frequency Effects (1998).

Wire loss equation

$$V = 4.44 NABf$$

$$P_{input} = VI \propto BfI = (const)$$

$$I \propto \frac{1}{Bf} \quad = \quad ^{2} \quad \propto \frac{1}{B^2 f^2}$$

Total Loss Equation

Republic of Korea

- Total loss = (Wire loss) + (Core loss)
- So,

$$P_{loss} = \frac{a}{B^2 f^2} + b f^{\alpha} B^{\beta}$$

Other parameters : Core metal, shape, etc.

$$P_{loss} = \frac{a}{B^2 f^2} + b f^{\alpha} B^{\beta}$$

$a = 0.001$, b=3, α=2, β=2, B= 0.04,

- → V_1/V_2 is maximized when P_{loss} is minimized

Frequency Graph

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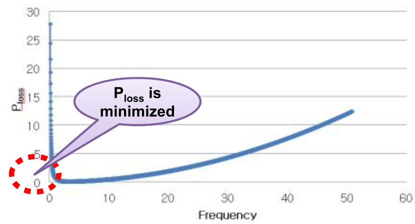

Changing parameters

1. Frequency→ *Optimal frequency?*
2. Core material
3. Number of turns ratio
4. Waveform

Optimal frequency

Republic of Korea

$$\frac{dP}{df} = \frac{d}{df}\left(\frac{a}{B^2 f^2} + b f^{\alpha} B^{\beta}\right) = 0$$

So, P minimizes(→V_1/V_2 maximizes) at

$$f_{optimum} = \left(\frac{2a}{\alpha b B^{\beta+2}}\right)^{\frac{1}{\alpha+2}}$$

Frequency $-V_1/V_2$ Graph

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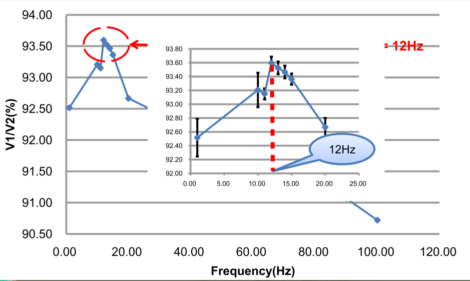


Calculating the optimal frequency

Republic of Korea

$a =$ 0.001942, $b =$ 0.0005,
$\alpha =$ 1.7, β = 1.9

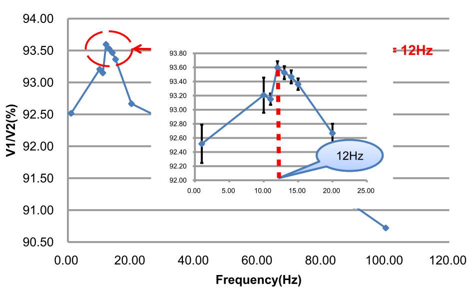
W.G. Hurley et al. Optimized Transformer Design: Inclusive of High-Frequency Effects (1998).

so,

$$f_{optimum} = \left(\frac{2a}{\alpha b B^{\beta+2}}\right)^{\frac{1}{\alpha+2}} = 17\text{Hz}$$

Frequency-V_1/V_2 Graph

Republic of Korea



Settings

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- At $f_{optimum} =$ 12Hz,

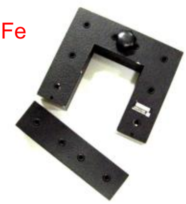
$$P_{loss} = \frac{a}{B^2 f^2} + b f^{\alpha} B^{\beta}$$

$$(core\ loss):(coil\ loss) \approx 1:1$$

Ratio between core loss & coil loss

Changing core metal

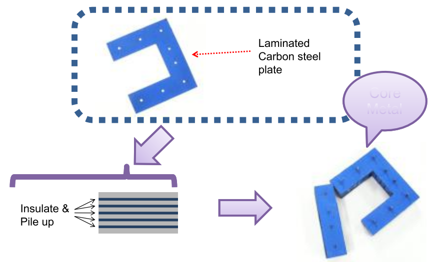
Fe



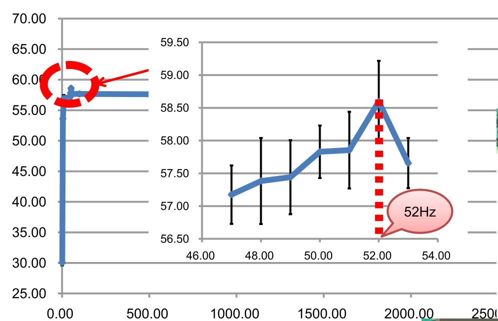
Smaller core loss

$a = 0.001942,\ b = 0.0005,\ \alpha = 1.7,\ \beta = 1.9 \Rightarrow f_{optimal} = 17Hz$

Changing core metal



Carbon steel core



Pure iron vs. carbon steel

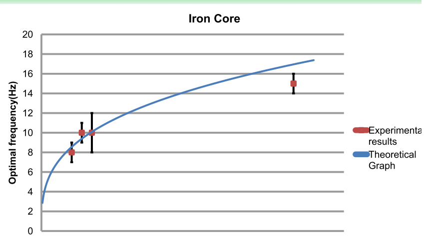
Values	Iron core	Carbon steel core
Theoretical Optimal frequency	17Hz	46Hz
Experimental results	12Hz	52Hz
Efficiency range	88~94%	53~57%

Changing number of turns

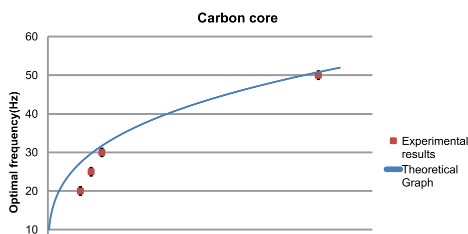
- $a \propto N$, So. The graph of

$$f_{optimum} = \left(\frac{2a}{\alpha b B^{\beta+2}}\right)^{\frac{1}{\alpha+2}}$$ Would be

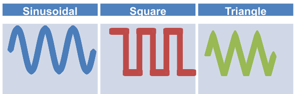
Number of turns Vs. optimum Frequency



Number of turns Vs. optimum Frequency



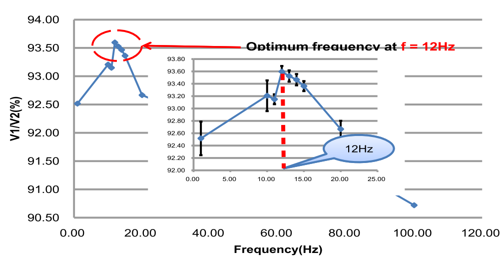
Changing waveforms

	Sinusoidal	Square	Triangle
Shape			
Waveform factor(K)	4.44	4.0	4.62

Since $\frac{1}{K^2} \propto a$

| | f_{opt}=17Hz | f_{opt}=18Hz | f_{opt}=17Hz |

Sinusoidal waveform



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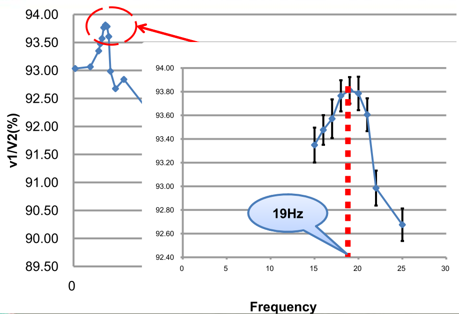
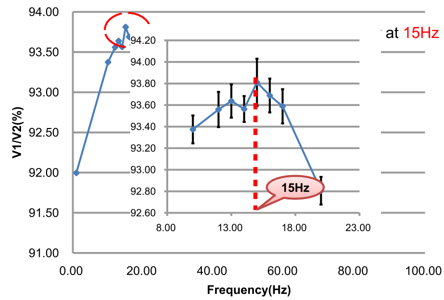
Square waveform



Triangle waveform



Theory Vs. Experimental data

	Sinusoidal	Square	Triangle
Theoretical Frequency	17Hz	18Hz	17Hz
Experimental data	12Hz	19Hz	15Hz

Conclusion

- Core loss
- Coil loss

$$\Big\} \; f_{optimum} = \left(\frac{2a}{\alpha b B^{\beta+2}} \right)^{\frac{1}{\alpha+2}}$$

– Core material

– Number of turns

– Waveform

3. String of Beads

Rojin Anbarafshan

IYPT 2012 BAD SAULGAU - GERMANY

IYPT 2012 Germany, National team of I. R. Iran

Problem

- A long string of beads is released from a beaker by pulling a sufficiently long part of the chain over the edge of the beaker. Due to gravity the speed of the string increases.
- At a certain moment the string no longer touches the edge of the beaker (see picture).
- Investigate and explain the phenomenon.

The Approach

Basic explanation → Understanding the physics

Theory for tensions → Governing equations → Contradiction → Resolving the contradiction

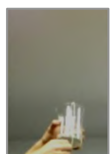
Designing different experiments → Understanding the problem

Observations

Interesting Phenomenon

Initial Prediction : The string doesn't go up (No peaks)

Observations : The string goes up and we see a peak



The interesting question is that why this string goes up and rises?!

Experimental Setup

String of beads : Several beads threaded together on a string [1]





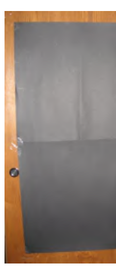
[1] Audio English Online Dictionary

Experimental Setup

- Blacking the beads each 1 meter to recognize them

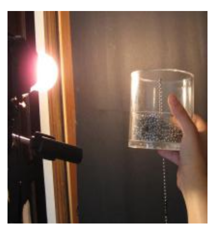
- Placing the string of beads in the beaker


- Fixing black papers on the wall for background



- Placing a high-speed camera for capturing the motion of the string (with high fps)


- Having a light while capturing video



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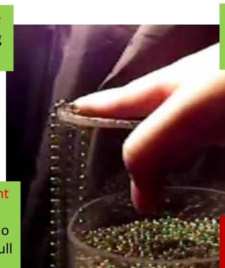
What happens?!

We need a force to pull the string down

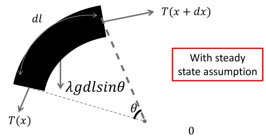
The string velocity is increasing due to gravity



In a certain moment the tensions and weight forces are no longer enough to pull the string down

In that moment the string rises to reduce its velocity

Is that possible theoretically?!

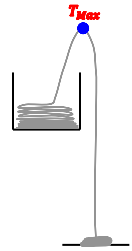


With steady state assumption

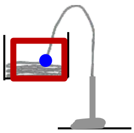
$$dT - \lambda g dl \sin\theta = ma \Rightarrow dT = m\overset{0}{\cancel{a_y}} + \lambda g dh$$

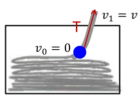
$$\frac{dT}{dh} > 0$$

T in the peak is maximum



Is that possible theoretically?!
Tensions





Second law of Newton: $T = \frac{dP}{dt} = \frac{d}{dt}(m.v)$

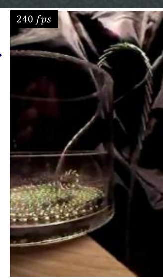
$$\left.\begin{array}{l} m = \lambda l \\ l = vt \end{array}\right\} \Rightarrow \frac{dm}{dt} = \lambda v, \quad \frac{dv}{dt} = v \quad \Rightarrow \boldsymbol{T = \lambda v^2}$$

A Contradiction?!...

$$\left.\begin{array}{r} T_{Max} = T_{peak} = \lambda v^2 - \lambda g r \\ T = \lambda v^2 \end{array}\right\} \Rightarrow T > T_{peak} \;\; ✗$$

The theory has an incorrect initial assumption: Steadiness

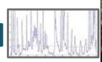
Unsteady Tension in Time



Arrangement → Unsteady tension

Rising & peak

The Approach



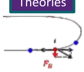
Experiment — Theories — Numerical

The unsteady tension Due to arrangement

Resolving the Contradiction

The extra force Due to bending resistance

Experiment — Theories — Numerical

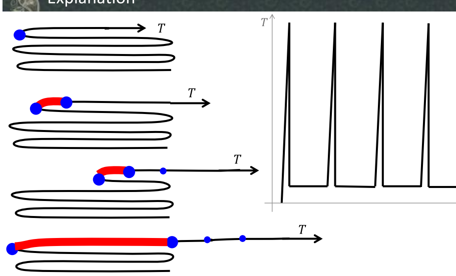


Unsteady Tension in Time
Resolving the contradiction Theoretically

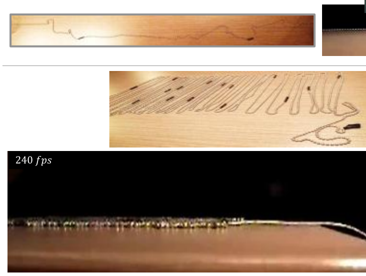
T_{peak} can be bigger than T in most of the times

$$\left.\begin{array}{l} P(t) = \lambda l v \\ P(t + \Delta t) = \lambda\left(l + \frac{\Delta x}{2}\right) v \\ T = \frac{dP}{dt} \end{array}\right\} \Rightarrow \boldsymbol{T = \frac{\lambda v^2}{2}}$$

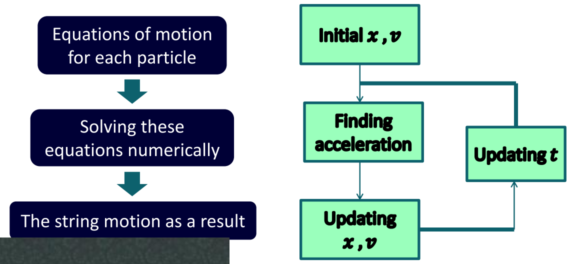
Unsteady Tension
Explanation



Arrangement



Numerical Method
Flowchart

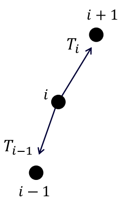


Numerical Scheme

Numerical Method
Modeling String

Assumption : 1. The distance between the beads is constant

2. No bending resistance



Known : $x_i, x_y, v_{xi}, v_{yi}, B_{xi}, B_{yi}, m$

Unknowns : a_{xi}, a_{yi}, T_i

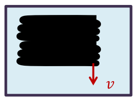
Equations : $1. \Sigma F_x = ma_x$

$2. \Sigma F_y = ma_y$

$3. \Delta a_x \, \Delta x + \Delta a_y \, \Delta y = \Delta {v_x}^2$

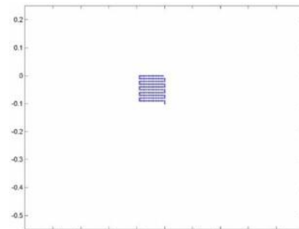
[Equations][Unknowns] = [known]

Unsteady Tension in Time
Resolving the Contradiction Numerically

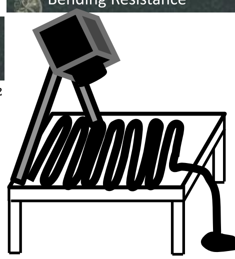


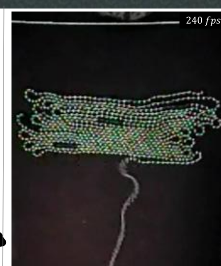
Initial condition : $B = 0$

$a(n) = 0$

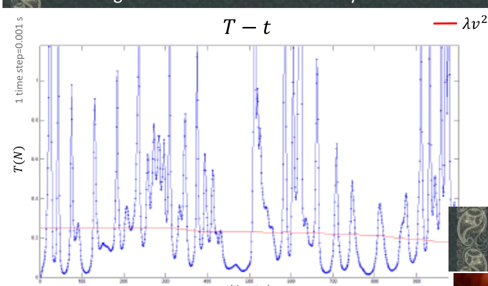


Extra Force
Bending Resistance

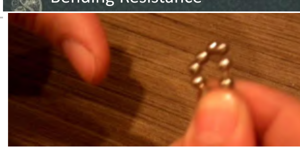




Unsteady Tension in Time
Resolving the contradiction Numerically



The Source of the Extra Force
Bending Resistance

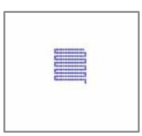


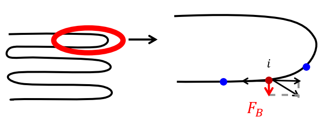
String of beads has bending resistance

The Source of the Extra Force
Numerical Solution

- In the numerical results : No extra force
- The thing was neglected was the bending resistance

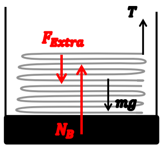
Due to the numerical solution we guess the reason of this extra force is the bending resistance





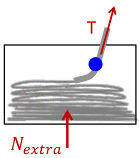
$$\sum_{i=1}^{n} F_B = F_{Extera}$$

The Bottom Force



No bending resistance: $N = mg$

Bending resistance: $N_B = mg + F_{extr}$

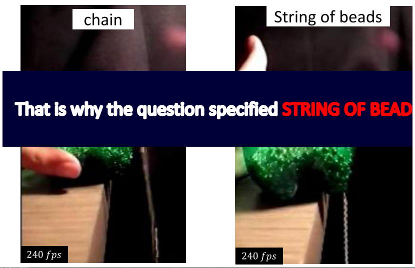


$N_B - N = N_{Extra}$ → Appli bottom (

$T + N_{Extra} = \lambda v^2 \quad \Rightarrow T < \lambda v^2$

In the Case of No Bending Resistance

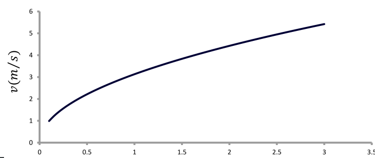
Chain or string of beads?!



Velocity vs. Height

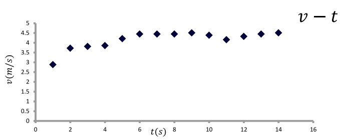
- If the string reaches a steady state we have:

$$\lambda gh = \lambda v^2 \quad \Rightarrow v = \sqrt{gh}$$

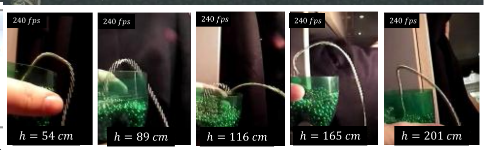


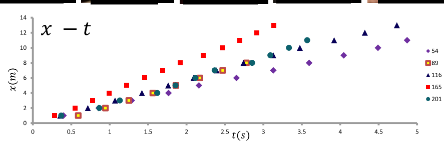
Experiments
Velocity

- Counting the number of frames between each 2 black beads



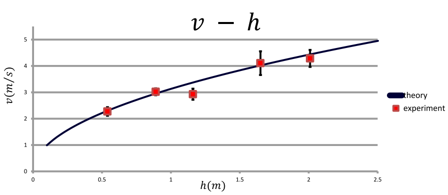
Experiment
Velocity





Theory vs. Experiment

- Defining an average velocity for each height
- Plotting the average velocity vs. height

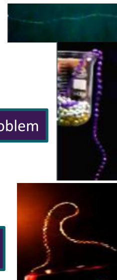


Reproducibility?!...
Waves and initial conditions

ART AN AMAZING FACT IN SCIENCE

The approach

- Wave point of view
  - Principles of waves
  - Source of waves in our problem
  - Different waves → Different shapes



Principles of Wave
Speed

- The speed of the wave on a string : $u = \sqrt{\frac{T}{\lambda}}$ [1]

The string with speed v
- v, u Same direction → The wave speed increases
- v, u Opposite direction → The wave speed decreases

[1] Fundamentals of physics , David Halliday , Robert Resnick , Jearl Walker , Volume 2, page 67

The Waves in Our Problem
Source and Speed

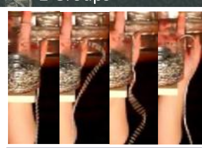
The impact of the beads with the rim → Producing waves

Pending the string is in touch with the rim → 2 groups of waves are produced

Speed of the waves: $u = \sqrt{\frac{T}{\lambda}} \xrightarrow{T=\lambda v^2} u = v$

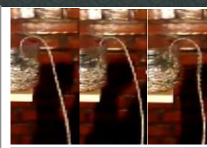
The speed of the waves is equal to the speed of the string

The waves in our problem
2 Groups



Group 1 : Waves in the direction of the string speed

$u = 2v$

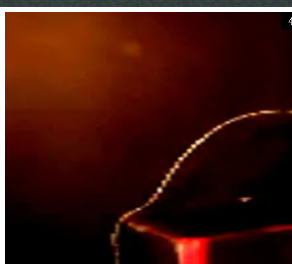


Group 2 : Waves in the opposite direction of the string speed

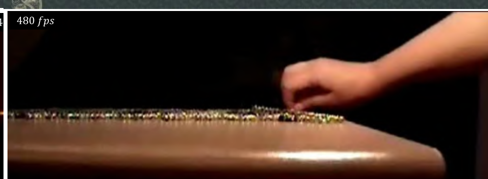
$u = 0$

Static waves

Different waves



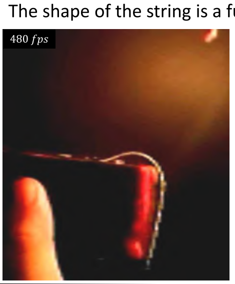
A Peak or A Wave?!



The shape of the string is the waves
Group 2 remain and they get similar to a peak

Interesting Waves - Interesting Shap

The shape of the string is a function of many things



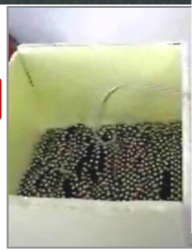
Conclusion

- The arrangement and the bending resistance of the string of beads cause unsteady tension and extra force
- The waves are exist in our problem due to the impact with the rim
- The shape of the string of beads is this waves

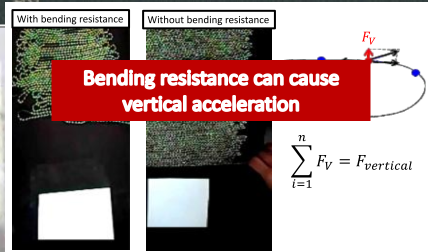
The whole phenomenon is a wave that rises because of this unsteady tension and extra force

Conclusion

$DOF\ system : 2n\ \ ,n \approx 1000$



Vertical Acceleration
Due to Bending Resistance



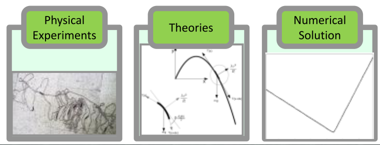
$$\sum_{i=1}^{n} F_V = F_{vertical}$$

The string dynamics is chaotic thus a direct solution seems impossible

Methods for understanding

Generally the string motion is chaotic → We present different methods for studying string motion

Methods

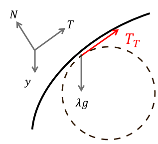


Theory Method
Base of the theory

Governing equations for chain dynamics → Solving the equations → Understanding the answer of the problem

1. Lagrangian point of view : The motion of every single particle
2. Eulerian point of view : The motion of a considered volume

Chain Theory
Lagrangian Point of View



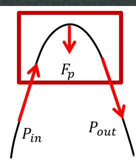
For each bead
↑
Base Lagrangian method: $F = ma$
↓
The motion of each bead

Force on the unit of length : $\frac{d\vec{F}}{ds} = \frac{dT_T}{ds}\hat{T} - \frac{T_T}{R}\hat{N} - \lambda g\hat{y}$

$$\frac{d\vec{F}}{ds} = \frac{dm}{ds}\vec{a} \overset{\frac{dm}{ds}=\lambda}{\Longrightarrow} \vec{a} = \frac{d\vec{F}}{\lambda ds}$$

Chain Theory
Eulerian Point of View

Base Eulerian method: *Control Volume*
↓
Study the whole volume

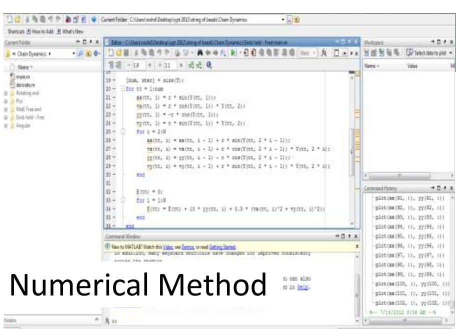


Momentum conservation:

Applied forces to the volume

$$P_{in}(t) - P_{out}(t) + P_{produced}(t) = P_{cm}(t) = ma$$

Input momentum — Output momentum — Center of mass momentum



Numerical Method

References

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- W. Tomaszewski ,P. Pieranski, The motion of the freely falling chain tip (2005)
- Anoop Grewal, Phillip Johnson,Andy Ruina, A chain that accelerates, rather than slows, due to collisions: how compression can cause tension(2011)
- Jean-Christophe Géminard ,Loïc Vanel, The motion of a freely falling chain tip. Force measurements.(2007)
- Pawe Fritzkowski ,Henryk Kaminski, Journal of Mechanics of Materials and Structures, volume 3, DYNAMICS OF A ROPE AS A RIGID MULTIBODY SYSTEM (2006)
- Waldemar Tomaszewski ,Piotr Pieranski, Dynamics of ropes and chains: I. the fall of the folded chain (2005)

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Problem no.6

Woodpecker Toy

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IYPT 2012
BAD SAULGAU - GERMANY

IYPT 2012 Germany, National team of I. R. Iran

Background
- Motion steps
- Woodpecker toy details

Theory
- Defined parameters
- Degrees of freedom
- Governing equation + Impact dynamics
- Numerical equation solving
- Simulation
- Results

Experiment
- Setup
- Image processing
- Sound Processing
- Frequency analysis (FFT Vs LSHE)

Main Approach

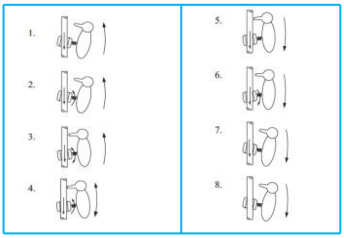
Theory Vs. Exp
- Diagrams

Conclusion

The Problem

A woodpecker toy (see picture) exhibits an oscillatory motion. Investigate and explain the motion of the toy.

Background-Motion Steps



Background-WP details

➢ *Consist of 3 rigid bodies :*

- *The woodpecker*
- *The sleeve*
- *The pole*

Theory- Defined parameters

➢ ***parameters:***

The woodpecker :

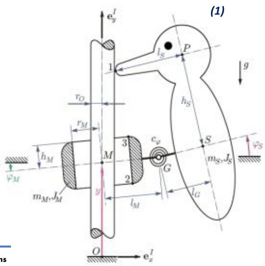
- S center of mass
- J_s moment of inertia
- ω_s angular velocity
- φ_s angular displacement

The sleeve :

- M center of mass
- m_M mass
- J_M moment of inertia
- ω_M angular velocity
- φ_M angular displacement
- y vertical displacement
- v vertical velocity

Others :

- C_φ angular stiffness

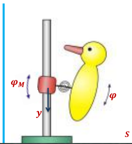


(1) Glocker, Ch.: Dynamics of Structure-Variant Systems

Theory- Degrees of freedom

➢ ***DOF of the system:***

$$q = \begin{pmatrix} y \\ \varphi_M \\ \varphi_S \end{pmatrix} \qquad u = \begin{pmatrix} v \\ \omega_M \\ \omega_S \end{pmatrix}$$



Theory-Equations

➢ ***Main equation of motion (second law of NEWTON) :***

$$F = m\ddot{q}$$

$$F + m\ddot{q} = 0$$

❖ ***Considering F for spring***

$$F = kq$$

❖ ***Matrix Form***

$$[k][q] + [M][\ddot{q}] = 0$$

$$kq + m\ddot{q} = 0$$

❖ ***Considering dt***

$$Fdt + md\dot{q} = 0$$

❖ ***Considering contact forces (dR)***

$$Mdu + Fdt + dR = 0$$

dR???

Theory-Impact dynamic

NOTE:
W is direction
Λ is force

$$dR = \Sigma(W_T\Lambda_T + W_N\Lambda_N)$$

➢ ***The way for finding Λ***

❖ ***We need ΔV*** $(v_{after\ impact} - v_{before\ impact})$

$$\xi = \Delta V$$

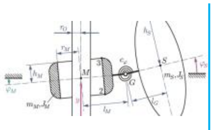
$$\xi = \gamma^+ + COR\,\gamma^-$$

$$\xi = \gamma^+ - COR\,\gamma^-$$

NOTE:
γ is Velocity

Theory-Impact dynamic

- ***Three gap Functions :***



$$g_{N1} = (l_M + l_G - l_S - r_O) - h_S \varphi_S$$
$$g_{N2} = (r_M - r_O) + h_M \varphi_M$$
$$g_{N3} = (r_M - r_O) - h_M \varphi_M$$

- ***Direction in WPT :***

$$w_{T1} = \begin{pmatrix} 1 \\ l_M \\ l_G - l_S \end{pmatrix} \quad w_{N1} = \begin{pmatrix} 0 \\ 0 \\ -h_S \end{pmatrix}$$
$$w_{T2} = \begin{pmatrix} 1 \\ r_M \\ 0 \end{pmatrix} \quad w_{N2} = \begin{pmatrix} 0 \\ h_M \\ 0 \end{pmatrix}$$
$$w_{T3} = \begin{pmatrix} 1 \\ r_M \\ 0 \end{pmatrix} \quad w_{N3} = \begin{pmatrix} 0 \\ -h_M \\ 0 \end{pmatrix}$$

- ***Contact Forces :***

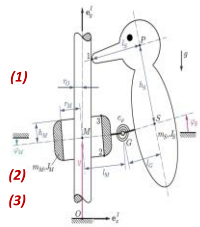
$$d\Lambda_{Ni} \in Upr(\zeta_{Ni})$$

$$\Lambda_{n_i}\xi_{N_i} = 0 \qquad \textbf{(1)}$$

$$d\Lambda_{Ti} \in \mu_i d\Lambda_{Ni} Sgn(\zeta_{Ti})$$

$$(\mu\Lambda_N + \Lambda_T)\xi_{Right} = 0 \qquad \textbf{(2)}$$
$$(\mu\Lambda_N - \Lambda_T)\ \xi_{Left} = 0 \qquad \textbf{(3)}$$

$$\xi_T = \xi_{Right} - \xi_{Left} \qquad \textbf{(4)}$$



Unknowns: $_T$ ξ_{Right} ξ_{Left} **dR**

Theory-Main equation

- ***Main equation of motion:***

$$Mdu + Fdt + dR = 0$$

$$F = \begin{pmatrix} -(m_S + m_M)g \\ -C_{\varphi}(\varphi_M - \varphi_S) - m_S l_M g \\ -C_{\varphi}(\varphi_S - \varphi_M) - m_S l_G g \end{pmatrix} \qquad M = \begin{pmatrix} m_S + m_M & m_S l_M & m_S l_G \\ m_S l_M & J_M + m_S l_M{}^2 & m_S l_M l_G \\ m_S l_G & m_S l_M l_G & J_S + m_S l_{G}{}^2 \end{pmatrix}$$

ω

NOTE:

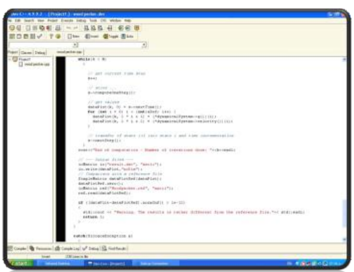
The mass matrix ***M***
The vector ***F***

Theory-Solving equation (Euler Method)

- ***Algorithm:***
- ❑ Choose a time step Δt and compute the midpoint and the endpoint
$$t^M = t^A + \frac{1}{2}\Delta t \qquad t^E = t^A + \Delta t$$
- ❑ Compute the midpoint displacements
$$q^M = q^A + \frac{1}{2}\Delta t . u^A$$
- ❑ Matrix calculations according to slide 7
$$M(u^E - u^A) - F\Delta t - \sum (w_{Ni}\Lambda_{Ni} + w_{Ti}\Lambda_{Ti}) = 0$$
- ❑ Computation of :
$$\gamma_i^E = w_i^T u^E \qquad \zeta_i = \gamma_i^E + \varepsilon_i \gamma_i^A$$
- ❑ 5. Computation of
$$\gamma_i^A = w_i^T u^A$$
$$q^E = q^M + \frac{1}{2}\Delta t . u^E$$

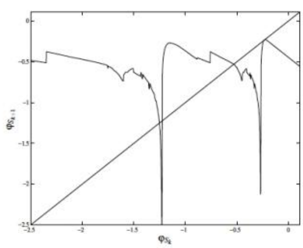
Theory-Numerical solution

- ***Programming:***



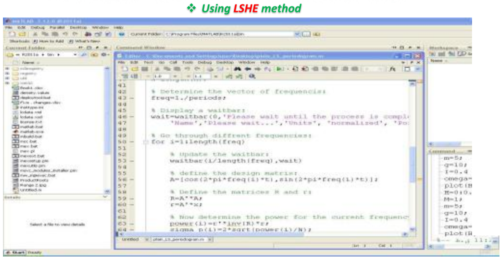
Theory-Poincare map (FFT)

- ***Poincare map:***

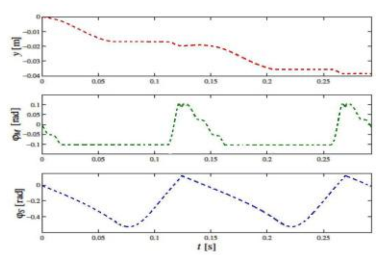


Theory-Finding frequency

- ***FFT Vs. LSHE :***
  - ❖ ***Gaps in trajectory + Discontinuity***
  - ❖ ***FFT is not accurate enough***
  - ❖ ***Using LSHE method***



Theory-Results

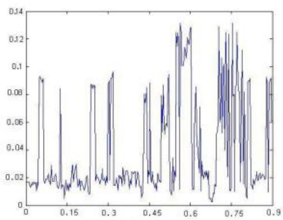


Experiment-Setup

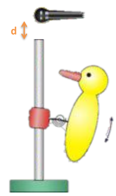
Specifications of WPT

geometry	radius of pole	$r_O = 0.0025\,\mathrm{m}$
	inner radius of sleeve	$r_M = 0.0031\,\mathrm{m}$
	$\frac{1}{2}$ height of sleeve	$h_M = 0.0058\,\mathrm{m}$
	distance M–G	$l_M = 0.010\,\mathrm{m}$
	distance G–S	$l_G = 0.015\,\mathrm{m}$
	distance S–P	$h_S = 0.02\,\mathrm{m}$
	length of beak P–1	$l_S = 0.0201\,\mathrm{m}$
inertias	mass, sleeve	$m_M = 0.0003\,\mathrm{kg}$
	mass, woodpecker	$m_S = 0.0045\,\mathrm{kg}$
	moment of inertia, sleeve	$J_M = 5.0 \cdot 10^{-9}\,\mathrm{kg\,m^2}$
	moment of inertia, woodpecker	$J_S = 7.0 \cdot 10^{-7}\,\mathrm{kg\,m^2}$

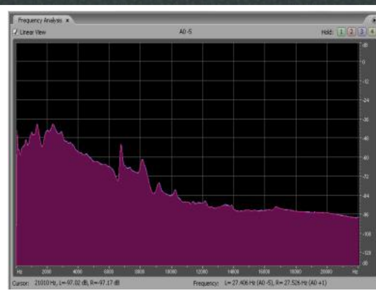
Experiment-Image Processing RESULTS



Experiment-Sound processing

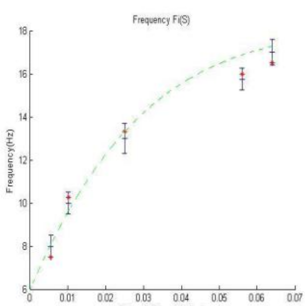


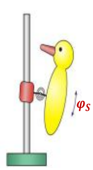
Experiment-Frequency analysis



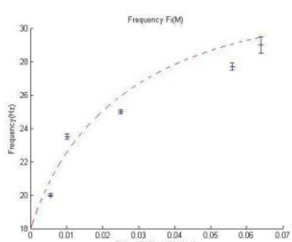
Experiment-Sound processing

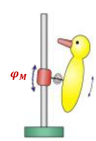
Theory Vs. Exp-Diagrams





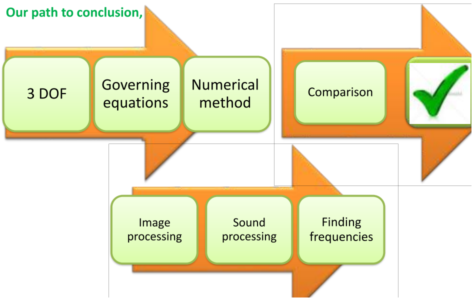
Theory Vs. Exp-Diagrams



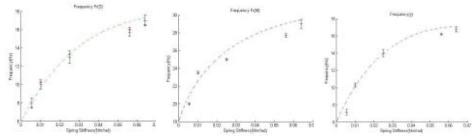


Conclusion

Our path to conclusion,



Conclusion



✓ Our Theoretical assumptions & calculations were almost accurate !

References

- Glocker, Ch.: Dynamics of Structure-Variant Systems. Graduate lecture for mechan- ical engineers at ETH Zurich.
- Glocker, Ch.: Set-Valued Force Laws: Dynamics of Non-Smooth Systems. Springer Verlag, Berlin, Heidelberg 2001.
- Leine, R.I., Glocker, Ch., van Campen, D.H.: Nonlinear Dynamics and Modeling of Various Wooden Toys with Impact and Friction. Journal of Vibration and Control 9, pp. 25–78, 2003.

Problem #17: DIDGERIDOO

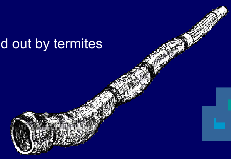
The 'didgeridoo' is a simple wind instrument traditionally made by the Australian aborigines from a hollowed-out log. It is, however, a remarkable instrument because of the wide variety of timbres that it produces. Investigate the nature of the sounds that can be produced and how they are formed.

Diogo Bercito

17th IYPT - AUSTRALIA - Brisbane - 24th June to 1st July

1.0 INTRODUCTION

- Origin: Australian aborigines
  - Eucalyptus branches hollowed out by termites
- Accoustic behavior: Cylinder



ular breathing (air enters through the nose and goes out through the

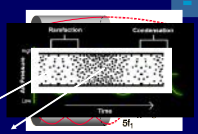
2.0 NECESSARY KNOWLEDGE

a) Resonance

- Natural frequencies of vibration
- Amplification of the sound
- Improvement of some frequencies (usually behind 200Hz)

b) Closed cylinders behavior

- Only produces odd harmonics (1,3,5...)
- Frequency is inversly proportional to the cylinder lenght



Rarefaction

Condensation

2.0.1 NECESSARY KNOWLEDGE (cont.)

c) Didgeridoo functioning

- Many different techniques and individual styles
- The sound is created by the vibration of the player's lips, being amplified by the resonance in some frequencies.
- Result: A bass intense sound that differs from the inicial buzz sound

3.0 EXPERIENCE

- 3.1 Material
- 3.2 Procedure

3.1 MATERIAL

- Three PVC Tubes
- Two Hollow Bamboos
- Sound analysis softwares
- Microphone
- Didgeridoo
- Measuring tape

3.2 PROCEDURE

- Measure each cylinder
- Play each cylinder
- Analyze each sound in the computer

3.3 MEASURE EACH CYLINDER

	Lenght (m)
PVC Cylinder #1	1.230 +/- 0.005
PVC Cylinder #2	1.170 +/- 0.005
PVC Cylinder #3	0.610 +/- 0.005
Bamboo #1	1.000 +/- 0.005
Bamboo #2	0.610 +/- 0.005
Didgeridoo	0.610 +/- 0.005

3.4 PLAY EACH CYLINDER

After playing them, it is possible to make some qualitative statements.

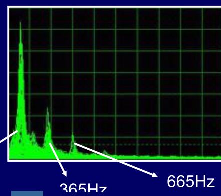
- The bigger is the cylinder, the basser is the sound

- Playing it with more intensity, the sound is more hig pitched

- The lips must be relaxed in order to have a bass an clear sound.

3.5 ANALYZE EACH SOUND ON THE COMPUTER

- Opened or closed cylinder?



127/127 =1
365/127=3
665/127=5

(approximated values)

Odd harmonics!

3.5.1 ANALYZE EACH SOUND ON THE COMPUTER (cont.)

Procedure:

- Create a white noise in the mouthpiece of the didgeridoo

- Close the other extremity

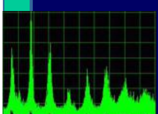
- Measure the sound using the microphone

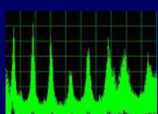
3.5.2 ANALYZE EACH SOUND ON THE COMPUTER (cont.)

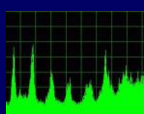
	FUNDAMENTAL FREQUENCY	1ST HARMONIC	2ND HARMONIC	3RD HARMONIC	4TH HARMONIC
PVC Cylinder #3	132Hz	390Hz	678Hz	921Hz	1216Hz
Bamboo #2	125Hz	412Hz	671Hz	950hz	1218Hz
Didgeridoo	133Hz	415Hz	678Hz	958Hz	1231Hz

Software error: 12 Hz

3.5.3 ANALYZE EACH SOUND ON THE COMPUTER (graphs.)


PVC #3


Bamboo #2


Didgeridoo


The white noise

4.0 DATA ANALYSIS

- It was already said that the bigger the cylinder is, the basser the sound is. We can apply the formula for closed cylinders to explain it.

$f = v/4L$ (Velocity of sound, at 16 ºCelcius, is 341m/s)

- Comparing the theoretical values with the experiment data for the three tubes of 0.61m (PVC #3, Bamboo #2 and Didgeridoo).

	Theoretical values	Experiment Data
PVC TUBE #3	139Hz	132Hz
Bamboo #2	139Hz	125Hz
Didgeridoo	139Hz	133Hz

4.0.3 DATA ANALYSIS (CONT. 3)

- The most intense it was played, the most high-pitched was the sound. We can apply the formula to explain it

$v = \lambda . f$

- It is possible to calculate the wave lenght for the three cylinders of 0.61 (PVC #3, Bamboo #2 and Didgeridoo)

341 = λ132 → λ = 341/132 = 2.6m
341 = λ125 → λ = 341/125 = 2.7m
341 = λ133 → λ = 341/133 = 2.6m

6.0 CONCLUSIONS

- Timbre

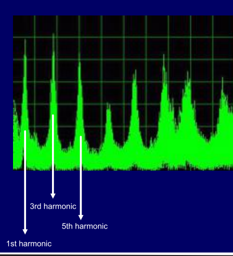
- Intensity of each harmonic

- It is possible to vary the timbre by:

-Varying the material of the didgeridoo

- Playing it with different intensities

- Varying the lenght of the didgeridoo



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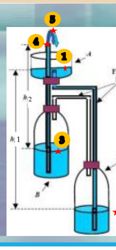
$H = h_1 - h_2 - (h_{loss1} + h_{loss2})$



$\lambda = \frac{64}{Re}$

$h_{loss} = \frac{\lambda L V_4^2}{dg}$

$Re = \frac{\rho V D}{\mu}$

$h_{loss} = \frac{32\,\mu\,V}{\rho d^2 g}\,l$

$H = h_1 - h_2 - (h_{loss1} + h_{loss2})$

$h_{loss} = \frac{32\mu V}{\rho d^2 g} l$

$H = (h_1 - h_2) - \frac{32\,\mu(l_1 + l_2)}{\rho d^2 g} V_4$

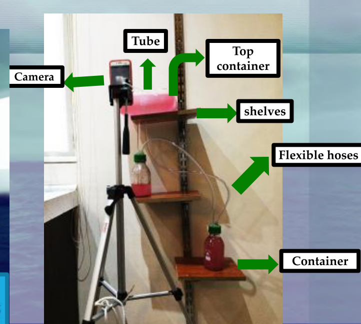
Height
- h1 (height from top of water in bottom container to middle container)
- Amount of water in bottom container
- Amount of water poured
- Length of tube 1

Pressure loss
- Kinematic viscosity
- Length of the hoses
- Temperature
- Diameter of tube 1

Did the experiment → Recorded the experiment

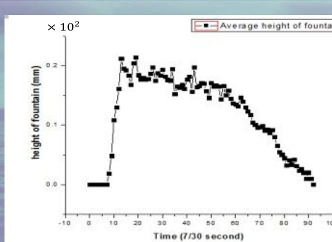
Repeated the process for several times

Got data of the videos using PASCO Capstone software → Analyzed the final data using Microsoft Excel → Fit the data using Origin8 software



Results and Effect of parameters

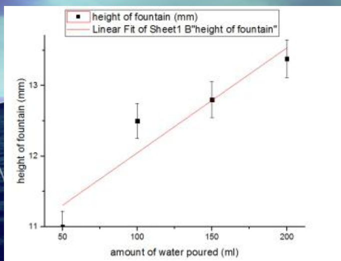
Height of the fountain



Parameters with a positive effect
h1 (height from top of water in bottom container to top container)
Amount of water poured
Temperature of water
Diameter of tube

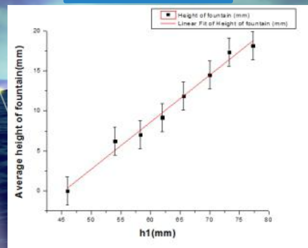
Parameters with a negative effect
Kinematic viscosity of fluid
Length of hoses
Amount of water in bottom container
Length of tube 1

Amount of water poured in the system



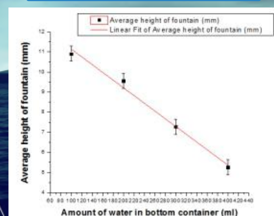
h1

$$H = 5.9h_1 - 2.7$$



Amount of water in bottom container

$$H = -2V_{water} + 1.3$$



Conclusion

Conclusion

We were able to construct a heron's fountain and investigate the relevant parameters on the height of the fountain, one of the most important parameters was h1, we were also able to predict the height of the fountain.

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PROBLEM

Problem NO.13
It is commonly known that an hourglass **changes its weight** (as measured by a scale)**while flowing**. Investigate this phenomenon.

FLOW CHART

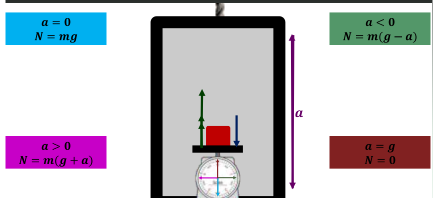
Introduction
- Does the scale shows the real weight?
- How does the hourglass works?
- Set up
- Flow Rate

Theory
- The first approximation
- Experiment prediction
- Center of mass
- The second approximation
- Experiment prediction

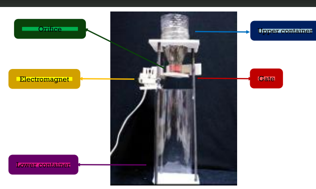
Experiment
- The effect of density
- The effect of orifice
- The effect of height
- The effect of shape of upper container
- The effect of lower container
- Sand behavior

Results
- Theory and experiment comparison
- Effective parameters
- Conclusion

DOES THE SCALE SHOWS THE REAL WEIGHT?

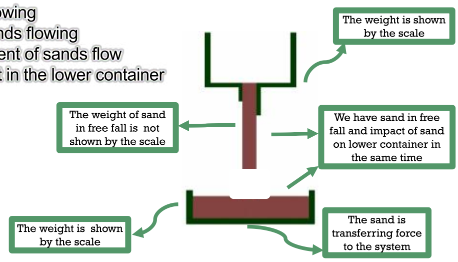


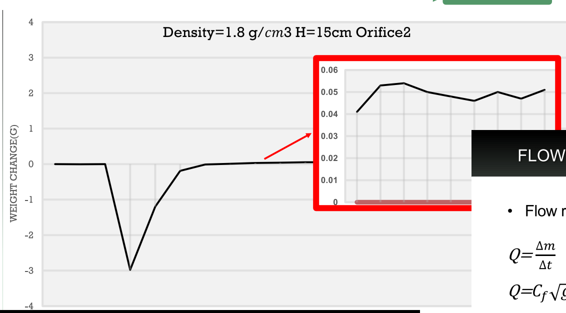
SET UP



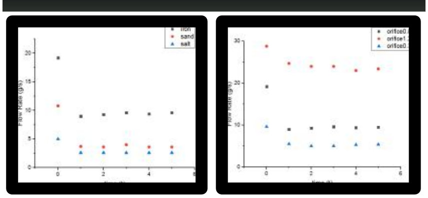
HOW DOES THE HOURGLASS WORKS?

1) Sand starts flowing
2) During the sands flowing
3) The last moment of sands flow
4) Sand is at rest in the lower container





FLOW RATE



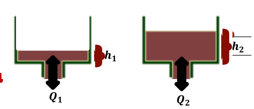
FLOW RATE

- Flow rate

$$Q=\frac{\Delta m}{\Delta t}$$

$$Q=C_f\sqrt{g}\rho_b(D_O-kd_p)^{\frac{5}{2}}$$

$Q_1=Q_2$



[2]Beverloo W. A., Leniger H. A. and Van de Velde J.: The flow of granular material through orifices. J. Chem. Eng. Sci. 15, (1961).

THEORY

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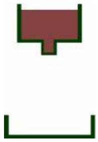


The falling phase: ΔW_1 The steady phase: ΔW_2 The impact pha

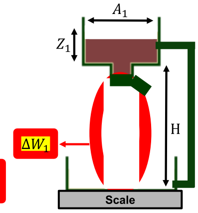
THE FALLING PHASE:

$$h = \frac{1}{2} g \Delta t_1^{\ 2} \Rightarrow \Delta t_1 = \sqrt{\frac{2h}{g}}$$

$$\Delta W_1 = -\Delta M_1 g$$

Flow Rate: $Q = \frac{\Delta m}{\Delta t}$

$$\Delta W_1 = -Q\sqrt{2gH}$$

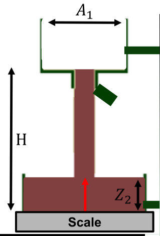


THE STEADY PHASE:

$$F = \frac{\Delta P}{\Delta t} = Qv = Q\sqrt{2gH}$$

* Non-elastic collision

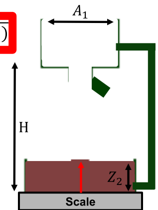
$$\Delta W_2 = Q\sqrt{2g(H - Z_2)} - Q\sqrt{2g(H - Z_2)} = 0$$



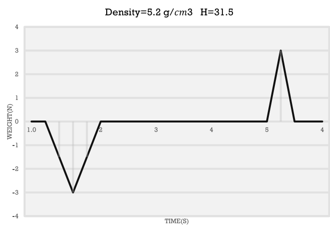
THE IMPACT PHASE:

$$\frac{1}{2}mv^2 = mg(H - Z_1) \Rightarrow v = \sqrt{2g(H - Z_2)}$$

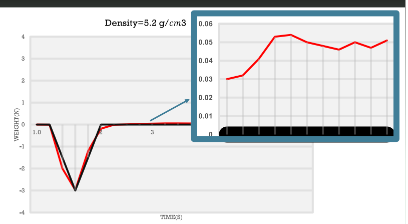
$$\Delta W_3 = \frac{\Delta P}{\Delta t} = Qv = Q\sqrt{2g(H - Z_2)}$$



EXPERIMENT PREDICTION



EXPERIMENT PREDICTION



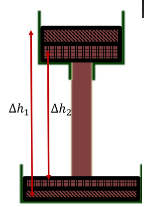
CENTER OF MASS

$$V_1 = \frac{\Delta h_1}{\Delta t}$$

$$V_2 = \frac{\Delta h_2}{\Delta t}$$

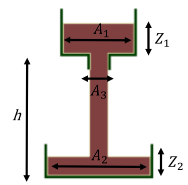
$$V_1 > V_2$$

$$a = \frac{(-V_2) - (-V_1)}{\Delta t} = \frac{V_1 - V_2}{\Delta t}$$



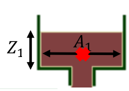
THE STEADY PHASE

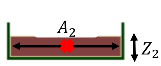
- $\Delta W_2 = M\left(g + \frac{d^2 z_{cm}}{dt}\right)$

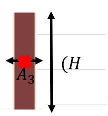


THE STEADY PHASE

- $Z_{cm} = \frac{\rho A_1 Z_1\left(\frac{Z_1}{2} + h\right)}{M} + \frac{\frac{1}{2}\rho A_2 Z_2^{\ 2}}{M} + \frac{\rho A_3 (h - Z_2)(\frac{h - Z_2}{2}}{M}$



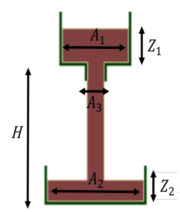




THE STEADY PHASE

- $\frac{dZ_1}{dt} = -\frac{Q}{\rho A_1}$

- $\frac{dZ_2}{dt} = \frac{Q}{\rho(A_1 - A_3)}$



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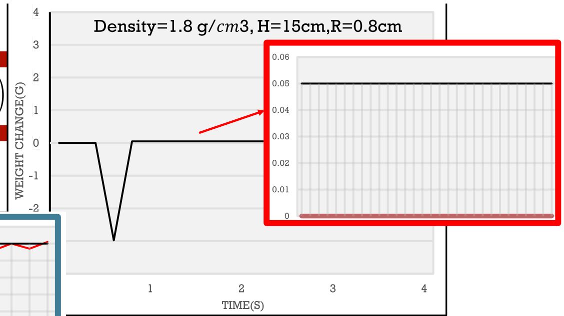
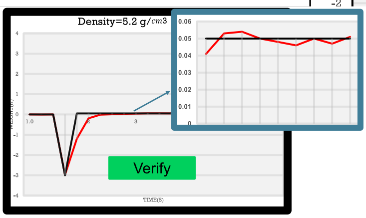
STEADY PHASE

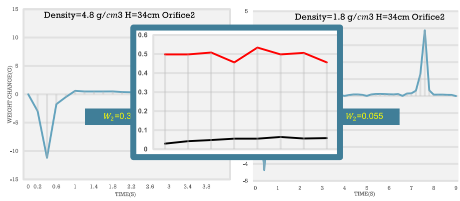
$$\Delta W_2 = M\frac{d^2 z_{cm}}{dt^2 \rho} = \frac{Q^2}{\rho}\left(\frac{A_1}{(A_1 - A_3)^2} + \frac{1}{A_2} - \frac{A_3}{(A_1 - A_3)}\right)$$



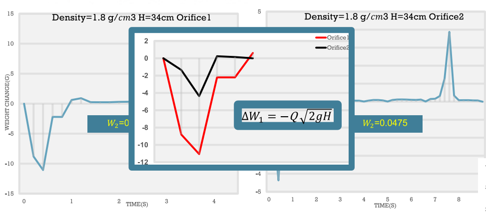
EXPERIMENT PREDICTION



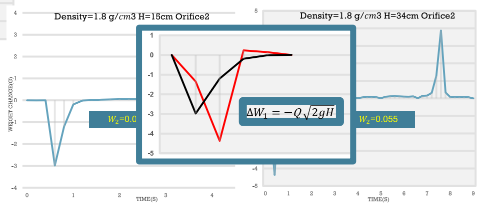
THE EFFECT OF DENSITY

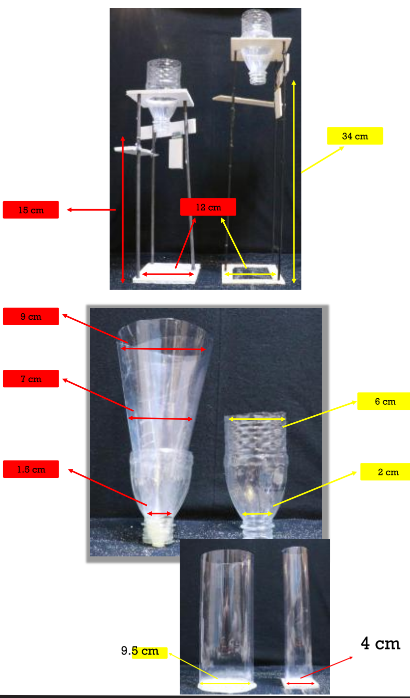


THE EFFECT OF ORIFICE

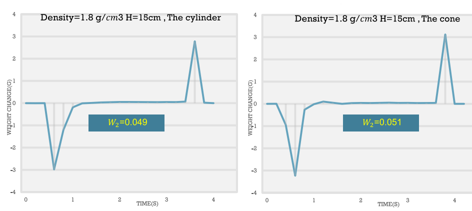


THE EFFECT OF HEIGHT

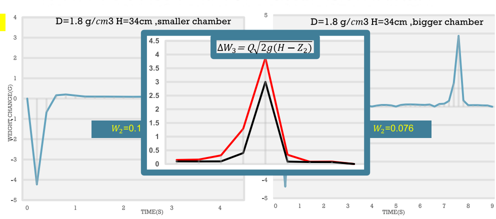




THE EFFECT OF SHAPE OF UPPER CONTAINER



THE EFFECT OF AREA OF LOWER CONTAINER



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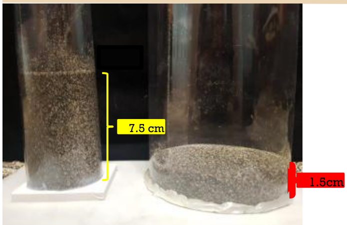
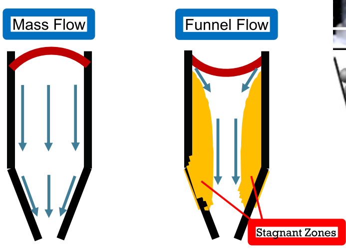
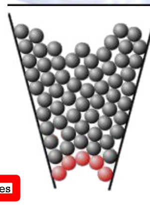
SAND BEHAVIOR

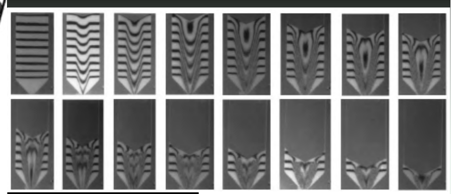






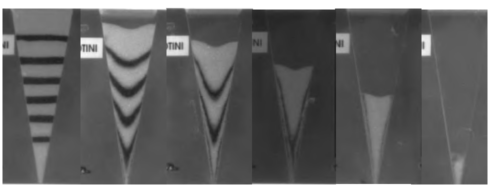
[3]http://www.sciencedirect.com/science/article/pii/S0032591017306496

SAND BEHAVIOR



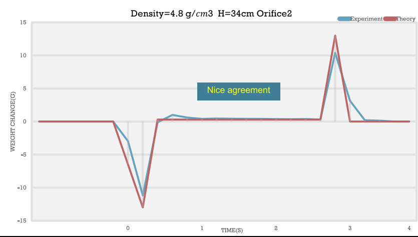
[3]A Mills, S Day and S Parkes , Department of Geology, The University, Leicester LE1 7RH, UK, (1996)

SAND BEHAVIOR

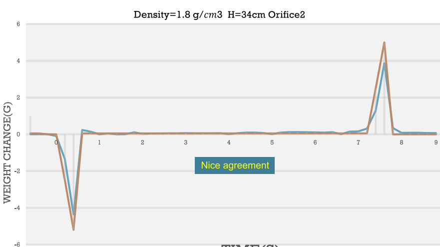


[3]A A Mills, S Day and S Parkes , Department of Geology, The University, Leicester LE1 7RH, UK, (1996)

THEORY AND EXPERIMENT COMPARISON

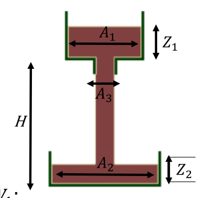


THEORY AND EXPERIMENT COMPARISON



EFFECTIVE PARAMETERS

- The effective parameters for W_2:
  - $A_1 \uparrow \Rightarrow W_2 \downarrow$
  - $A_2 \uparrow \Rightarrow W_2 \downarrow$
  - $Q \uparrow \Rightarrow W_2 \uparrow$



The effective parameters for W_3, W_1:

- $Q \uparrow \Rightarrow W_3, W_1 \uparrow$
- $H \uparrow \Rightarrow W_3, W_1 \uparrow$
- $Z_2 \uparrow \Rightarrow W_3 \downarrow$

- The effective parameters for Q:
  - Density
  - Diameter of the orifice
  - Diameter of the Particle
  - Diameter of the container
  - Angle of upper container

CONCLUSION

- We have two competing effects while hourglass following , the weight of sand in free fall and the impact of sand in the base of the hourglass.
- For determining the magnitude of forces we presented a theory and the relevant parameters were detected.
- We perform the experiment using even sub-standard equipment and achieve quantitative agreement with theory.


ART AN AMAZING FACT IN SCIENCE
6. Hurricane Balls
Reporter: Negar Rahimi
Two steel balls that are joined together can be spun at incredibly high frequency by first spinning them by hand and then blowing on them through a tube, e.g. a drinking straw. Explain and investigate this phenomenon.
IYPT 2019 Team Iran
Explanation
TOP VIEW
Outline
Experimental setup
Qualitative explanation and theory
Quantitative explanation and results
Conclusion
Rising
Speeding
Steady State
Explanation
Surface Inconsistencies
Blower increases angular velocity
TOP VIEW
Experimental setup and data processing
Glass surface
Hurricane balls
15 L/min from oxygen capsule
Camera filming at 240fps
3mm
5 mm
7mm
7.5mm
Tracker
Angle (degrees)
Angular velocity (rad/s)
Why does the ball rise?
Qualitative explanation
Steady State
Speeding
One point momentarily loses contact
Lagrangian approach
3 rotational degrees of freedom
$U = (2mgrcos\theta)$
$\mathcal{L}(\dot\theta, \dot\psi, \dot\varphi) = K - U$
3 rotational degrees of freedom
$\theta = 90$
$I_3\ddot\theta = 2mgr - I_1\dot\psi\dot\phi$
$\ddot\theta > 0$
$2mgr > I_1\dot\psi\dot\phi$
$\dot\psi\dot\phi > \frac{2mgr}{I_1}$
Increases N_A
Decreases $\dot\psi$
Rises center of mass
Speeding
$\vec{\alpha} = \vec{\dot\varphi} \times \vec{\dot\psi}$
Γ_{N_A} causes the balls to speed up (α)
$F_b > F_A$
TOP VIEW
$v_r = v_0 - v$
Drag force

ART AN AMAZING FACT IN SCIENCE

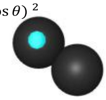
Steady state

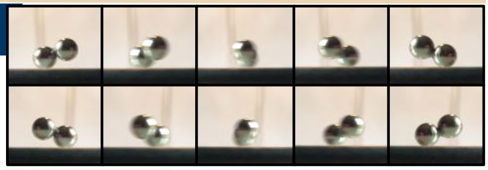
$U = 2mgrcos\theta$

$K = \frac{1}{2} I_2 \dot{\varphi}^{\,2} \sin^2\theta + \frac{1}{2} I_3 \dot{\theta}^{\,2} + \frac{1}{2} I_1 (\dot{\psi} + \dot{\varphi}\cos\theta)^2$

$\mathcal{L}(\dot{\theta}, \dot{\psi}, \dot{\varphi}) = K - U$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = \frac{\partial L}{\partial \theta} = 0$





2 *Rolling without sliding*

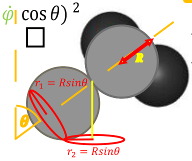
Steady state

$U = 2mgrcos\theta$

$K = \frac{1}{2} I_2 \dot{\varphi}^{\,2} \sin^2\theta + \frac{1}{2} I_3 \dot{\theta}^{\,2} + \frac{1}{2} I_1 (\dot{\psi} + \dot{\varphi}\cos\theta)^2$

$\mathcal{L}(\dot{\theta}, \dot{\psi}, \dot{\varphi}) \quad K - U$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = \frac{\partial L}{\partial \theta} = 0$



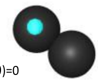
Rolling without sliding

Steady state

$\dot{\theta} = 0$

Rolling without sliding

constraints



$I\ddot{\theta} - (I_1(\dot{\psi} - \dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta + \frac{1}{2}\dot{\varphi}^2\sin\theta\cos\theta + 2mgrsin\theta) = 0$

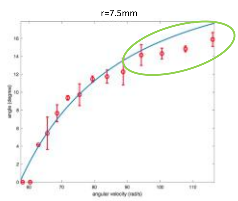
$I\ddot{\theta} = -I_1(\dot{\psi} - \dot{\varphi}\cos\theta)\dot{\varphi}\sin\theta + \frac{1}{2}\dot{\varphi}^2\sin\theta\cos\theta + 2mgrsin\theta$

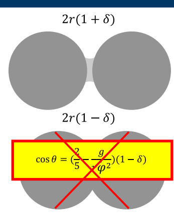
$I_1 = 2 \times (\frac{2}{5} mr^2)$

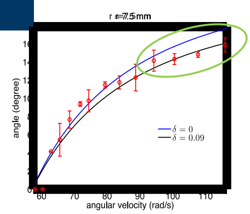
$I_2 = 2 \times (\frac{7}{5} mr^2)$

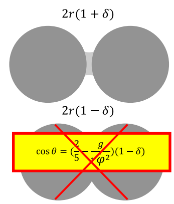
$\cos\theta = \frac{2}{5} - \frac{g}{r\dot{\varphi}^2}$

Corrections for imperfect balls

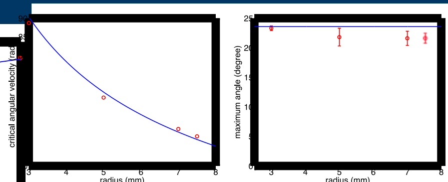




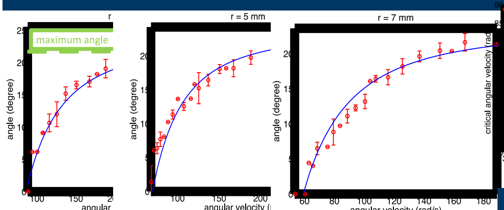




Critical angular velocity and maximum angle



Theory-experiment comparison



Conclusion

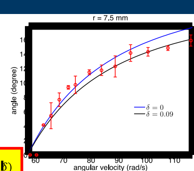
1 3 phases:

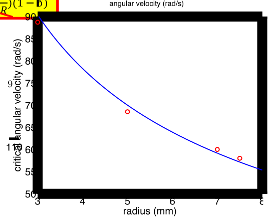
- Rising
- Speeding
- Steady state

2 Theoretical predictions

$\dot{\psi}\dot{\varphi} > \frac{2mgr}{I_1}$

$\cos\theta = (\frac{2}{5} - \frac{g}{\dot{\varphi}^2 R})(1-\delta)$

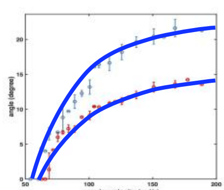




Triple-balls experiment

$I_x = 2\left(\frac{2}{5} m_1 r_1^{\,2}\right) + \frac{2}{5} m_2 r_2^{\,2}$

$I_y = 2\left(\frac{2}{5} m_1 r_1^{\,2} + m_1 (r_1 + r_1)^2\right) + \frac{2}{5} m_2$



References

1 Jackson, David P., David Mertens, and Brett J. Pearson. "Hurricane Balls: A rigid body-motion project for undergraduates." American Journal of Physics 83.11 (2015)

2 Andersen, W. L., and Steven Werner. "The dynamics of hurricane balls." European Journal of Physics 36.5 (2015)

3 Cross, Rod. "The rise and fall of spinning tops." American Journal of Physics 81.4 (2013)

4 Tornado Spheres by the National British Science museum

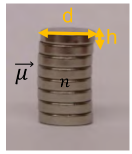
16

Magnetic Brakes

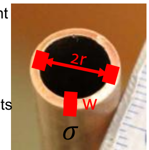
Matej Badin

Task

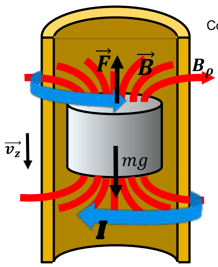
When a strong magnet falls down a non ferromagnetic metal tube, it will experience a retarding force. Investigate the phenomenon.



$\vec{\mu}$ Magnetic moment
σ El. conductivity
d Diameter
h Magnet height
n Number of magnets
r Radius of tube
w Width of wall



Qualitative Explanation



Consequence of an electromagnetic induction

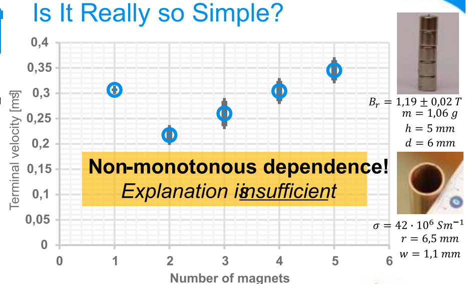
$$I \sim -\sigma \frac{d\phi_z}{dt} \sim \sigma v_z B_\rho$$

$$F \sim B_\rho I 2\pi r \sim v_z \sigma B_\rho^2 2\pi r$$

Terminal velocity

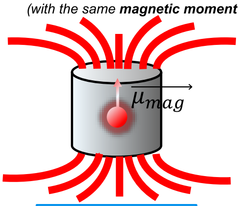
$$v_z \sim \frac{mg}{\sigma B_\rho^2 r}$$

Is It Really so Simple?



Magnetic Dipole Model

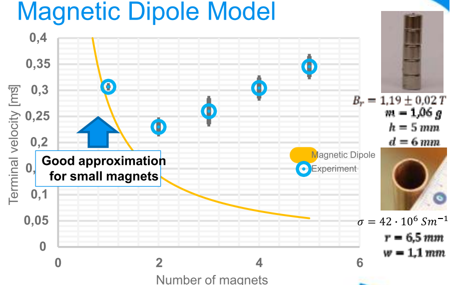
Threatening the magnetic field as the one created by **magnetic dipole**
*(with the same **magnetic moment**)*



$$F = \frac{45\mu_0^2 \sigma \mu_{mag}^2 v}{1024} \frac{w}{r^4}$$

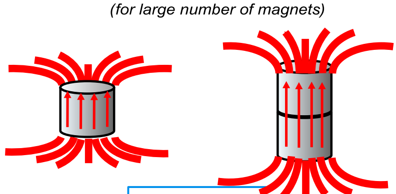
(Derivation in appendices

Magnetic Dipole Model



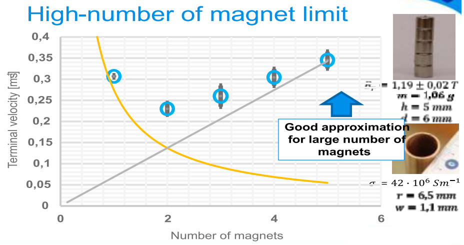
High-number of magnet limit

(for large number of magnets)

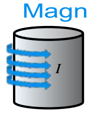


$$v \sim n \frac{mg}{B_{eff}^2} \sim n \frac{mg}{\mu_{eff}^2}$$

High-number of magnet limit



Magnetic Dipole Model: Too rough



Instead we used expressions for **solenoid field**
(Equivalent to uniformly magnetized cylindrical magnets)

Electromagnetism **9.2 pg.319**
(Proof in appendix)

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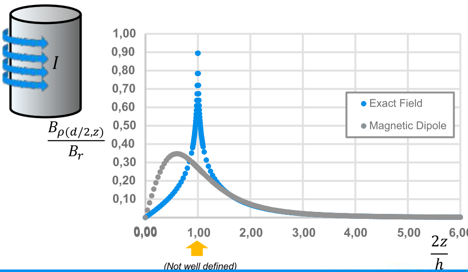
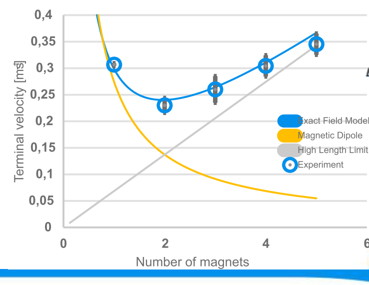
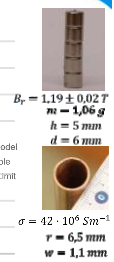
Magnetic Dipole Model: Too rough



Exact Field Model



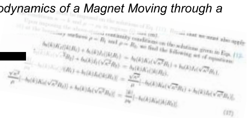


Summary of (Good) Existing Work

M. Hossein Partovi, Eliza J. Morris, *Electrodynamics of a Magnet Moving through a Conducting Pipe*, Can. J. Phys. 84, (2006)

\+ **Exact solution** [redacted]

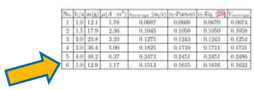
Cumbersome to handle



Norman Derby, Stanislaw Olbert, *Cylindrical Magnets and Ideal Solenoids* 78, Issue 3, pp. 229-235 (2010)

\+ **Exact solution of the field of** [redacted]

Experimental drawback

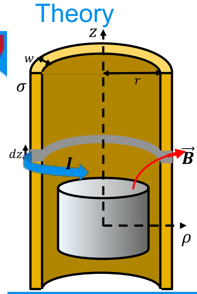


Goals of Our Work

1 Simpler theory, numerical approach

2 Experimentally **investigate** large number of parameters

Theory



&

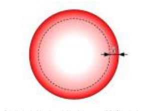
Calculating infinitesimal forces from each ring

$$F = -v_z \sigma 2\pi \left(r + \frac{w}{2}\right) w \int_{-\infty}^{\infty} B^2_{\rho(\rho,z)} dz$$

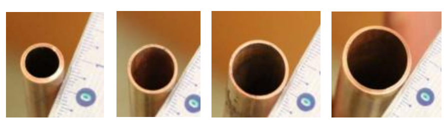
Numerical approach with Exact Magnetic Field

Our Approximations

- Quasi-static limit ($v \ll c$)
- Skin effect neglected
- Inducted currents in magnet neglected (Self-inductance)
- Cylindrical symmetry of the system
- Magnet falls down always in the



Cu Tubes



Tube	N.1	N.2	N.3	N.4
Inner radius [mm]	4,75	6,5	7,8	10,1
Width [mm]	1,25	1,1	1,2	1,02
Conductivity [10^6 Sm^{-1}]	42 2	42 2	42 2	42 2

Measured using Kelvin bridge

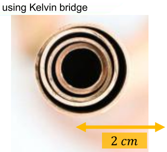
Uniformly Magnetized NdFeB Magnets

(Photographs not in scale)

(Data for one magnet. Remanence given by the manufactor)

	N.1	N.2	N.3	N.4	N.5
Mass [g]	1,06	1,88	2,09	6,54	9,54
Remanence [T]	1,19 0,02	1,33	1,19 0,02	1,19 0,02	1,19 0,02
Magnetic moment [Am^2]	0,13 0,001	0,25 0,1	0,54 0,1	0,835 0,15	1,22 0,01
Diamater [mm]	6	8	12	15	18
Height of the magnet [mm]	5	5	5	5	5

Cu Tubes



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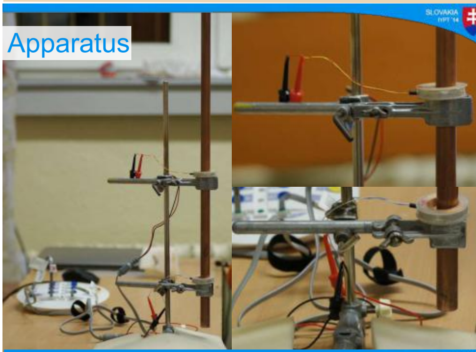

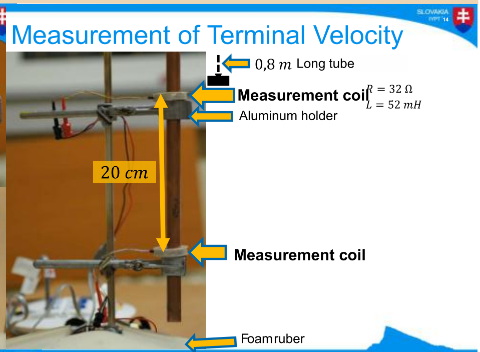

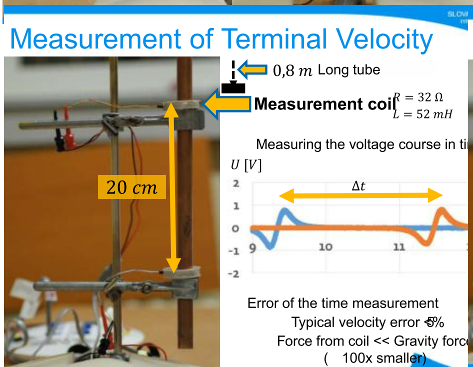

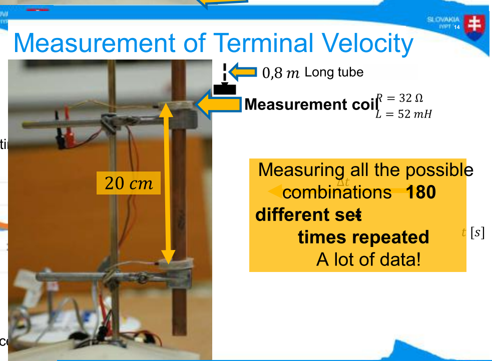

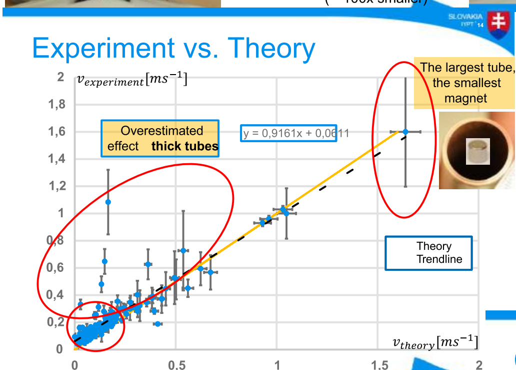

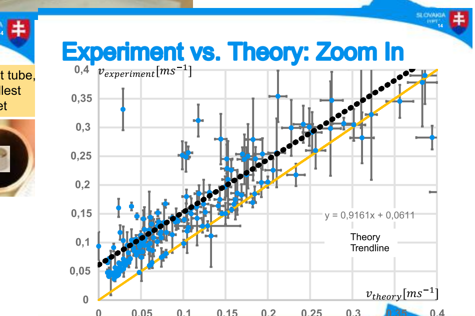

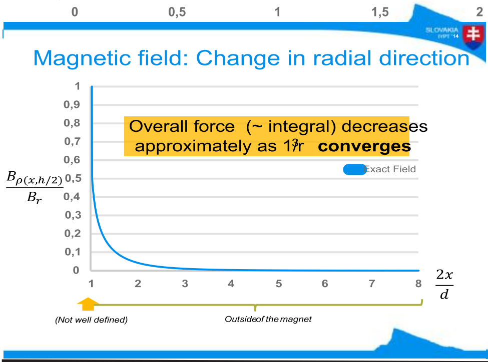

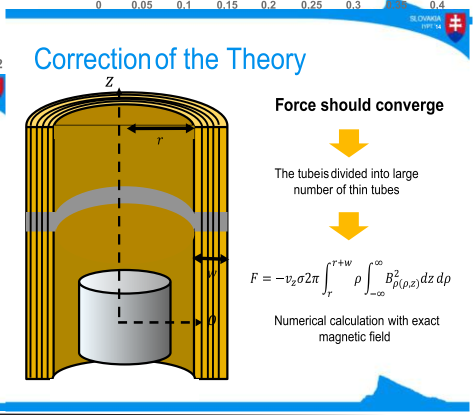

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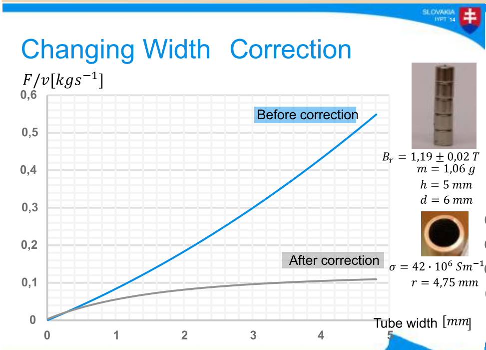
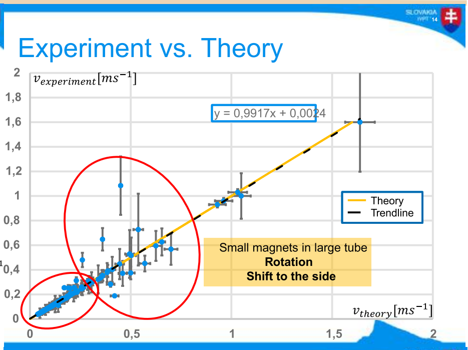
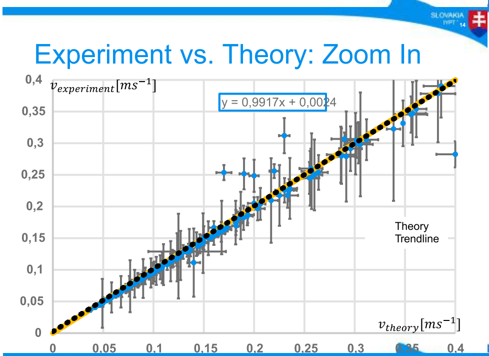
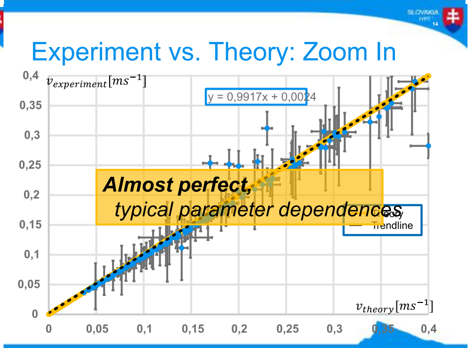
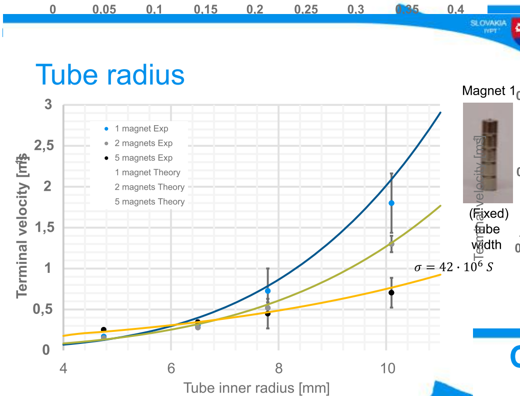
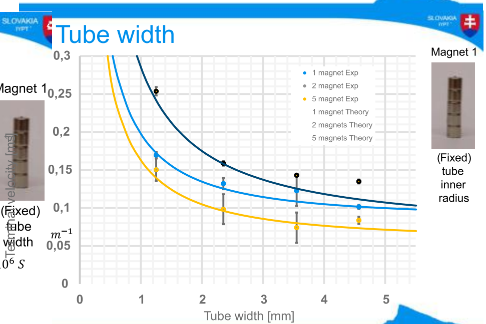


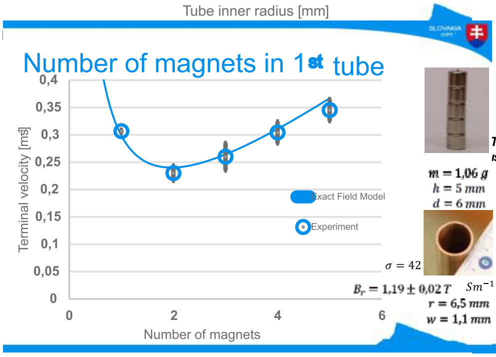


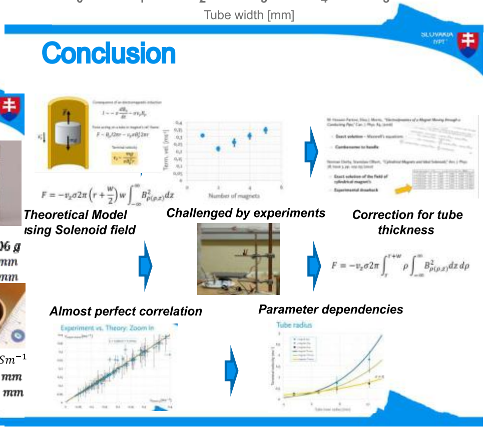






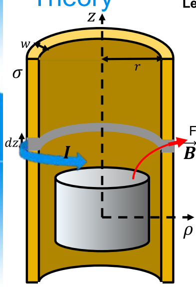






Appendix

Theory



Lenz law:

$$U_{ind} = -\frac{d\emptyset}{dt} = v_z \frac{d\emptyset}{dz} \qquad \emptyset = \oint \vec{B} \cdot \vec{dS}$$

$$\nabla \cdot \vec{B} = 0) :$$

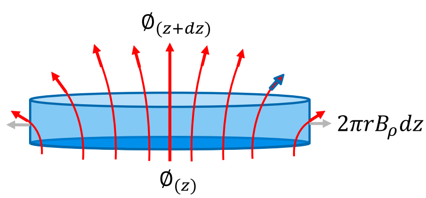
$$\frac{d\emptyset}{dz} = -B_{\rho(\rho,z)} 2\pi r$$

For the every ring in the height z above the magnet

$$dI = \frac{U_{ind}}{dR} = -v_z \sigma w B_{\rho(\rho,z)} dz$$

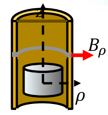
$$dF = -2\pi r B_{\rho(\rho,z)} dI$$

$$F = v_z \sigma 2\pi r w \int_x^{L-x} B^2_{\rho(\rho,z)} dz$$



$$\nabla \cdot \vec{B} = 0 \Rightarrow \emptyset_{(z+dz)} + 2\pi r B_\rho dz = \emptyset_{(z)} \Rightarrow \frac{d\emptyset}{dz} = -2\pi r B_\rho$$

Magnetic Dipole Model



Field of magnetic dipole $\vec{B}_{(r)} = \frac{\mu}{4\pi}\left(\frac{3\vec{r}(\vec{\mu}\cdot\vec{r})}{|r|^5} - \frac{\vec{\mu}}{|r|^3}\right)$

Radial field $\vec{B}_\rho = \frac{\mu}{4\pi}\frac{3\rho z|\vec{\mu}|}{\sqrt{\rho^2+z^2}^5}$

Resistance of small ring

$$dI = -2\pi\rho B_\rho v \frac{\sigma w dz}{2\pi\rho}$$

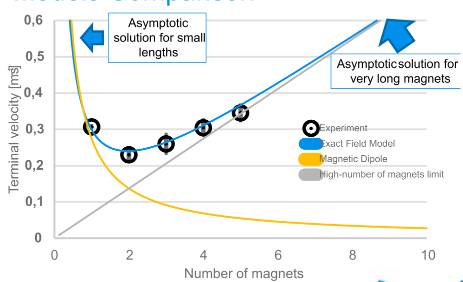
Inducted voltage

$$F = \int_{-L/2+x}^{L/2-x} 2\pi\rho B_\rho dI = -\int_{-L/2+x}^{L/2-x} 2\pi\rho B_\rho^2 v\sigma w dz$$

$$F \approx -\left(\frac{\mu_0}{4\pi}\right)^2 18\rho^3 \mu_{mag}^2 \pi v\sigma w \int_{-\infty}^{\infty} \frac{z^2}{\left(z^2+\rho^2\right)^5} dz = -\frac{45\mu_0^2\sigma\mu_{mag}^2 v}{1024}\frac{w}{r^4}$$

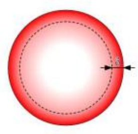
$$\frac{5\pi}{128\rho^7}$$

Models Comparison



Justifications of Our Approximations

- Not considering:
  - Skin effect $\delta = \sqrt{\frac{2\rho}{\omega\mu_0}} \approx 10\ mm \gg r$
    - ρ - Conductivity
    - ω - AC Frequency



Characteristic frequency

$$\omega \sim \frac{v}{r} \qquad \sim 10^2 s^{-1}$$

Tube radius

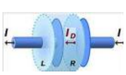
  - Inducted currents in magnet (Self-inductance)

$$\frac{dz}{v} \gg \frac{L_{(dz)}}{R_{(dz)}}$$

Self-inductance — Characteristic time of eddy currents decay — Ohmic resistance

Another Assumptions

- Not considering:
  - Displacement currents $\frac{\partial E}{\partial t} \approx 0$
    (Consequence of quasistatic fields)



Displacement/Conduction currents $\frac{\epsilon_0 v}{\sigma r} \approx 10^{-16}$

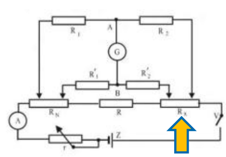
  - Dipole radiation

Dipole radiation vs Ohmic dissipation

$$(\mu_0 m^2/6\pi c^7)(\dot{v}^2 + v\ddot{v}) \ll \frac{45\mu_0^2\sigma\mu_{mag}^2 v^2}{1024}\frac{w}{r^4}$$

Conductivity Measurement

- Using **Kelvin bridge** with
  $R_1 = R_1'$
  $R_2 = R_2'$
- In $I_G = 0$ state:
  $R_x = R_N \frac{R_1}{R_2}$
- Used resistance:
  $R_N = 10^{-4}\ \Omega$
  $R_{1,2} \approx 10 - 100\ \Omega$



Conductivity of our copper pipes: $\sigma = (42 \pm 2)10^6\ Sm^{-1}$

Pure copper conductivity $\sigma = 56 \cdot 10^6\ Sm^{-1}$

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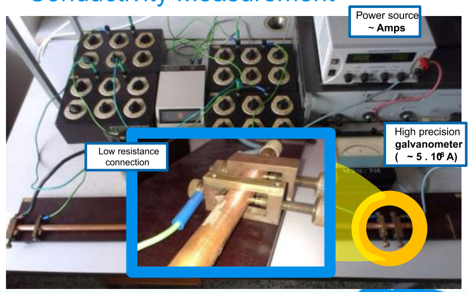
Conductivity Measurement

Conductivity Measurement



Equivalence of the Fields

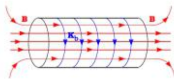
- Simple proof from ***Electromagnetism*** **9.2 pg.319**

Uniformly magnetized cylinder $M = M_0 \hat{k}$

$$A_{(x)} = \int \frac{\mu_0 M \times (x - x')}{4\pi |x - x'|^3} \mathrm{d}x'$$

By definition

$$B_{(x)} = \nabla \times A_{(x)}$$



The same **B** as if it was created by (bound) currents

$$J_b = \nabla \times M$$
$$K_b = M \times \hat{n}$$

In our case

$$J_b = \nabla \times M = 0$$
$$K_b = M \times \hat{n} = M_0 \hat{\varphi}$$

Magnetic field is caused by azimuthal bond surface currents

Ttherefore the **same field as Coil**

Magnetic Field of Cylindrical Magnet

[Cylindrical Magnets and Ideal Solenoids]

$$B_\rho = B_0\left(\alpha_+ C_{(k_+,1,1,-1)} - \alpha_- C_{(k_-,1,1,-1)}\right)$$

$$B_z = \frac{B_0 d}{d+2\rho}\left(\beta_+ C_{(k_+,\gamma^2,1,\gamma)} - \beta_- C_{(k_-,\gamma^2,1,\gamma)}\right)$$

Where $C_{(k,p,c,s)} = \int_0^{\pi/2} \frac{c\cos^2\varphi + s\sin^2\varphi}{(\cos^2\varphi + p\sin^2\varphi)\sqrt{\cos^2\varphi + k\sin^2\varphi}} d\varphi$ Generalized complete elliptic integral

$$B_0 = \mu_0 \frac{4\vec{\mu}}{\pi^2 h d^2}$$

$$z_\pm = z \pm \frac{h}{2}$$

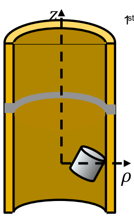
$$\alpha_\pm = \frac{d}{2\sqrt{z_\pm^2 + \left(\rho + \frac{d}{2}\right)^2}}$$

$$\beta_\pm = \frac{z_\pm}{\sqrt{z_\pm^2 + \left(\rho + \frac{d}{2}\right)^2}}$$

$$\gamma = \frac{d-2\rho}{d+2\rho}$$

$$k_\pm = \sqrt{\frac{z_\pm^2 + \left(\frac{d}{2} - \rho\right)^2}{z_\pm^2 + \left(\frac{d}{2} + \rho\right)^2}}$$

Modificated theory Shift & Rotation



1st step - Transformations of the coordinate system **Shift & Rota**

$$dI_{(z)} = -2\pi\rho v \frac{\sigma d\rho dz}{2\pi\rho} \int_0^{2\pi} (\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)}) d\beta$$

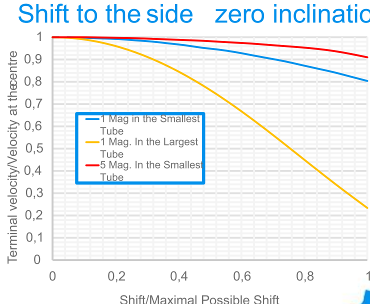
Magnetic field **flowing outside** the tube in **radial direction**

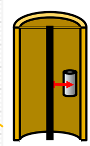
$$dF = -\frac{2\pi}{4\pi^2}\rho\left(\int_0^{2\pi}(\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)})d\beta\right)^2 v$$

$$F = \int_{-\infty}^{\infty}\int_r^{r+w} dF_{(z,\rho)} d\rho dz$$

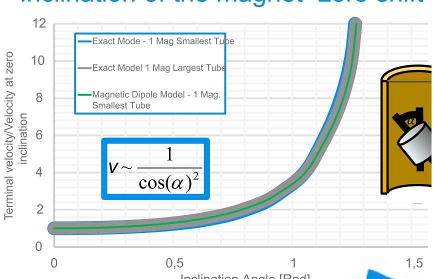
$$F = -\int_{-\infty}^{\infty}\int_r^{r+w} \frac{1}{2\pi}\rho v\left(\int_0^{2\pi}(\vec{B}_{\rho(\beta)} \cdot \vec{n}_{(\beta)})d\beta\right)^2 d\rho dz$$

Shift to the side zero inclination





Inclination of the magnet zero shift



$$v \sim \frac{1}{\cos(\alpha)^2}$$

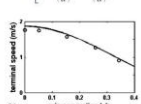
Shift Magnetic Dipole Model

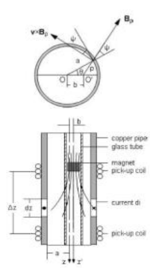
Elliptic integrals No advantage against our model

- Solved in **G. Donoso, C. L. Ladera, and P.** *Damped fall of magnets inside a conducting* Am. J. Phys. 79, 193 (2011)

$$F_z = \frac{36\pi\sigma\tau\bar{\mu}^2}{\alpha a^4}\int_0^\infty [G(u,b)]^2 du = \frac{36\pi\sigma\tau\bar{\mu}^2}{\alpha a^4} f(b/a)$$

$$G(u,b) = \int_0^{2\pi} \frac{u[1-(b/a)\cos\theta]}{2\pi\left[1+\left(\frac{b}{a}\right)^2 - 2\left(\frac{b}{a}\right)\cos\theta + u^2\right]^{5/2}} d\theta$$





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Rotation Magnetic Dipole

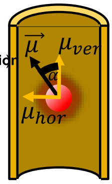
Analytical solution possible only for magnetic dipole model

Magnetic moment **could be divided** into two (**with the same position**)

$$\mu_{ver} = |\vec{\mu}|\cos(\alpha)$$
$$\mu_{hor} = |\vec{\mu}|\sin(\alpha)$$

Resulting magnetic field is the superposition of the two partial

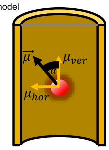
Thanks to: $(\vec{a}+\vec{b})\cdot\vec{c} = (\vec{a}\cdot\vec{c}) + (\vec{b}\cdot\vec{c})$



Rotation Magnetic Dipole

Analytical solution possible only for magnetic dipole model

$$F = \frac{45\mu_0^2\sigma\mu_{mag}^2 v}{1024}\frac{w}{r^4}\cos(\alpha)^2$$



Associated radial fields (Flowing perpendicular to tube)

$$\mu_{ver} \quad \vec{B}_\rho = \frac{\mu}{4\pi}\frac{3\rho z|\mu_{ver}|}{\sqrt{\rho^2+z^2}^5}$$

$$\mu_{hor} \quad \vec{B}_\rho = \frac{\mu}{4\pi}\frac{3\rho^2|\mu_{hor}|}{\sqrt{\rho^2+z^2}^5}\left(\cos(\beta)^3 + \sin(\beta)^2\cos(\beta) - \frac{\rho^2+z^2}{3\rho^2}\cos(\beta)\right)$$

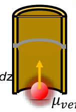
Induced voltages (over infinitesimal ring)

$$U = \int_0^{2\pi} B_\rho \rho v d\beta \neq 0$$

Torque on Magnet

Induced voltage (in infinitesimal ring) due to vertical component of the magnetic moment:

$$U = -2\pi\rho\frac{\mu}{4\pi}\frac{3\rho z|\mu_{ver}|}{\sqrt{\rho^2+z^2}^5}v \Rightarrow dI_{(z)} = -\frac{\mu}{4\pi}\frac{3\rho z|\mu_{ver}|}{\sqrt{\rho^2+z^2}^5}v\sigma w dz$$



Inducted current induces **magnetic field** in the position of the dipole:

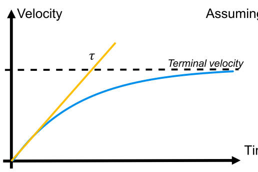
$$dB_{(z)} = \frac{\mu}{4\pi}\frac{2\pi\rho^2 dI_{(z)}}{\sqrt{\rho^2+z^2}^3} \Rightarrow B = \int_{-L/2+x}^{L/2+x} dB_{(z)} = \left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu_{ver} w\sigma v\left[\frac{1}{6}(z^2+\rho^2)^{-3}\right]_{-L/2+x}^{L/2+x}$$

(Biot-Savart law for current loop at distance z)

Which exerts **torque on magnetic dipole:**

$$\vec{\tau} = \vec{\mu}\times\vec{B} = \mu B\sin(\alpha) = \left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu^2 w\sigma v\left[\frac{1}{6}(z^2+\rho^2)^{-3}\right]_{-L/2+x}^{L/2+x}\cos(\alpha)\sin(\alpha)$$

Motion of the magnet



Assuming braking force from infinite tube all time:

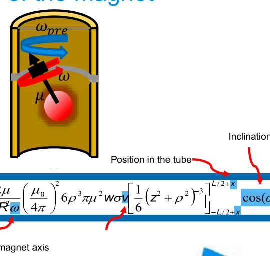
$$v_{(t)} = \frac{mg}{C}\left(1 - e^{-\frac{C}{m}t}\right)$$

Distance travelled ($v_{(t=0)} = 0$):

$$s_{(t)} = \frac{mg}{C}t + \left(\frac{m}{C}\right)^2 g\left(e^{-\frac{C}{m}t} - 1\right)$$

Worst case scenario	Characteristic time	95 % of terminal velocity	Distance
$v_{terminal} = \frac{mg}{C} = 1{,}6ms^{-1}$	$\tau = \frac{m}{C} = 0{,}16\ s$	$3\tau = 0{,}48\ s$	$s_{(3\tau)} = 52\ cm$

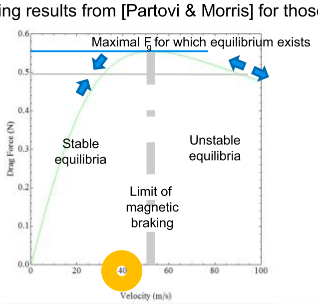
Precession of the magnet

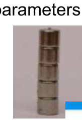


$$\omega_{pre} = \frac{\mu B}{L} = \frac{2\mu B}{mR^2\omega} = \frac{2\mu}{mR^2\omega}\left(\frac{\mu_0}{4\pi}\right)^2 6\rho^3\pi\mu^2 w\sigma v\left[\frac{1}{6}(z^2+\rho^2)^{-3}\right]_{-L/2+x}^{L/2+x}\cos(\alpha)$$

Large velocities Skin effect

- Using results from [Partovi & Morris] for those parameters:





$B_r = 1{,}19 \pm 0{,}($
$m = 1{,}0$
$h = 5\ m$
$d = 6\ m$

$\sigma = 42\cdot 10^6\ S$
$r = 6{,}5\ m$

[Partovi & Morris] Result

- Obtained the **general solution** for uniformly magnetized **cylinder** pipe system:

$$\mathbf{F}^{uni} = -\hat{\mathbf{v}}\frac{\mu_0 m^2}{2\pi^2}\int_0^{+\infty} dk\,k^3\left[\frac{\sin(kL/2)}{(kL/2)}\right]^2\left[\frac{I_1(ka)}{(ka/2)}\right]^2 \mathrm{Im}[Q(k)]$$

Where **Q(k)** is ratio of $b_1(k)/b_0(k)$

$$b_1(k) = \{[K_0(|k|R_1)K_0(|k|R_2)T_{11} + \beta K_0(|k|R_1)K_1(|k|R_2)T_{10} - \beta K_1(|k|R_1)K_0(|k|R_2)T_{01} - \beta^2 K_1(|k|R_1)K_1(|k|R_2)T_{00}] + [I_0(|k|R_1)K_0(|k|R_2)T_{11} + \beta I_0(|k|R_1)K_1(|k|R_2)T_{10} + \beta I_1(|k|R_1)K_0(|k|R_2)T_{01} + \beta^2 I_1(|k|R_1)K_1(|k|R_2)T_{00}]\}b_0(k),$$

Exact, but **very cumbersome** to handle

$$T_{00} = K_0(\alpha|k|R_1)I_0(\alpha|k|R_2) - I_0(\alpha|k|R_1)K_0(\alpha|k|R_2)$$
$$T_{01} = K_0(\alpha|k|R_1)I_1(\alpha|k|R_2) + I_0(\alpha|k|R_1)K_1(\alpha|k|R_2)$$
$$T_{10} = K_1(\alpha|k|R_1)I_0(\alpha|k|R_2) + I_1(\alpha|k|R_1)K_0(\alpha|k|R_2)$$
$$T_{11} = K_1(\alpha|k|R_1)I_1(\alpha|k|R_2) - I_1(\alpha|k|R_1)K_1(\alpha|k|R_2),$$

$$\alpha = \sqrt{\kappa^2}/|k| = \sqrt{1 - i\frac{\mu_0\mu_{rel}\sigma v}{k}},$$
$$\beta = \frac{\alpha}{\mu_{rel}} = \frac{1}{\mu_{rel}}\sqrt{1 - i\frac{\mu_0\mu_{rel}\sigma v}{k}}.$$

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10.Rocking Bottle

Reporter: Shiva Azizpour

IYPT 2012
BAD SAULGAU - GERMANY

Question

- Fill a bottle with some liquid. Place it on a horizontal surface and give it a push. The bottle may first move forward and then *oscillate* before it comes to rest. Investigate the bottle's motion.

Theory
- Fixed Bottle
  - Pendulum
- Moving Bottle
  - Formulation

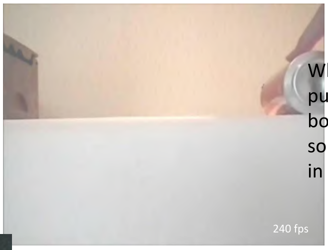
Experiments
- Fixed bottle
- Moving Bottle
  - image processing
  - X - t
  - V – t
- Frequency vs. height

Discussion
- Theory Vs. Experiment
  - Fixed Bottle
- Theory Revision
  - Numerical Theory
  - Results
- Non-Dimensional
- Theory Vs. Experiment
  - Fixed Bottle
  - Moving Bottle

Main Approach

Conclusion
- Frequency Vs. Height
- Pendulum vs. Numerical
- Frequency Comparison

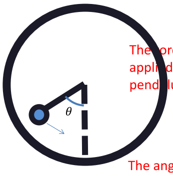
Observation



When giving a push to a half filled bottle we can see some oscillations in the motion

Observation
motion of water in bottle

Theory
Pendulum



The torque applied to the pendulum

$$\tau = mgl\sin(\theta) = I\ddot{\theta}$$

The angular speed of pendulum

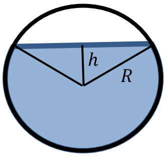
$$\omega = \sqrt{\frac{mgl}{I}}$$

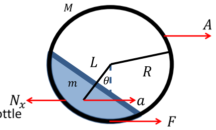
Theory
Moment of Inertia

Now assuming the water in the bottle oscillating like a pendulum:

$$I = \frac{1}{6}\rho L \left[h\sqrt{R^2 - h^2}(2h^2 + R^2) + 3R^4\tan^{-1}\left(\frac{h}{\sqrt{R^2 - h^2}}\right) + \frac{3}{2}\pi R \right.$$

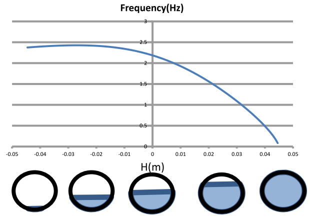
Where ρ is the density and L is the length of the bottle.





Moving Partially Filled Bottle

Theory
Water as a Pendulum



H is the deviation of water free level from bottle's center.

$$forces \begin{cases} -MA + F - N_x = 0 \\ -ma + N_x = ma \end{cases}$$

$$torques \begin{cases} I_b\ddot{\varphi} = F \times R \\ I_w\ddot{\theta} = (-mA - mg) \times L \end{cases}$$

Theory
Moving Partially Filled Bottle

$$I_b\ddot{\varphi} = mR^2\left[\left(1+\frac{M}{m}\right)\ddot{\varphi} + \frac{L}{R}(\ddot{\theta}cos\theta - \dot{\theta}^2 sin\theta)\right]$$

$$I_w\ddot{\theta} = -mRL\left(\ddot{\varphi}cos\theta + \frac{g}{R}sin\theta\right)$$

Substituting $I_b = mR^2$ →

$$0 = mR^2\left[\ddot{\varphi} + \frac{L}{R}(\ddot{\theta}cos\theta - \dot{\theta}^2 sin\theta)\right]$$

$$I_w\ddot{\theta} = -mRL\left(\ddot{\varphi}cos\theta + \frac{g}{R}sin\theta\right)$$

Mass of the bottle disappears from the equations.

Theory
Moving Partially Filled Bottle

Frequency is not dependent on mass of the bottle.

$m \to \infty$

We can assume that the bottle is fixed.

Experiments
Less Than Half Filled Bottle

We change the images into black & white by detecting the colored water.

Experiments
More Than Half Filled Bottle

The waves are seen at the top when the bottle is more than half filled.

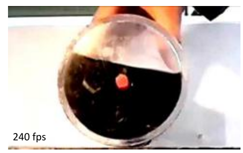
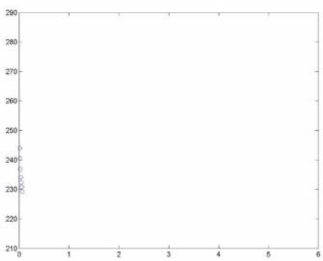
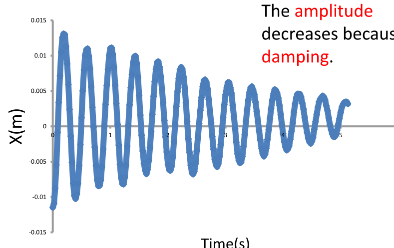


Image Processing
X vs. t

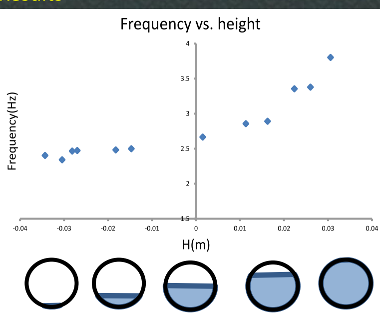


Experiments
Results



The amplitude decreases because of damping.

Experiments
Results



Experiments
Moving Bottle

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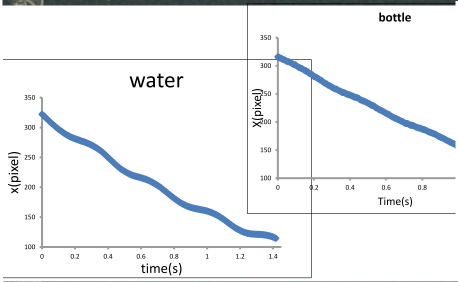
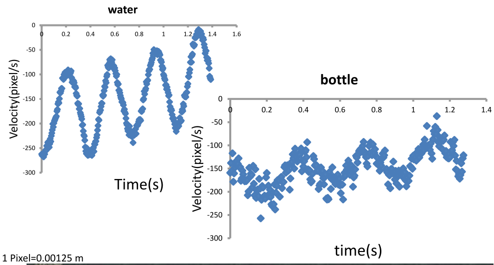
Experiments
Moving Bottle

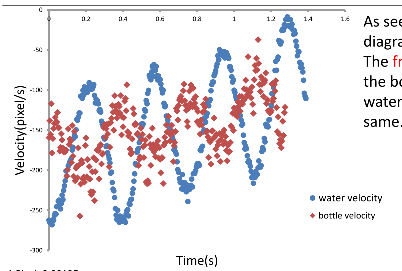


Experiments
Moving Bottle



1 Pixel=0.00125 m

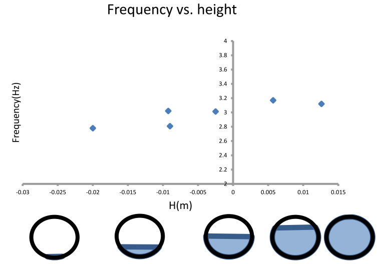
Experiments
frequency



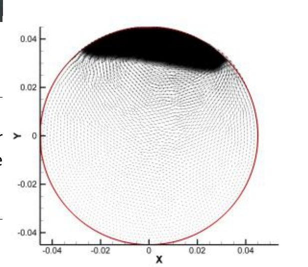
1 Pixel=0.00125 m

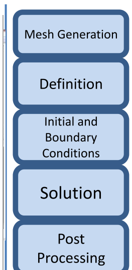
As seen in the diagram ,
The frequencies o the bottle and water are the same.

Experiment
Moving Bottle

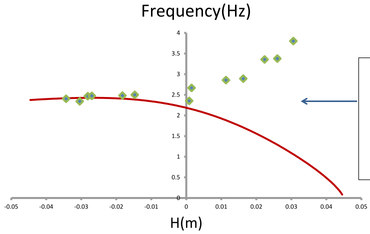


Theory
Numerical Theory



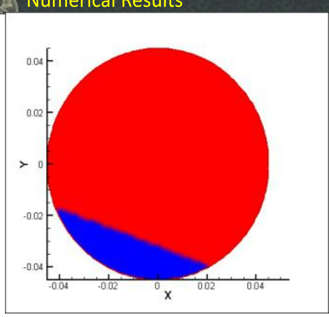


Theory vs. Experiment
Pendulum



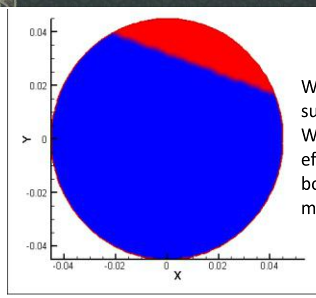
Here's a mismatch in the positive values of h.

Theory
Numerical Results



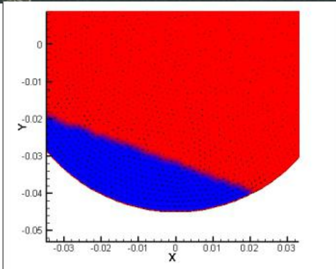
The motion similar to a pendulum when h is negative.

Theory
Numerical Results



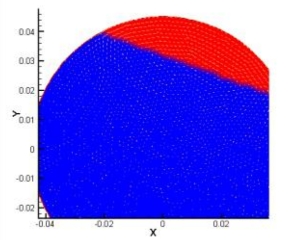
Waves on the surface
Will have a greater effect when the bottle is filled more than half.

Theory
Numerical Results



When the bottle is less than half filled , All of the water sections are oscillating.

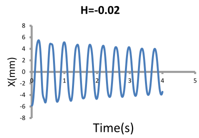
Theory
Numerical Results

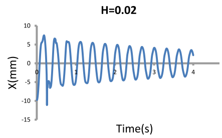


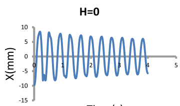
When the bottle is more tan half filled , Not all parts share the same velocity.

The water on the surface has more velocity

Theory
Numerical Results

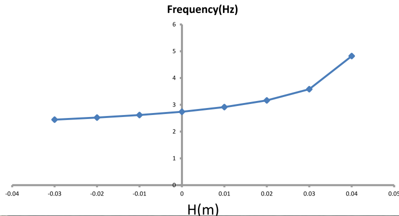






Theory
Numerical Results

The frequency increases by increasing the water fraction.



Theory
Non Dimensionals

Parameters

h

R

g

f

Non dimensionals

$$\pi_1 = \frac{h}{R}$$

$$\pi_2 = \frac{R f_r^{\,2}}{g}$$

$$\pi_2 = f(\pi_1)$$

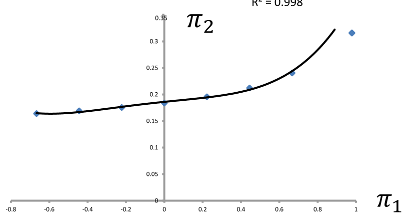
The frequency is derived using this function

$$f_r = \sqrt{f\left(\frac{h}{R}\right)\frac{g}{R}}$$

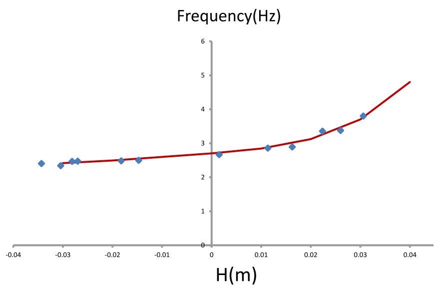
Theory
Non Dimensionals

Here using the numerical results ;

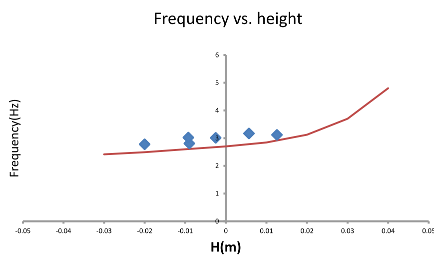
We get $\pi_2 = f(\pi_1)$ → $y = 0.1308x^4 + 0.0491x^3 - 0.017x^2 + 0.0376x + 0.1861$, $R^2 = 0.998$



Theory vs. Experiment
fixed Bottle

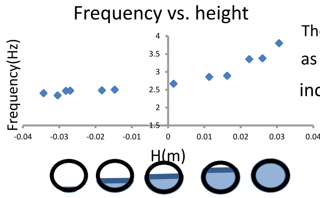


Experiments
Moving Bottle



Conclusion

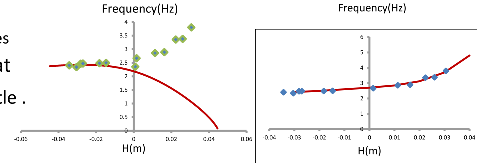
Frequency vs. Height



The frequency increases as the amount of wat increases in the bottle .

Conclusion

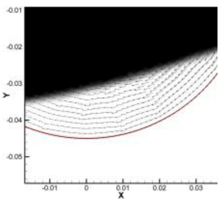
Pendulum Vs. Numerical



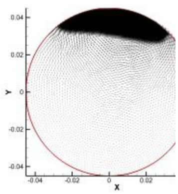
The mismatch between the initial theory and experiment can be explained in the revised theory.

Conclusion

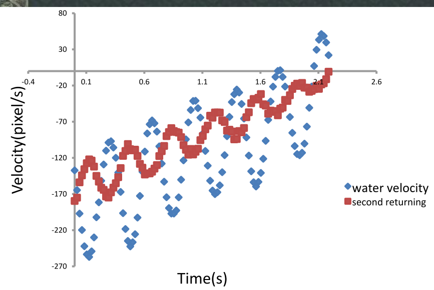
Pendulum Vs. Numerical



All the water will move when the bottle is less than half filled.



Just the water near th surface will move whe the bottle is more tha half filled.



Problem

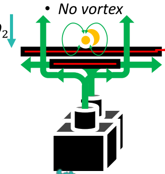
It is stated that:
Put a lit candle behind a bottle. If you blow on the bottle from the opposite side, the candle may go out, as if the bottle was not there at all. Explain the phenomenon.

Fire Triangle

- O_2 ✓
- $T(\theta)$ ✗
- $Fuel$ ✗

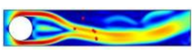
$v\uparrow \rightarrow P\downarrow \rightarrow O_2\downarrow$

Separations



- No vortex
- Vortex
  - Candle condition: Won't go out
  - Motion of flame of the candle : Will circulate
  - Flow motion: Will separate from each other and weak flow will circulate behind the plate

Coanda effect

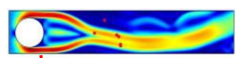


$v \rightarrow P_f < P_0 \rightarrow F \rightarrow$ Will change the direction of the flow

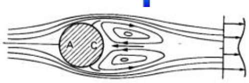
- Why happen?
  - $\Delta P(Pressure)$
  - $\Delta\vec{p}(Momentum)$

$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{p} \propto \vec{F} \rightarrow$ Will change the direction of the flow

Coanda effect



$v\uparrow$

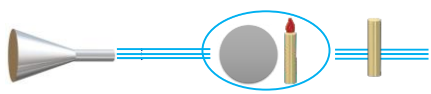


- Weak flow
- Circulate
- Placed between two points

Types of flow

2 kinds
- Flow jet
  - Become from narrow pipe
  - Earlier and easier wrap around back of cylinder
  - $v_{jet} > v_{ordinary}$
  - $t_{jet} < t_{ordinary}$
- Ordinary flow

Explanation



Reynold number

$Re = \frac{\rho u D_H}{\mu}$

- u: Flow velocity
- μ: Viscosity
- ρ: Density

$D_H = \frac{4A}{P}$

$\rightarrow$ Circle $\rightarrow D_H = \frac{4A}{P} = \frac{4\pi r^2}{2\pi r} = 2r$

- Types of flow
  - Laminar $Re < 5 \times 10^5$
  - Transient $Re = 5 \times 10^5$
  - Turbulent $Re > 5 \times 10^5$

$\rightarrow$ $Re = \frac{2\rho u r}{\mu}$

Continuity equation

- Green: $A_1 = \pi {r_1}^2, v = u_{x1}$
- Red: $A_2 = \pi r_2^2, v = u_{x2}$

$\rightarrow A_1 u_{x1} = A_2 u_{x2} \rightarrow A_n \propto u_x^{-1}$

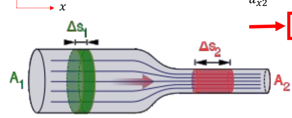
$\rightarrow \pi r_1^2 u_{x1} = \pi r_2^2 u_{x2}$

$\rightarrow r_1^2 u_{x1} = r_2^2 u_{x2}$

$\rightarrow \frac{u_{x1}}{u_{x2}} = (\frac{r_2}{r_1})^2 = \alpha^2$

$\rightarrow \log_\alpha \frac{u_{x1}}{u_{x2}} = 2$

$\rightarrow \log_\alpha u_{x1} - \log_\alpha u_{x2} = 2$



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Continuity equation

$A_1 u_{x1} = A_2 u_{x2} \rightarrow \pi r_1^2 u_{x1} = \pi r_2^2 u_{x2} \rightarrow r_1^2 u_{x1} = r_2^2 u_{x2}$

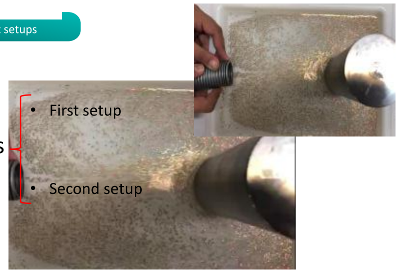
$\rightarrow (r_1^2 u_{x1} = r_2^2 u_{x2}) \times \frac{\rho D_H}{\mu} \rightarrow \frac{\rho D_H u_{x1}}{\mu} \times r_1^2 = \frac{\rho D_H u_{x2}}{\mu} \times r_2^2$

$\rightarrow Re_1 \times r_1^2 = Re_2 \times r_2^2 \rightarrow \frac{Re_1}{Re_2} = (\frac{r_2}{r_1})^2 = \alpha^2 \rightarrow \log_\alpha \frac{Re_1}{Re_2} = 2$

$\rightarrow \boxed{\log_\alpha Re_1 - \log_\alpha Re_2 = 2}$

Experiment setups

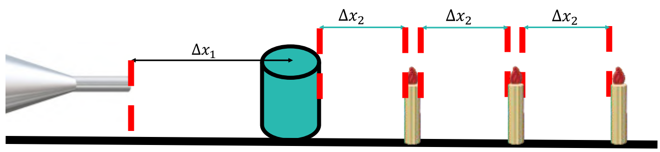
2 Setups
- First setup
- Second setup



Experiment setups

$\Delta x_1 = 18.5cm \pm 0.1$ $\Delta x_2 = 7.0cm \pm 0.1$

- 2 Setups
  - First setup
  - Second setup



Second setup

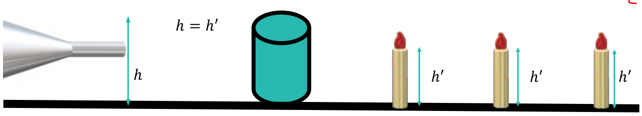
A_n:
- $A_{n1} = 3.14cm^2 \pm 0.31$
- $A_{n2} = 9.62cm^2 \pm 0.31$
- $A_{n3} = 15.90cm^2 \pm 0.31$

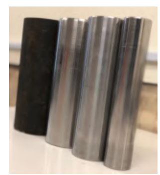
r_n:
- $r_{n1} = 1.0cm \pm 0.1$
- $r_{n2} = 1.75cm \pm 0.1$
- $r_{n3} = 2.25cm \pm 0.1$

u_x:
- u_{xl}
- u_{xm}
- u_{xh}

R_c:
- $1.0cm \pm 0.1$
- $2.0cm \pm 0.1$
- $3.0cm \pm 0.1$
- $4.0cm \pm 0.1$
- $5.0cm \pm 0.1$

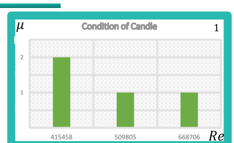
Cylinders:
- Smooth
- Rough



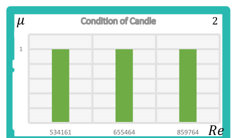


Experiment results

$Re_l = B$
$Re_m = A$
$Re_h = A$

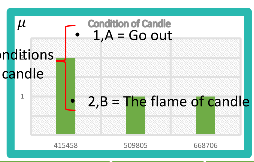


$A_n (cm^2)$	$R_c (cm)$
$15.90 \pm 0.31 (cm^2)$	$2.0 \pm 0.1 (cm)$
Type of cylinder	Δx
Rough	7.0 ± 0.1



$A_n (cm^2)$	$R_c (cm)$
$9.62 \pm 0.31 (cm^2)$	$2.0 \pm 0.1 (cm)$
Type of cylinder	Δx
Rough	7.0 ± 0.1

- Conditions of candle
  - 1,A = Go out
  - 2,B = The flame of candle circulate



Distance between candle and cylinder

$A_n (cm^2)$	$R_c (cm)$	Type of cylinder	Δx
$15.90 \pm 0.31 (cm^2)$	$2 \pm 0.1 (cm)$	Rough	$7.0 \pm 0.1 (cm)$

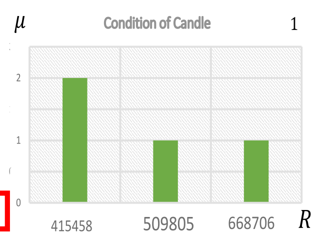
$Re \uparrow \rightarrow u_x \uparrow \rightarrow A_w \downarrow \rightarrow \boxed{u_x \propto A_w^{-1}}$

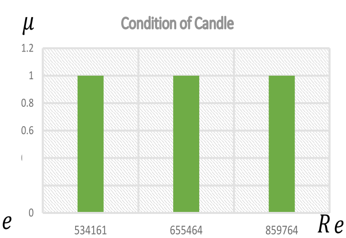
Experiment results

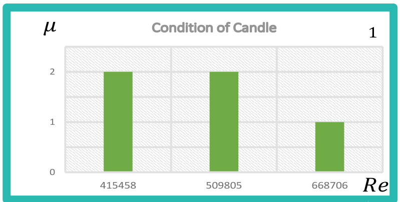
1:
- $u_{xl} = B$
- $u_{xm} = A$
- $u_{xh} = A$
- $A_n = 15.90cm^2 \pm 0.31$

2:
- $u_{xl} = A$
- $u_{xm} = A$
- $u_{xh} = A$
- $A_n = 9.62cm^2 \pm 0.31$

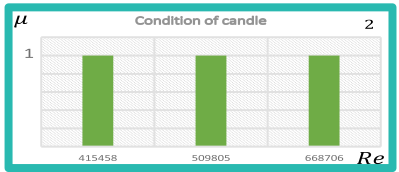
$\boxed{A_{ws} < A_{wr}}$





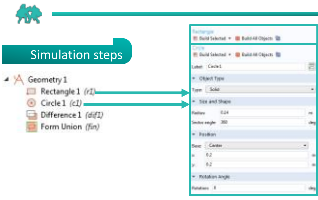


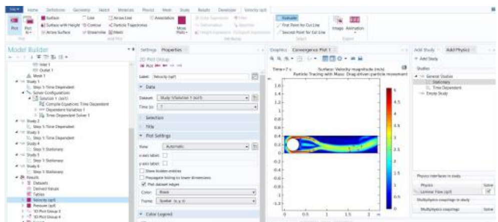
$A_n(cm^2)$	$R_c(cm)$
$15.90 \pm 0.31(cm^2)$	$3.0 \pm 0.1(cm)$
Type of cylinder	Δx
Rough	7.0 ± 0.1



$A_n(cm^2)$	$R_c(cm)$
$15.90 \pm 0.31(cm^2)$	$3.0 \pm 0.1(cm)$
Type of cylinder	Δx
Smooth	7.0 ± 0.1

Simulation steps





- Theory: Navier - stokes equation

$u_x = c = 1.5\, {}^{m}/_{s}$

$\rho = c$

$Roughness\ of\ cylinder\ = c$

$A_n = c$

- 2 types of simulation
  - First simulations
  - Second simulations

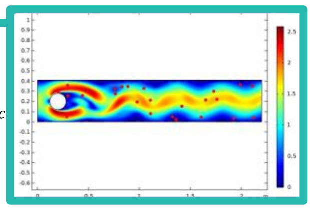
$R_c = c$

$\rho = c$

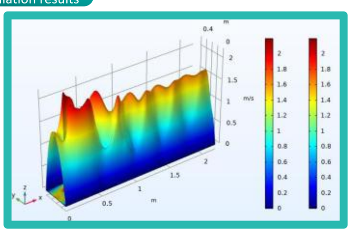
$Roughness\ of\ cylinder\ = c$

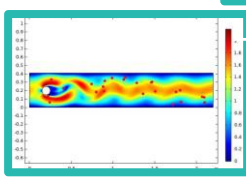
$A_n = c$

$Place\ of\ cylinder = c$



Simulation results

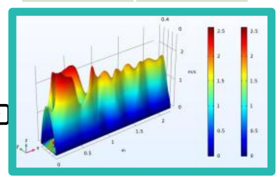


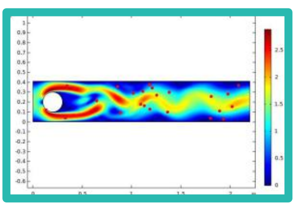


Distance between center of cylinder and blower

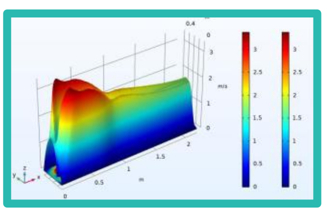
$\Delta x_1(cm)$	$R_c(cm)$
20	5

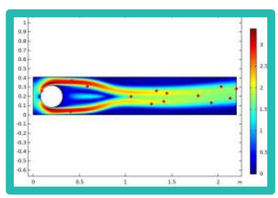
$\Delta x_1(cm)$	$R_c(cm)$
20	8





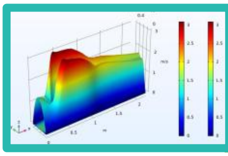
$\Delta x_1(cm)$	$R_c(cm)$
20	10

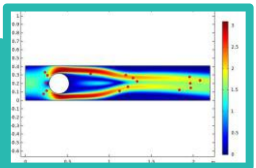




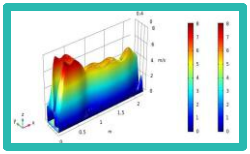
$\Delta x_1(cm)$	$R_c(cm)$
20	12

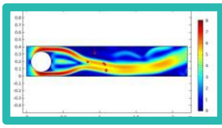
Simulation results





$\Delta x_1(cm)$	$R_c(cm)$
40	12



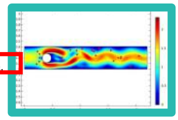


$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	2

$R_c \propto A_w$

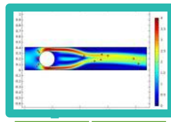
Simulation conclusion

u_{Max}: It' s happen in the moment of adhesion

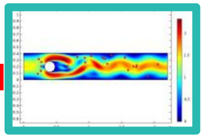


$A_w^{-} \Delta x_1$

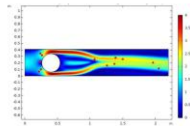
$\Delta x_1(cm)$	$R_c(cm)$
40	8



$\Delta x_1(cm)$	$R_c(cm)$
40	14



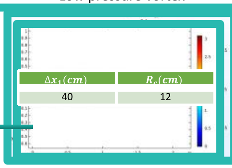
$\Delta x_1(cm)$	$R_c(cm)$
40	8



$\Delta x_1(cm)$	$R_c(cm)$
40	14

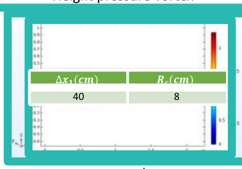
laminar Flows

Low pressure vortex



$\Delta x_1(cm)$	$R_c(cm)$
40	12

Height pressure vortex

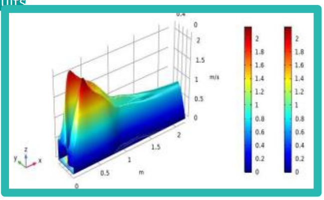


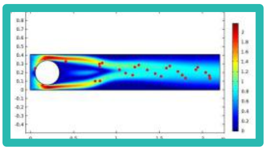
$\Delta x_1(cm)$	$R_c(cm)$
40	8

lv

Part 2

Simulation results





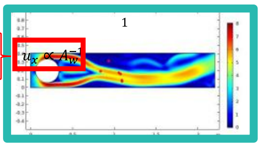
$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	0.5

Simulation conclusion

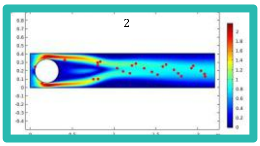
$u_{x1} > u_{x2}$

$A_{w1} < A_{w2}$

$u_x \propto A_w^{-1}$



$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	2



$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	0.5

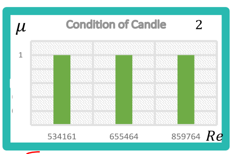
Theory vs experiment

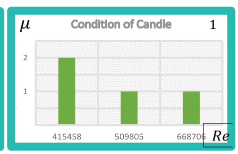
Theory

$u_x \propto A_w^{-1}$ Simulation

$u_x \propto A_n^{-1}$ Continuity equation

Experiment





1 $u_{xl} = B$, $A_n = 15.90cm^2 \pm 0.31$

2 $u_{xl} = A$, $A_n = 9.62cm^2 \pm 0.31$

$A_n \propto A_w$

Theory vs experiment

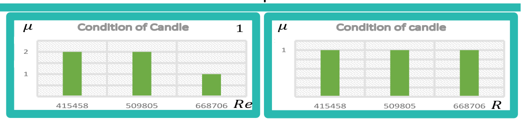
Theory

$\vec{f_s} < \vec{f_r}$ → $\vec{u}_s > \vec{u}_r$

$u_x \propto A_w^{-1}$

→ $A_{wr} > A_{ws}$

Experiment

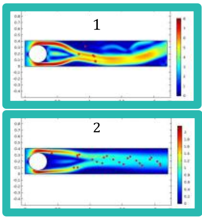


1 $u_{xm} = B$; Type of cylinder: Rough

2 $u_{xm} = A$; Type of cylinder: Smooth

$A_{wr} > A_{ws}$

Theory



$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	2

$u_{x1} > u_{x2}$

$A_{w1} < A_{w2}$

$\Delta x_1(cm)$	$R_c(cm)$	$u_x(m/s)$
20	14	0.5

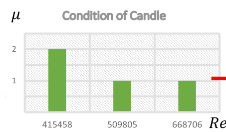
→ $u_x \propto A_w^{-1}$

Experiment

$u_{xl} = B$

$u_{xm} = A$

$u_{xh} = A$



→ $A_n \propto A_w$

$A_n(cm^2)$	$R_c(cm)$	Type of cylinder	Δx
$15.90 \pm 0.1(cm^2)$	$2 \pm 0.1(cm)$	Rough	7.8 ± 0.1

Re ↑ → u_x ↑ → A_w ↓ → $u_x \propto A_w^{-1}$

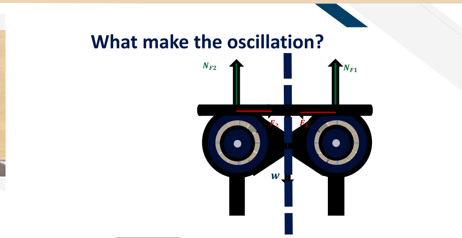
Conclusion

- Introduction
- Why candle will go out
- Separations
- Coanda effect →
  - Why Coanda effect will happen
  - Wake area
- Types of flow
- Phenomenon explanation
- Reynold number
- Continuity equation
- Setups
- Experiment results
- Simulation

Investigation

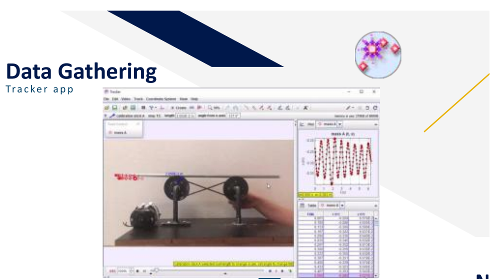
- $u_x \propto A_w^{-1}$
- $Re \propto A_w^{-1}$
- $A_n \propto A_w$
- $A_{ws} < A_{wr}$
- $R_s \propto A_w$
- $A_w^{-1} \propto \Delta x$

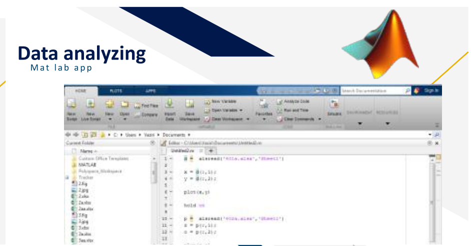

Friction Oscillator
Problem#13
A massive object is placed into two identical parallel horizontal cylinders.
The two cylinders each rotate with the same angular velocity, but in opposite directions.
Investigate how the motion of the object on the cylinders depends on the relevant parameters.
Reporter :Sahar Semsarha

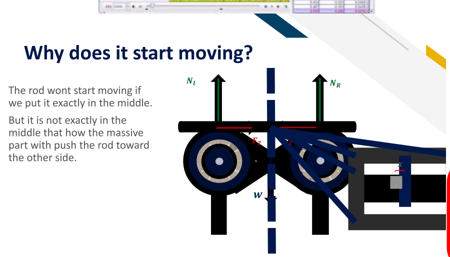

What make the oscillation?
N_{F2}
N_{F1}
w

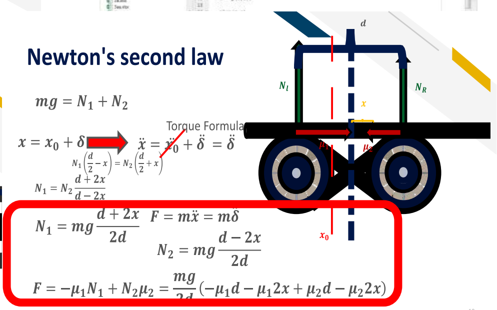

Setup
Rod
Van de Graaff generator belt
Iron Rods
Sewing motor
holes

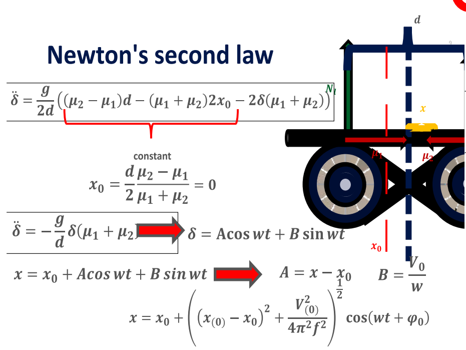

How to control the speed of the engine?
Speed Controlling Device

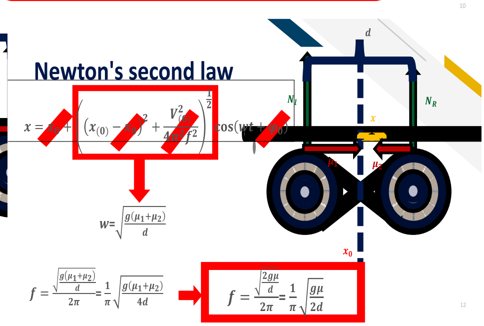

Data Gathering
Tracker app


Data analyzing
Mat lab app


Why does it start moving?
The rod wont start moving if we put it exactly in the middle.
But it is not exactly in the middle that how the massive part with push the rod toward the other side.
N_l
N_R
w


Newton's second law
$mg = N_1 + N_2$
$x = x_0 + \delta$
$\ddot{x} = \ddot{x}_0 + \ddot{\delta} = \ddot{\delta}$
Torque Formula
$N_1\left(\frac{d}{2}-x\right) = N_2\left(\frac{d}{2}+x\right)$
$N_1 = N_2 \frac{d+2x}{d-2x}$
$N_1 = mg\frac{d+2x}{2d}$
$F = m\ddot{x} = m\ddot{\delta}$
$N_2 = mg\frac{d-2x}{2d}$
$F = -\mu_1 N_1 + N_2\mu_2 = \frac{mg}{2d}(-\mu_1 d - \mu_1 2x + \mu_2 d - \mu_2 2x)$
d
N_l
N_R
x_0


Newton's second law
$\ddot{\delta} = \frac{g}{2d}\left((\mu_2-\mu_1)d - (\mu_1+\mu_2)2x_0 - 2\delta(\mu_1+\mu_2)\right)$
constant
$x_0 = \frac{d}{2}\frac{\mu_2-\mu_1}{\mu_1+\mu_2} = 0$
$\ddot{\delta} = -\frac{g}{d}\delta(\mu_1+\mu_2)$
$\delta = A\cos wt + B\sin wt$
$x = x_0 + A\cos wt + B\sin wt$
$A = x - x_0$
$B = \frac{V_0}{w}$
$x = x_0 + \left((x_{(0)}-x_0)^2 + \frac{V_{(0)}^2}{4\pi^2 f^2}\right)^{\frac{1}{2}}\cos(wt+\varphi_0)$
d
x_0


Newton's second law
$w = \sqrt{\frac{g(\mu_1+\mu_2)}{d}}$
$f = \frac{\sqrt{\frac{g(\mu_1+\mu_2)}{d}}}{2\pi} = \frac{1}{\pi}\sqrt{\frac{g(\mu_1+\mu_2)}{4d}}$
$f = \frac{\sqrt{\frac{2g\mu}{d}}}{2\pi} = \frac{1}{\pi}\sqrt{\frac{g\mu}{2d}}$
d
N_l
N_R
x_0

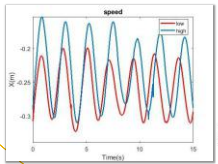
Effective Parameters

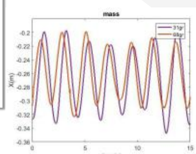
$$f=\frac{\sqrt{\frac{2g\mu}{d}}}{2\pi}=\frac{1}{\pi}\sqrt{\frac{g\mu}{2d}} \rightarrow T=\pi\sqrt{\frac{2d}{g\mu}}$$

parameters

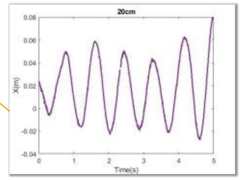
constant

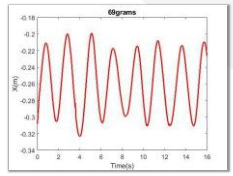
$$\uparrow T=\pi\sqrt{\frac{2d}{g\mu}}\downarrow \qquad \uparrow T=\pi\sqrt{\frac{2d}{g\mu}}\uparrow$$





No effective parameters

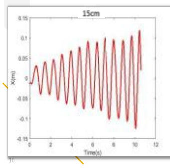


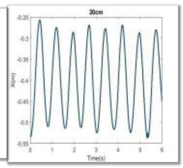


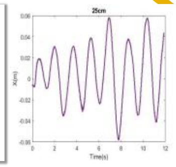
Period of oscillation given by theory

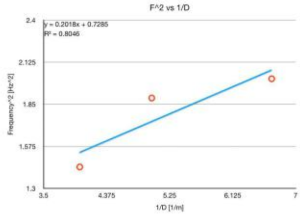
0.7586

Distance



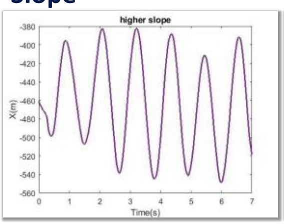


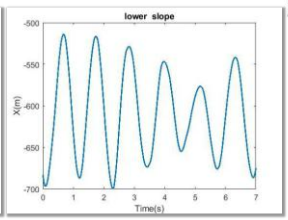


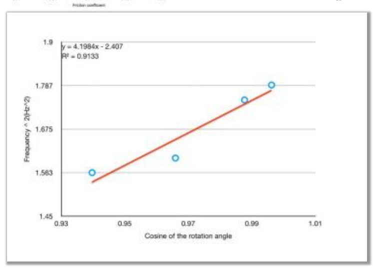


Friction Coefficient

Slope

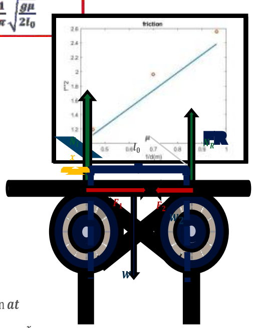






Conclusion

$$f=\frac{\sqrt{\frac{2g\mu}{l}}}{2\pi}=\frac{1}{\pi}\sqrt{\frac{g\mu}{2l_0}}$$



Newton's second law

$F=m\ddot{x} \rightarrow a=\ddot{x}$

$F_1-F_2=m\ddot{x}$

$F_1=\mu N_l=mg\mu\left(\frac{1}{2}-\frac{x}{l_0}\right)$

$F_2=\mu N_R=\mu gm\left(\frac{1}{2}+\frac{x}{l_0}\right)$ $\quad a=\sqrt{\frac{2g\mu}{l}}$

$x=\sin at$

$f=\frac{\sqrt{\frac{2g\mu}{l}}}{2\pi}=\frac{1}{\pi}\sqrt{\frac{g\mu}{2l_0}}$

$\ddot{x}=-2g\mu\frac{x}{l_0}$

$F_1-F_2=-2mg\mu\frac{x}{l_0}$

Normal Force

$N_l+N_R=mg$

$l_1w_1+N_Rl_R=w_2l_2$

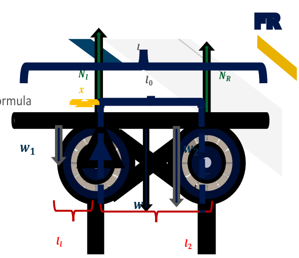
Torque Formula

$$w_1=\frac{\left(\frac{l-l_0}{2}\right)-x}{l}.mg$$

$$w_2=\frac{\left(\frac{l+l_0}{2}\right)+x}{l}.mg$$

$$l_1=\frac{\left(\frac{l-l_0}{2}\right)-x}{2}$$

$$l_2=\frac{\left(\frac{l+l_0}{2}\right)+x}{2}$$



How to guess the time coefficient

$\ddot{x} = -2g\mu \frac{x}{l_0}$ →

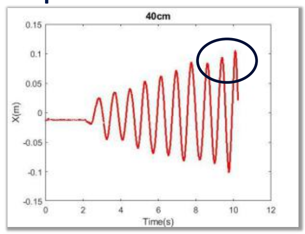
- The second derivate is negative
- X can be sinusoidal

$x = \cos at$ → $\dot{x} = a \sin at$ → $\ddot{x} = -a^2 \cos at$ x

$$-2g\mu\frac{x}{l_0} = -a^2 x$$

$$a^2 = \frac{2g\mu}{l_0} \qquad a = \sqrt{\frac{2gm}{l}}$$

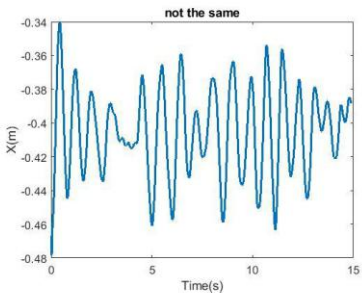
Theory vs Experiment



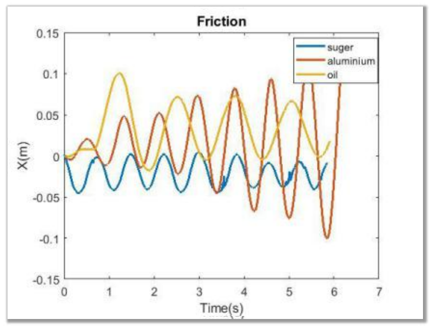
$$theory = T = \pi\sqrt{\frac{2l_0}{g\mu}} = 3.14 \times \sqrt{\frac{2\times 0.15}{9.8\times 0.7}} = 0.656974$$

$$Experiment = \frac{2}{3} = \mathbf{0.666666}$$

Not the same friction coefficient



Period of oscillation



References

[1] D. Russell. "*The Friction Oscillator* "[Video]. (Jul 23, 2013)

[2] Robin Henaff et al "A study on kinetic friction" (Oct 13, 2017)

[3] The Friction Oscillator by Enrique Zeleny"The Friction Oscillator" (July 23 2013)

[4] Robin Henaff, Gabriel Le Doudic, and Bertrand Pilette. A study of kinetic friction: The Timoshenko oscillator American Journal of Physics 86, 174 (2018)

ART AN AMAZING FACT IN SCIENCE

13. ROTATING SADDLE

Natália_Ružičková_Pavol_Kubinec_Michal_Hledík

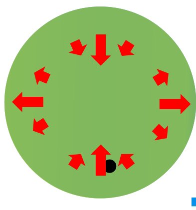
Task

A ball is placed in the middle of a rotating saddle

Investigate it's **dynamics** and explain the **conditions** under which the ball **does not fall off** the saddle.

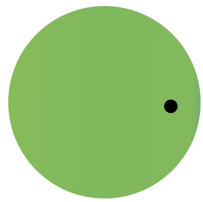
How can the rotation *help* the stability?

- Static saddle: just **rolls off**
- Rotating saddle: **rolls around the saddle** → effect of slopes cancels



(laboratory ref. frame)

1. Goes around with the saddle
   Centripetal force needed → **unstable**
2. Remains stationary
   (Rolls back quickly enough) → **stable**

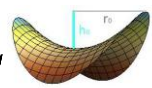


1. **Sufficient saddle rotation**
   – Cancels the effect of slopes
2. **Rolling backwards**
   – Avoids centrifugal force

How can these be achieved?

Existing theory

- **Thompson:** *The rotating-saddle trap: a mechanical analogy to RF-electricquadrupoleion trapping?*
  - Canadian journal of Physics, Vol. 80, 2002
- **Koch:** *Konzeption und Aufbau einer mobilen Experimentiereinheit fur Schuleräprsentationen zum Thema Teilchenfallen*
  - *Universität Stuttgart, 2004*
- Point mass in gravitational potential
  – Constrained to saddle's surface
  $U(x',y') = \frac{mgh}{r_0^2}(x'^2 - y'^2)$ $\quad F = -\nabla U$
- Mathematical trick:
  – coordinates in complex plane $z = x + iy$



$z = k(x^2 - y^2)$

Solution

b POSITION

$$z(\tau) = (Ae^{+\beta_+\Omega t} + Be^{-\beta_+\Omega t} + Ce^{+\beta_-\Omega t} + De^{-\beta_-\Omega t})$$

The only requirement for stability:
f > fc

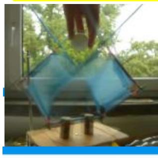
$$\frac{gh_0}{r_0^2\Omega^2} \le 0.5 \quad \Rightarrow \quad f \ge \frac{\sqrt{2gh_0}}{2\pi r_0} = f_{CRITICAL}$$

EXPERIMENTAL VERIFICATION

Apparatus: **Saddles**

h_0 = 1,5cm
r_0 = 8cm
- **f_c = 1,08 Hz**

material = plastic



h_0 = 6,5cm
r_0 = 8cm
- **f_c = 2,25 Hz**

material = nylons

Apparatus: **Balls**

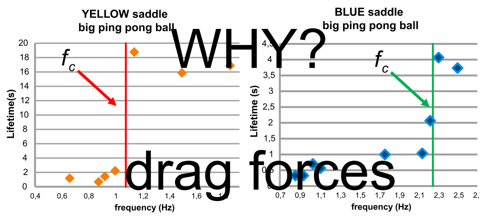
Radius range:
0,63 cm – 3,26 cm
Mass range
8,39 g 35,79 g

solid

Radius range:
1,88 cm – 5,0 cm
Mass range
2,46 g 26,56 g

hollow

Stability vs. Frequency



$f \geq f_c$ ➡ Significant increase in lifetime but clearly not infinite

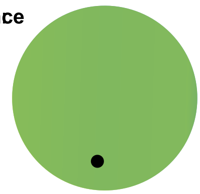
Second condition: Must roll backfast enough

Friction/rolling resistance

Drags the ball to rotate with the saddle

⬇

Ball becomes **unstable**



Literature: Effect of friction

- Thompson: $\vec{F}_{Friction} = -k\vec{v}$
  - Analytical solution: **always diverges**
- Koch: $\vec{F}_{Friction} = -k\frac{\vec{v}}{|\vec{v}|}$
  - Numerical solution: **no record of stability**

~~Stability~~ Maximal lifetime

Parametres affecting the lifetime

1. Drag forces & Friction
2. Frequency
3. Ball
4. Initial position

?

1. DRAG FORCES & FRICTION

Effect of friction

Thompson:

$$T_L = \frac{1}{\sigma\Omega}\ln\left(\frac{r_0}{R}\right)$$

- TL = trapping lifetime
- σ ~ friction coefficient
- R = initial distance from the center
- r_0 = trap's radius

Higher friction ➔ lower lifetime

1. Lifetime vs. Friction: Experim

DRY NYLON SADDLE		NYLON SADDLE SOAKED WITH WATEI
μ=0,25	>	μ=0,09
MAXIMUM LIFETIME: 2,25s	<	6,33s

PREDICTION CONFIRMEL

Not so simple

Koch's article:

	Teflonspray (lower friction)	Cleansaddle (higher friction)
Lifetime	10,1s	54,7s

HIGHER FRICTION

↓

HIGHER LIFETIME

?

What if the ball does NOT slip?

SUFFICIENT FRICTION

⇓

Avoids slipping

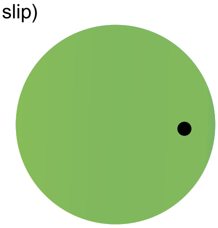
⇓

Sufficient friction: no slipping

Similar to zero friction (no slip)

Dragging effect:

only rolling resistance (much lower than dynamic friction)



Relatively stable:

- Zero friction
- Sufficient friction

Measurement: Slipping vs. Rolling

Slipping (dynamic friction)	Rolling (static friction)
AVERAGE LIFETIME: *(30 measurements)*	*AVERAGE LIFETIME:* *(30 measurements)*
2,7s 0,4 s	**8,8 s** 2,6s

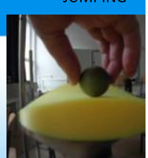
Parameters affecting lifetime

✓
1. Friction
   - dynamic: the lower, the longer lifetime
   - static: fulfills condition

?
2. Frequency
3. Moment of inertia
4. Initial position

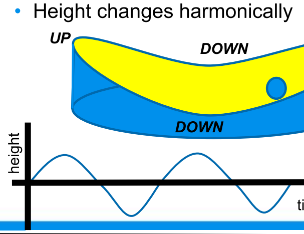
2. FREQUENCY

JUMPING



1. Ball free to move upwards

- Veryfast rotation:
- Height changes harmonically



Harmonic frequency = 2Ω

Condition of jumping

- Saddle shape in polar coordinates $h = kr^2 \cos(2\varphi$
- Vertical acceleration: $a = -(2\Omega)^2 kr^2 \cos(2\Omega t)$

$a \leq g$ Constrained to surface

$a > g$ **Jumps**

Critical frequency for jumping: $f > f_{jump} = \frac{1}{4\pi r}\sqrt{\frac{g}{k}}$

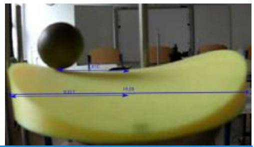
Estimation vs. reality

- We measured the distance in which jump occurred and estimated the frequency

$f = \frac{1}{4\pi r}\sqrt{\frac{g}{k}}$ → $f = 3{,}43 Hz \pm 0{,}14 Hz$

- Measured frequency:

→ $f = 3{,}69 Hz$



Parameters affecting lifetime

1. Friction
2. Frequency
   - Lower limit: rise of lifetime
   - Upper limit: jumping

?

3. Moment of inertia
4. Initial position

3. MOMENT OF INERTIA

EXPERIMENT

$J = \frac{2}{3} mR^2$

$J = \frac{2}{5} mR^2$

AVERAGE LIFETIME	AVERAGE LIFETIME:
(30 measurements)	*(30 measurements)*
8,96 s ± 1,74	**2,11 s ± 0,34 s**

Hollow VS. Solid Ball

Greater moment of inertia

More energy needed for rolling $(M = J\varepsilon)$

Lower speed

Longer lifetime

$J = \frac{2}{3} mR^2$ **Should have longer lifetime than** $J = \frac{2}{5} mR^2$

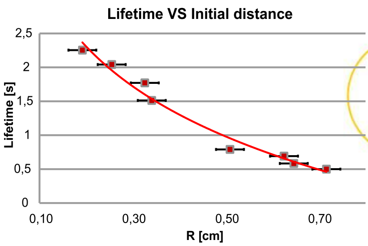
Parameters affecting lifetime

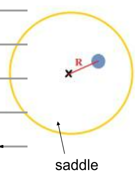
1. Friction
2. Frequency
3. Moment of inertia
   - Higher moment of inertia = longer lifetime

?

4. Initial position

INITIAL POSITION

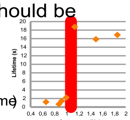




The **further** from the center we place the ball, the **sooner** it falls off

Conclusions

- Conditions under which the ball should be stable
  - Sufficient saddle rotation
    - Theory: Critical frequency fc
    - Experiment never stable (rise od lifetime)
  - Avoiding centripetal force
    - Theory: No or low drag force
    - Our contribution: by backward rotatio



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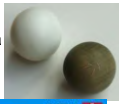
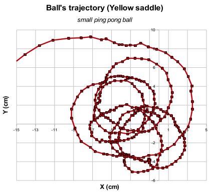
Conclusions

- Examined
  - Friction:
    - Theory: solution only for specific case
    - Our contribution: sufficient friction = more stable
  - Jumping (not mentioned in theory)
    - upper limit for frequency exists + estimation
  - Rotation of the ball (not mentioned)
    - Dependence on the moment of inertia

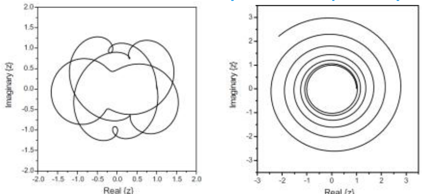




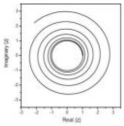
APPENDICES

SLOVAKIA IYPT 14

Prediction: Stability vs. Frequency

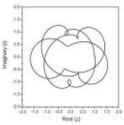


$f \geq f_{critical}$ $f < f_{critical}$

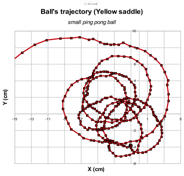


$f < f_c$

Similar to predicted behaviour

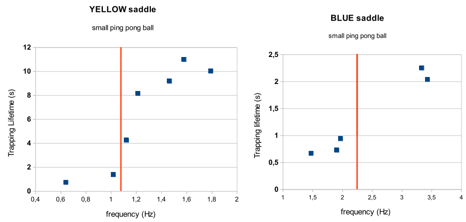


$f \geq f_c$



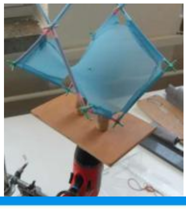
But always limited lifetime

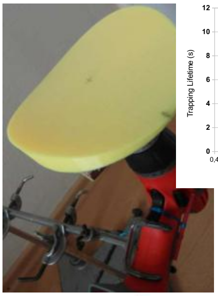
Small ping pong ball



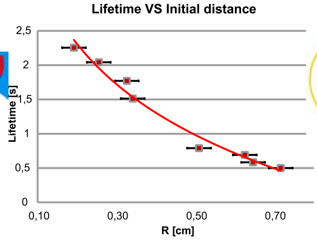
Apparatus:**Rotation**

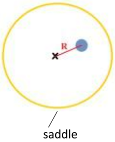
- Rotation: **driller**
  - Frequency range: 0.6Hz 3.7Hz





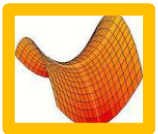
INITIAL POSITION



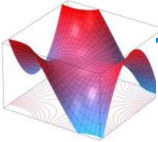


Saddle

- Convexin one direction; concavein the other
- Various saddle types:



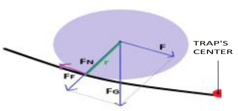
$z = k(x^2 - y^2)$



- Convenient for mathematical description

POINT MASS vs BALL

$M = Fr = J\epsilon$



DEEPER SADDLE
↓
THE BIGGER COMPONENT OF F_G CAUSES THE TORQUE
↓
THE LESS IT RESEMBLES POINT MASS

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'DEEP' vs 'SHALLOW' saddle

MAXIMUM LIFETIME: **4,07 s**

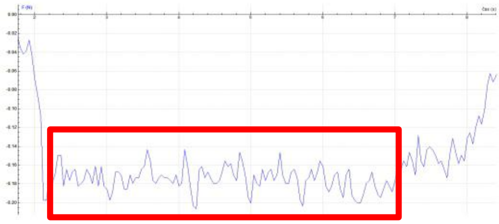
MAXIMUM LIFETIME: **23,91 s**

THE **SMALLER** THE SADDLE'S **HEIGHT** IS, THE **MORE STABLE** THE BALL IS

Friction

–blue saddle

$$f = \frac{F_{friction}}{F_N}$$



Theory

Gravitational potential:

- assigned to the rotating frame (fixed to U)

$$U(x', y') = \frac{mgh_0}{r_0^2}(x'^2 - y'^2)$$

- converted to the laboratory frame:

$$U(x, y) = \frac{mgh_0}{r_0^2}[(x^2 - y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)]$$

using the following formula $F = -\nabla U$ yields

$$\frac{\partial^2 x}{\partial t^2} = \frac{2mgh_0}{r_0^2}[-x\cos(2\Omega t) - y\sin(2\Omega t)]$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{2mgh_0}{r_0^2}[y\cos(2\Omega t) - x\sin(2\Omega t)]$$

using dimensionless parameters $\tau = \Omega t$ and $q = \frac{gh_0}{r_0^2 \Omega^2}$

converting to the complex plane $(z = x + iy)$,

the 2 equations are reduced into:

$$\frac{\partial^2 z}{\partial \tau^2} + 2q * e^{i2\tau} = 0$$

Applying another substitution $z(\tau) = f(\tau)e^{i\tau}$ *yields the solution:*

$$f(\tau) = Ae^{+\beta_+\tau} + Be^{-\beta_+\tau} + Ce^{+\beta_-\tau} + De^{-\beta_-\tau}$$

where A,B,C,D are real parameters depending on initial conditions

$$\beta_\pm = \sqrt{\pm 2|q| - 1}$$

ß± ∈ R- {0} => result will diverge in any case => particle is trapped only if ± ∈ I, thus

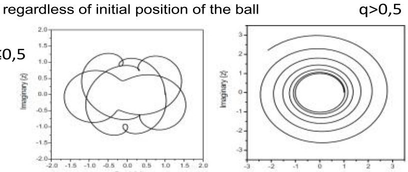
$$2|q| \le 1 \quad \Rightarrow \quad q \le 0{,}5$$

$$q = \frac{gh_0}{\Omega^2 r_0^2} \le 0{,}5$$

- **The condition for stability is:**
- $\Omega \ge \frac{\sqrt{2gh_0}}{r_0} \longrightarrow f \ge \frac{\sqrt{2gh_0}}{2\pi r_0}$

regardless of initial position of the ball

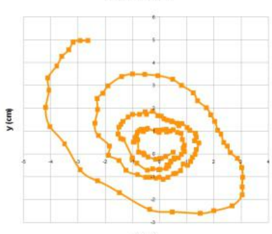
q≤0,5 q>0,5



STABLE TRAPPING PARAMETERS

Ball's Trajectory (BLUE saddle)

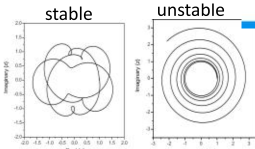
small ping pong ball



Ball's trajectory (Yellow saddle)

small ping pong ball

stable unstable



SOURCES

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The rotating-saddle trap: a mechanical analogy to RF-electricquadrupoleion trapping?
(Can. J. Phys. Vol. 80, 2002)

Wolfgang Rueckner, Justin Georgi, Douglass Goodale, Daniel Rosenberg, David Tavilla:
Rotating saddle Paul trap
(American Journal of Physics 63, 186 (1995); doi: 10.1119/1.17983)

A. K. Johnson and J. A. Rabchuk:
A bead on a hoop rotating about a horizontal axis: A one-dimensional ponderomotive trap
(Citation: American Journal of Physics 77, 1039 (2009); doi: 10.1119/1.3167216)

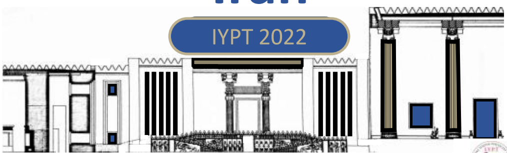
Tobias Koch:
Konzeption und Aufbau einer mobilen Experimentiereinheit fur Schuleräprsentationen zum Thema Teilchenfallen

Problem No. 13 Candle powered turbine

Reporter: Zahra Hosseini

Iran

IYPT 2022

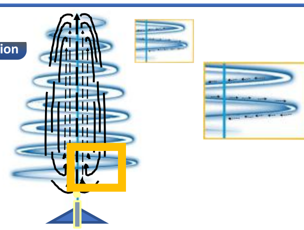


Problem

A paper spiral suspended above a candle starts to rotate. Optimize the setup for maximum torque.

Initial Observation

Initial Observation



Theoretical framework

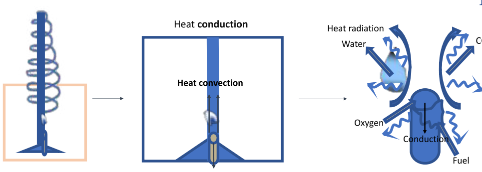
Theoretical framework

- Spiral
  1. Number of spiral cycles
  2. Radius of the spiral circle
  3. Paper width
- Convection
- Candle
  1. Candle wax combustion
  2. Candle flame
  3. Distance between candle and spiral

- Theoretical Framework

1.Macroscopic View
2.Microscopic View

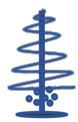
- Microscopic View :
Discuss the Partial force applied to the spiral

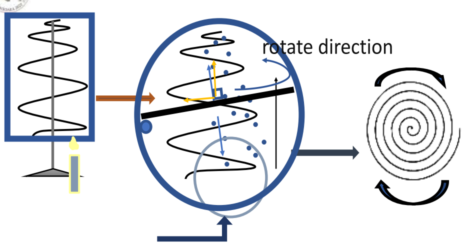


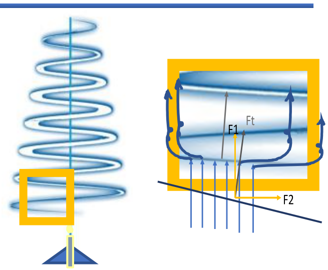
Equation for wax combustion :

$$C_{25}H_{52} + 38\, O_2 \rightarrow 25\, CO_2 + 26\, H_2O \text{ +Heat}$$

$$Q = mc\Delta\theta \rightarrow K = \frac{1}{2}mv^2$$







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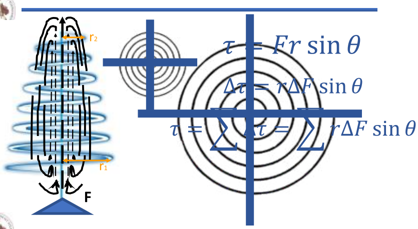
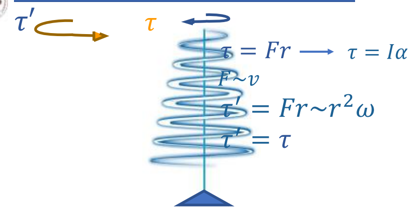


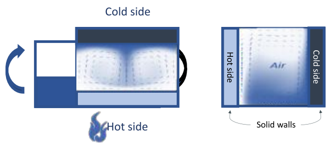


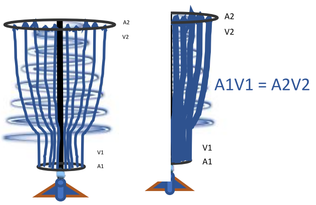
1.Macroscopic View
2.Microscopic View

- Microscopic View :

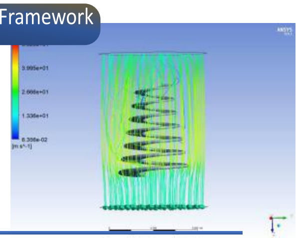
Discuss the Partial force applied to the spiral

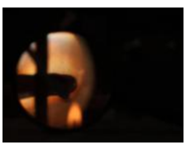
Convection is the **circular motion** that happens when warmer part of fluid rises, while the cooler part of fluid drops down.



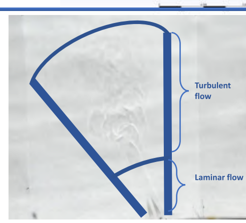


Theoretical Framework

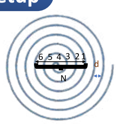


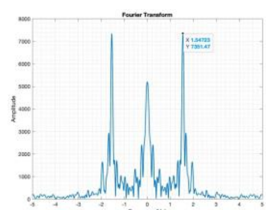


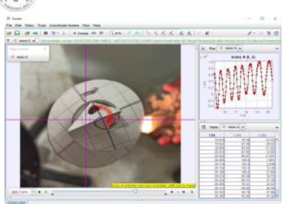
Schlieren photography

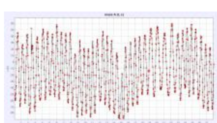


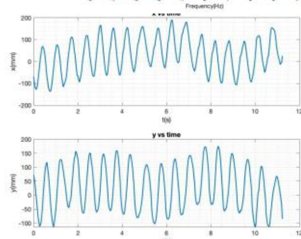
Experimental setup











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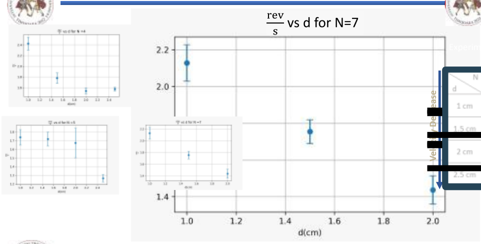
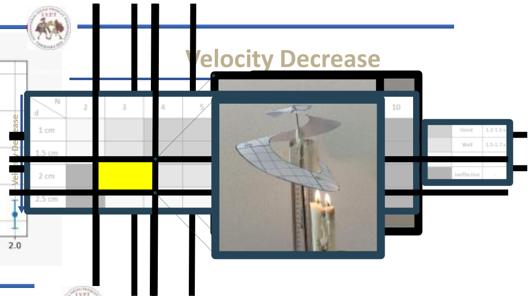
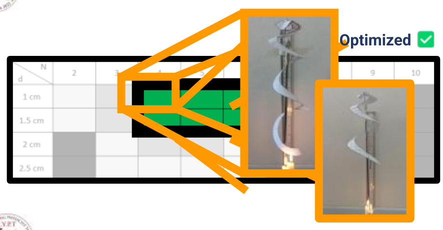
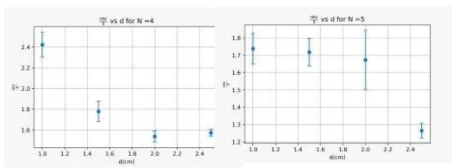
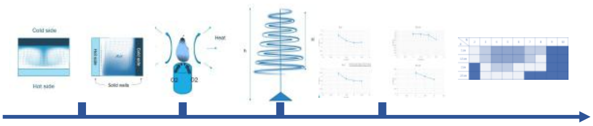








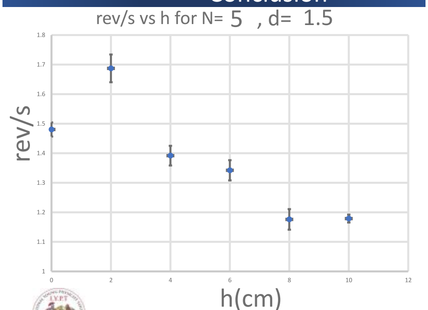
Solution Outline



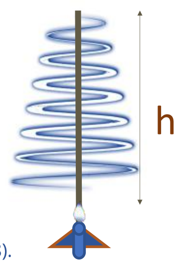
- **Theoretical framework**
  - ✓ Energy Transition
  - ✓ Drag Force
  - ✓ Force applied to system
  - ✓ Total torque equation
  - ✓ Torque limit definition
  - ✓ Convection Mechanism effects
  - ✓ Study of air flow transition

- **Experimental result**
  - ✓ 4 Number of rounds+4 Density observed
  - ✓ Study the rotation and angular velocity
  - ✓ Color code of each experiment
  - ✓ Optimization for best results

Conclusion



Conclusion



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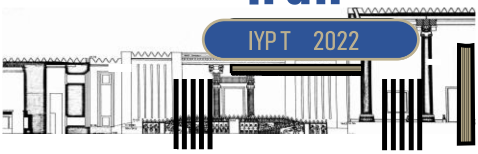
Problem No.3 Ring on the Rod

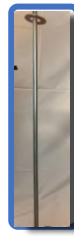
Ramin Abdollahzadeh Iran



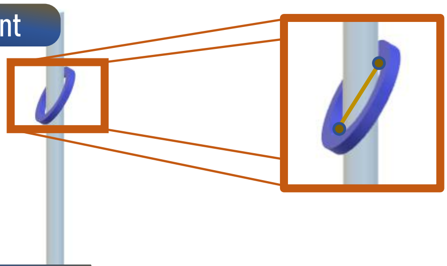
Problem

Ring on the Rod

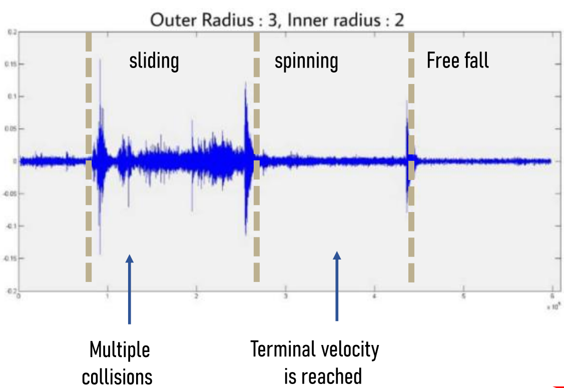
A Washer on a vertical steel rod may start spinning instead of simply sliding down. Study the motion of washer and investigate what determines the terminal velocity.



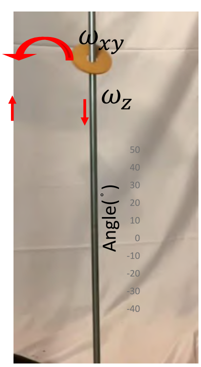
Contact point



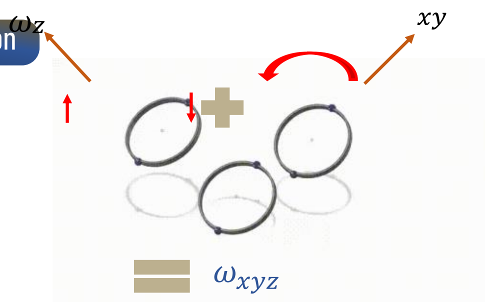
Transitions



ω



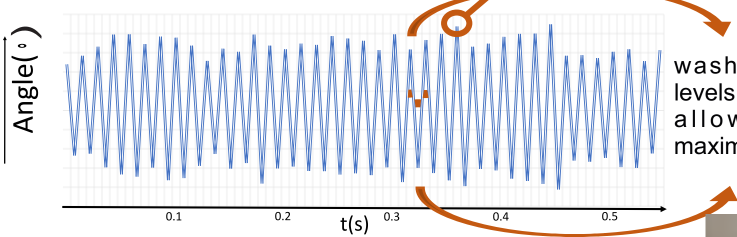
Spinning motion



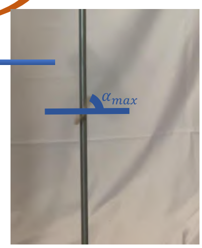
Motion Transition

SLIDING , SPINNING TRANSITION

$$\alpha_{max} = Cos^{-1}\left(\frac{D_{rod}}{D_{in.w}}\right)$$



washer slides and levels out before it is allowed to reach maximum tilt



Theory based on Energy

$$k_1 + k_2 + \frac{1}{2}mv_z^2 - mg(v_z t) + \mu N(r\omega_{xy}t) + W_{air} = \text{cte}$$

$$= \frac{1}{2}\rho v^2 C_D A.d$$

ρ: $1.31\frac{kg}{m^3}$

A: $\approx (1.25\pi, 24.5\pi) \times 10^{-4}\ m^2$

d: $\approx 7 \times 10^{-2}$

$\approx (300, 6000) \times 10$

$\Rightarrow$ ***negligible***

d = distance

ρ = density of fluid

v = speed of the object relative to the fluid

C_D = Drag coefficient

A = cross sectional area

v_z

$$v_z = \frac{D_{rod}}{2\pi}.(\omega_{xy}+\omega_z)$$

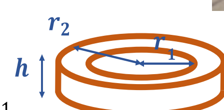
1cm

Term 1

$$\frac{1}{2}I\omega_{xy}^2 + \frac{1}{2}I\omega_z^2$$

$$I_{xy} = \frac{1}{12}m(3(r_2^2 - r_1^2) + h^2)$$

$$I_z = \frac{1}{2}m(r_2^2 - r_1^2)$$



$$k_1 + k_2 + \frac{1}{2}mv_z^2 - mg(v_z t) + \mu N(r\omega_{xy}t) = \text{const}$$

1cm

$$v_z = \frac{D_{rod}}{2\pi}.(\omega_{xy}+\omega_z)$$

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$$E = k_1 + k_2 + \frac{1}{2}mv_z^2 - mg(v_z t) + \mu N(r\omega_{xy} t)$$

$$E = k_1 + k_2 + \frac{1}{2}mv_z^2 - mg(v_z t) + \mu N(r\omega_{xy} t)$$

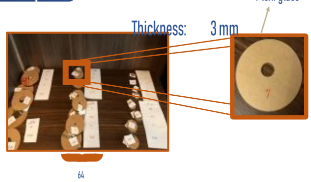
0.8

$mgcos\alpha$

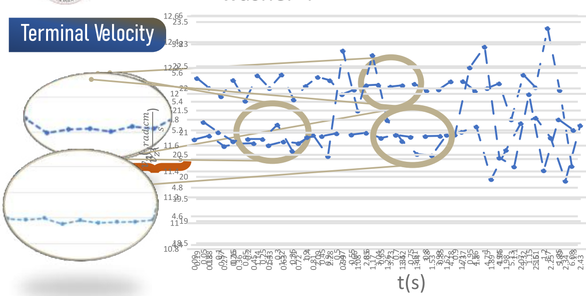
rotation time in α_{max} condition

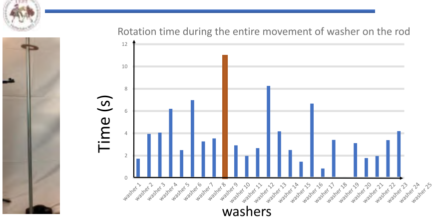
D_{rod} $\Rightarrow$ $(\omega_{xyz}^{2\pi} + \omega_{xyz}) - \mu N r \omega_{xy} = 0$

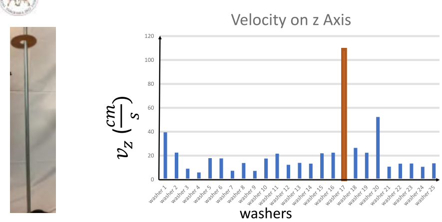
Setup

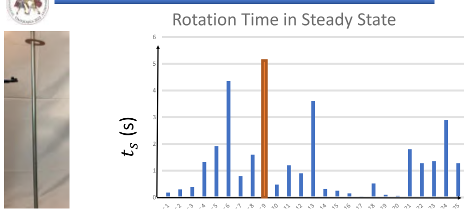


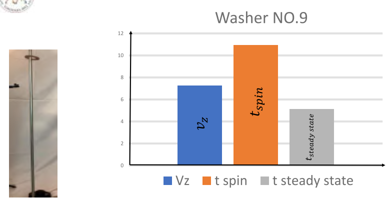
Terminal Velocity









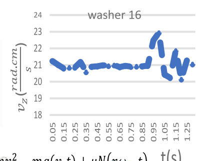


Washer NO.9

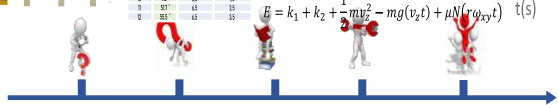
Washer NO.9
Washer Diameter: 6cm
Hole Diameter: 3 cm

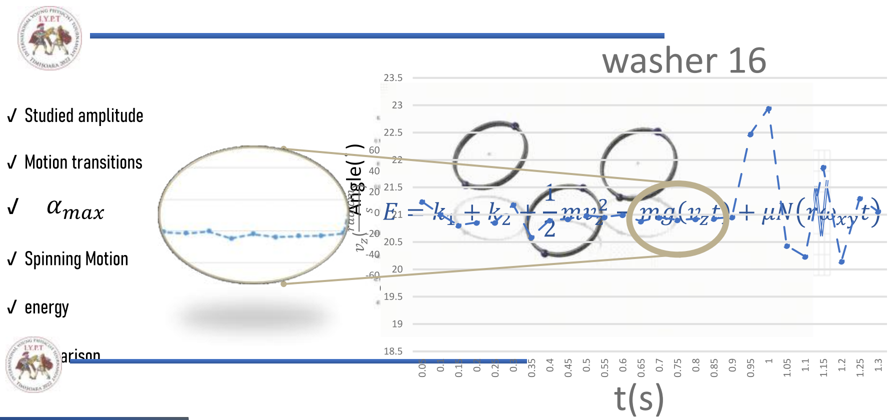
Solution Outline

washer	alpha max	washer diameter	hole diameter
1	48.4°	3	2
2	47.8°	4	2
3	47.4°	4.5	2.5
4	21.3°	4.5	3.5
5	44.8°	5	2
6	13.4°	5	3
7	49.1°	5.5	3.5
8	47.6°	6	2
9	18.8°	6	3
10	4.5°	6.5	1.5
11	51.7°	6.5	2.5
12	55.5°	6.5	3.5



$$E = k_1 + k_2 + \frac{1}{2}mv_z^2 - mg(v_z t) + \mu N(r\omega_{xy} t)$$





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- https://wwwyoutube.com/watch?v=ibe 2CaspGJY
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Problem No.10 Conducting Lines

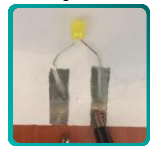
Arsha Niksa

IRAN

IYPT 2020

Problem

A line drawn with a pencil on paper can be electrically conducting. Investigate the characteristics of the conducting line.



Literature Review

Electrical conductivity of compacts of graphene, multi-wall carbon nanotubes, carbon black, and graphite powder

Bernardo Marinho, Marcos Ghislandi, Evgeniy Tkalya, Cor E. Koning, Gijsbertus de With

B. Marnho et al, Powder Tech. (2012)

Experimental Investigation

- Density, Pressure, Mass, and Orientation

Intuitive Theory

- Van der Waals Interactions

Influence of Bulk Graphite Density on Electrical Conductivity

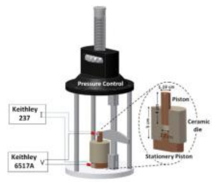
S. Rattanaweeranon, P. Limsuwan, V. Thongpool, V. Piriyawong, P. Asanithi

S. Rattanaweeranon et al, Procedia Eng. (2011)

Electrical conductivity of compacts of graphene, multi-wall carbon nanotubes, carbon black, and graphite powder

Bernardo Marinho, Marcos Ghislandi, Evgeniy Tkalya, Cor E. Koning, Gijsbertus de With

B. Marnho et al, Powder Tech. (2012)



Mode I: Conductivity during Compaction

Mode II: Conductivity on a Paper Film

Electrical conductivity of compacts of graphene, multi-wall carbon nanotubes, carbon black, and graphite powder

Bernardo Marinho, Marcos Ghislandi, Evgeniy Tkalya, Cor E. Koning, Gijsbertus de With

B. Marnho et al, Powder Tech. (2012)

- Referenced experimentally - deduced theory
- Experimented with a multitude of parameters
- Used a controlled experimental apparatus
- Did not introduce an independent theoretical framework that could have been compared with theory

Influence of Bulk Graphite Density on Electrical Conductivity

S. Rattanaweeranon, P. Limsuwan, V. Thongpool, V. Piriyawong, P. Asanithi

S. Rattanaweeranon et al, Procedia Eng. (2011)

- The chosen parameter (density) was investigated in detail.
- Material properties were measured in a number of ways
- A preliminary theoretical framework was introduced and was not theoretically developed

Literature Review

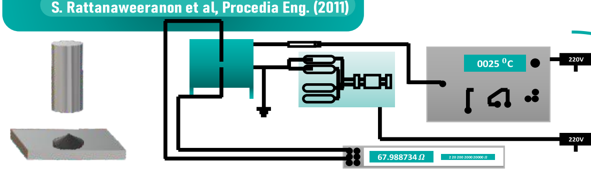
Influence of Bulk Graphite Density on Electrical Conductivity

S. Rattanaweeranon, P. Limsuwan, V. Thongpool, V. Piriyawong, P. Asanithi

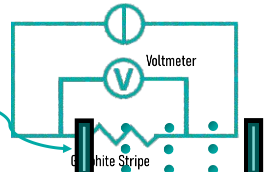
S. Rattanaweeranon et al, Procedia Eng. (2011)

Experimental Investigation

- Bulk Density



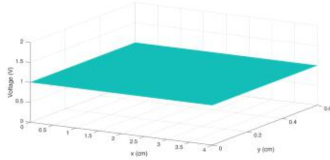
Pouillet's Law

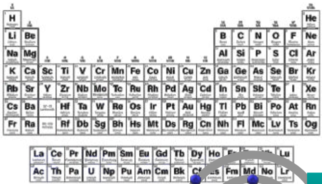


Pouillet's law is not applicable.

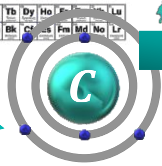
Structure of Carbon

$\sigma_{bond} = 1$





$Q_C = 6e \mid 1s^2, 2s^2, 2p^2$



Discretization

$$dR = \frac{dV}{dI}$$

$$-\frac{\rho}{\varepsilon_0} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}$$

$$-\frac{\partial q}{\partial t} = \oint_S J.d\vec{A}$$

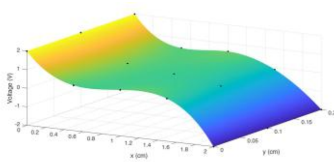
$$V(x,y) = \left(\frac{h_x^2}{2(h_x^2+h_y^2)}\right)(V(x,y+dy)+V(x,y-dy))$$

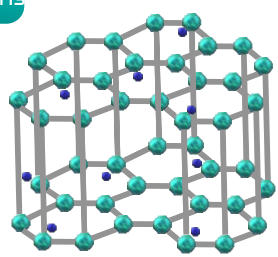
$h_x = \Delta x$

$h_y = \Delta y$

Graphite π Electrons

$\sigma_{bond\ 1,2,3} = 1$



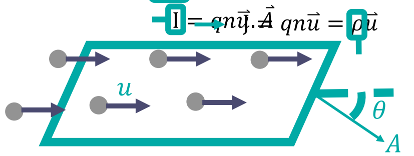


Current Density

$$\Delta N = n(|\vec{u}|\Delta t)\left(\left|\vec{A}\right|\right)\cos\theta = n\vec{u}.\vec{A}\Delta t \quad [3]$$

$$\Delta q = qn\vec{u}.\vec{A}\Delta t$$

$$I = qn\vec{u}.\vec{A} \quad qn\vec{u} = \rho\vec{u}$$



$$\vec{J} = \sum_k q_k n_k \vec{u}_k = \sum_k \rho_k \vec{u}_k = qnu = \rho u$$

$k \epsilon \mathbb{N}$ type

Voltage

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0}, where\ \rho = 0$$

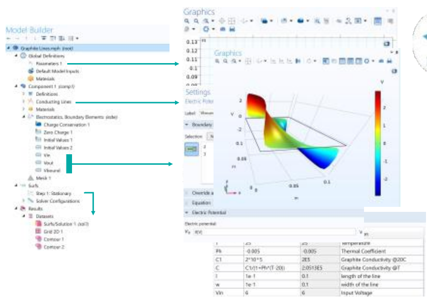
$$\nabla^2 V = -\frac{\rho}{\varepsilon} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$dR = \frac{dV}{dI}$$

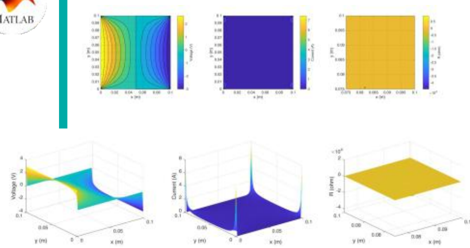
$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Cross checking

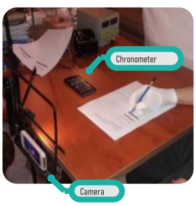
COMSOL



Simulation

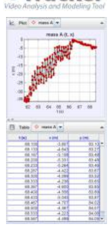


Drawing the Lines

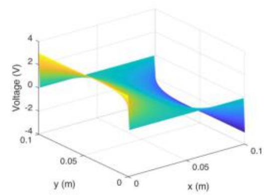


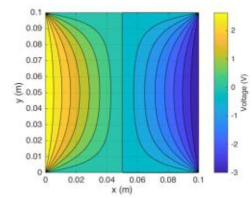
Tracker

Video Analysis and Modeling Tool

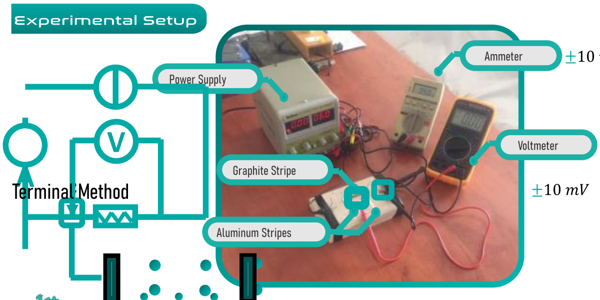


Error Calculation



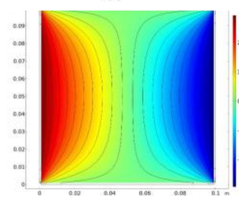


Experimental Setup

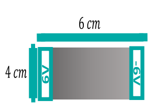


$\pm 10\ nA$

$\pm 10\ mV$



Parameters



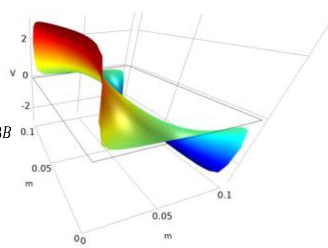
$0.2\ \times 2\ cm^2, 0.4\ \times 3\ cm^2$
$0.8\ \times 5\ cm^2, 1\ \times 6\ cm^2$

$0.2\ cm, 0.4\ cm, 0.8\ cm, 1\ cm$

$8H, 4H, 4B, 8B$

$2\ cm, 3\ cm, 5\ cm, 6\ cm$

$4\ V, 12\ V$



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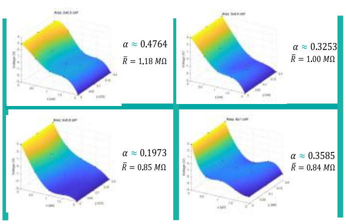
Parameter: Area

$2 \times 0.2\ cm^2$ | $3 \times 0.4\ cm^2$ | $5 \times 0.8\ cm^2$ | $6 \times 1\ cm^2$

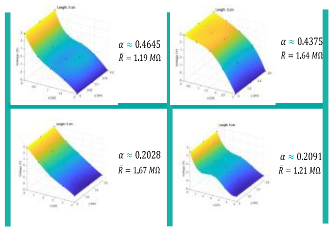
$\bar{I} = 2.485\ \mu A$



Parameter: Length

2 cm | 3 cm | 5 cm | 6 cm

$\bar{I} = 2.27\ \mu A$

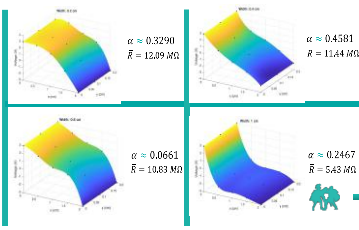


L↑
R↑

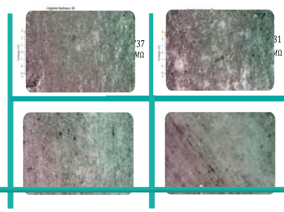
Parameter: Width

0.2 cm | 0.4 cm | 0.8 cm | 1 cm

$\bar{I} = 0.37\ \mu A$



Parameter:Hardness

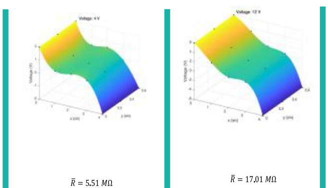


Graphite ↑
R ↓

Results

Parameter: Voltage

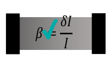
$\bar{I} = 0.4\ \mu A$



$R \propto V$

$R \propto P_{clay}$

V↑
R↑



$R \propto W^{-1}$

$R \propto P_{Graphite}^{-1}$

Length (cm)	2 cm: 0.62	3 cm: 0.48	5 cm: 0.92	6 cm: 0.39
Width (cm)	0.2 cm: 0.95	0.4 cm: 1.26	0.8 cm: 0.34	1 cm: 0.65
Surface Area (cm^2)	2×0.2: 2.77	3×0.4: 0.95	5×0.8: 0.95	6×0.8: 2.86
Hardness	4B: 2.16	8B: 0.62		
Voltage (V)	4 V: 0.475	12 V: 0.4625		

Scaling Ruler: $\pm 0.05\ cm$

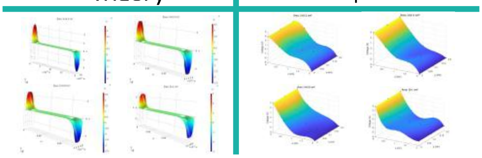
Protractor: $\pm 0.05^{\circ}$

Voltmeter: $\pm 10\ mV$

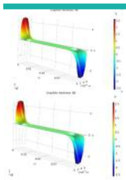
Ammeter: $\pm 10\ nA$

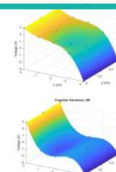
Theory VS. Experiment

Theory | Experiment

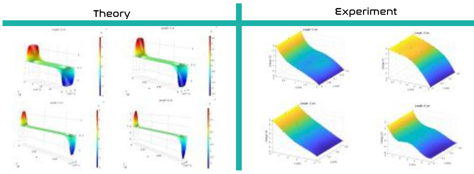


Theory | Experiment

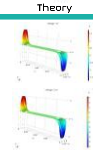


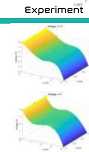


Theory | Experiment



Theory | Experiment





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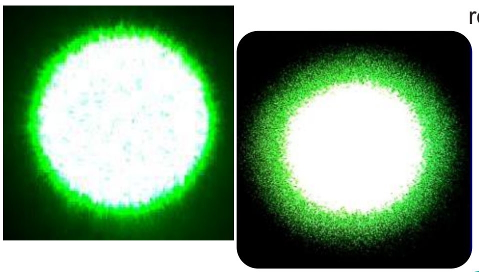
IRAN IYPT 2020

Arsha Niksa

Problem No. 11

Drifting Speckles

Shine a laser beam onto a dark surface. A granular pattern can be seen inside the spot. When the pattern is observed by a camera or the eye, that is moving slowly, the pattern seems to drift relative to the surface. Explain the phenomenon and investigate how the drift depends on relevant parameters.



Literature Review

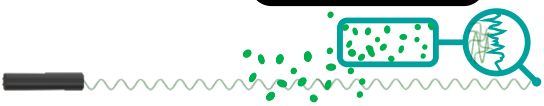
NEW APPROACHES

Laser speckle experiments for students

A E Ennos University of Surrey, UK

A. E. Ennos

DOI : 10.1088, 1964 reads



Algorithms for simulation of speckle (laser and otherwise)

Donald D. Duncan[a], Sean J. Kirkpatrick[b]

[a]Oregon Health & Science University, 3303 SW Bond Ave, Portland, OR USA 97239-4501

[b]Oregon Health & Science University, 20000 NW Walker Rd, Beaverton, OR USA 97006-8921

Algorithms for simulation of speckle (laser and otherwise)

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[b]Oregon Health & Science University, 20000 NW Walker Rd, Beaverton, OR USA 97006-8921

D. D. Duncan, S. J. Kirkpatrick

DOI : 10.1117 , 4337 reads

- Only inclusive of flat planes
- No comprehensive experiments

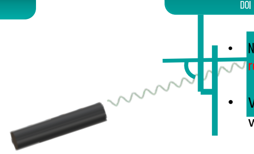
NEW APPROACHES

Laser speckle experiments for students

A E Ennos University of Surrey, UK

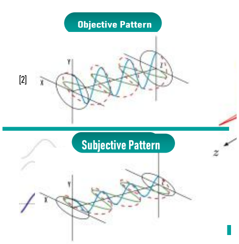
A. E. Ennos

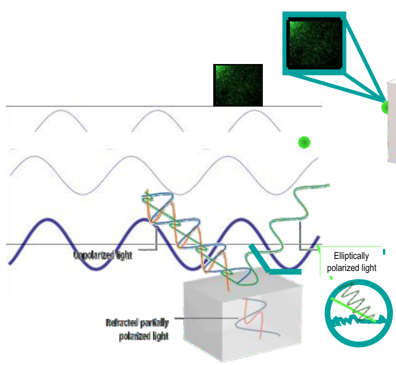
DOI : 10.1088, 1964 reads

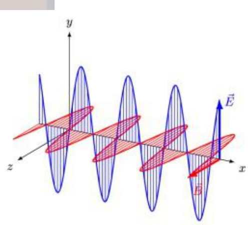


- Neglects the effect of roughness
- Wave polarization was not well-detailed

Polarization







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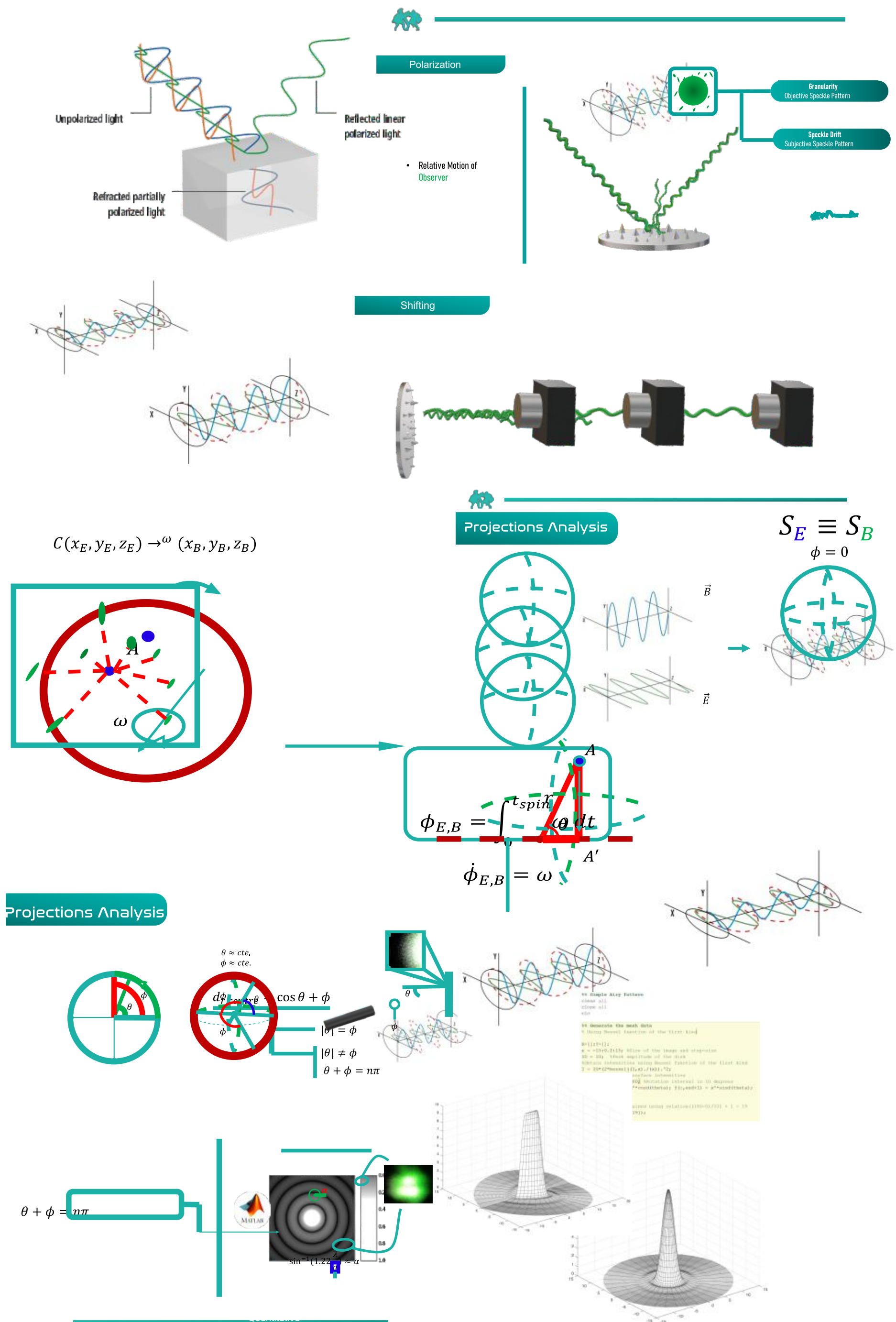

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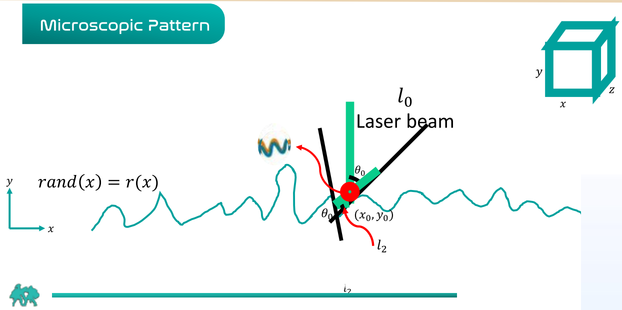
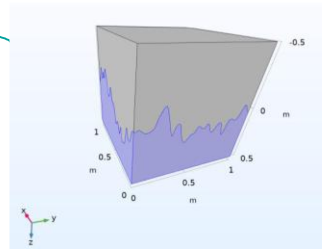
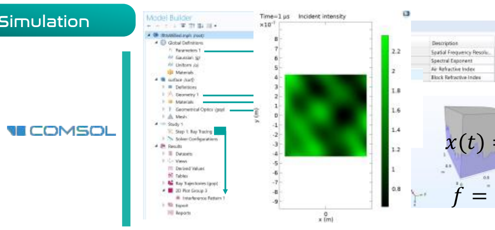
Microscopic Pattern





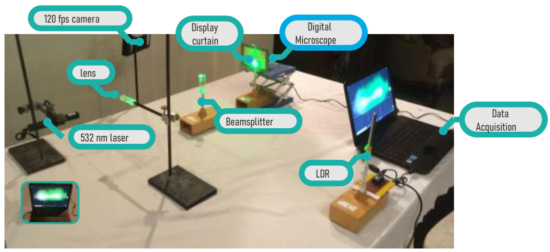
Simulation



$$x(t) = sd \sin \omega t$$

$$f = \left(\frac{c \pm v}{c}\right) f_0$$

Experimental Setup



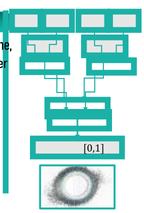
Accordant Parameters

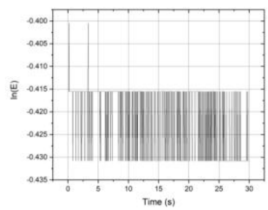
```
clear all
close all
clc

%% Importing the desired images
img1 = 'Control1.png';
img2 = 'Control2.png';

img1 = imread(img1);
img2 = imread(img2);

%% Creating a montage of both images
figure
montage({img1,img2})
title('Speckle Pattern 1 vs. Speckle Pattern 2')

%% Deducing SSIM values
[ssimval,ssimmap] = ssim(img1,img2);

%% Printing the differentiated image
figure
imshow(ssimmap,[])
title(['Local SSIM Map with Global SSIM Value: '
```

- $d_{laser} = 0.85\ m$
- $f = 120\ fps$
- $I = 0.8\ {}^{W}/_{cm^2}$
- $M_{microscope} = 58$
- $\lambda = 532\ nm$
- No external interference
- Rough paper

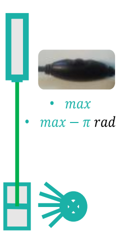
Control

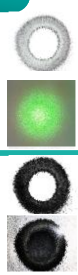
Parameter	Values
Distance	$0.55\ m, 1.15\ m$
Magnification	54 X, 64 X
Framerate	$60\ fps, 240\ fps$
Wavelength	$532\ nm$
Intensity	$0.12\ {}^{W}/_{cm^2}, 0.83\ {}^{W}/_{cm^2}$
Interference	ℓ_l, ℓ_h
Roughness	Cellophane, Thin Paper





Parameter: Interference





91.946%

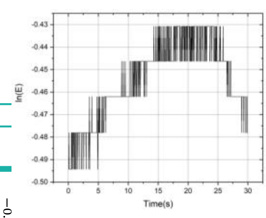
90.019%

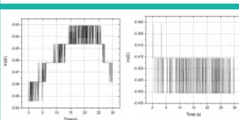
65.472%

52.784%

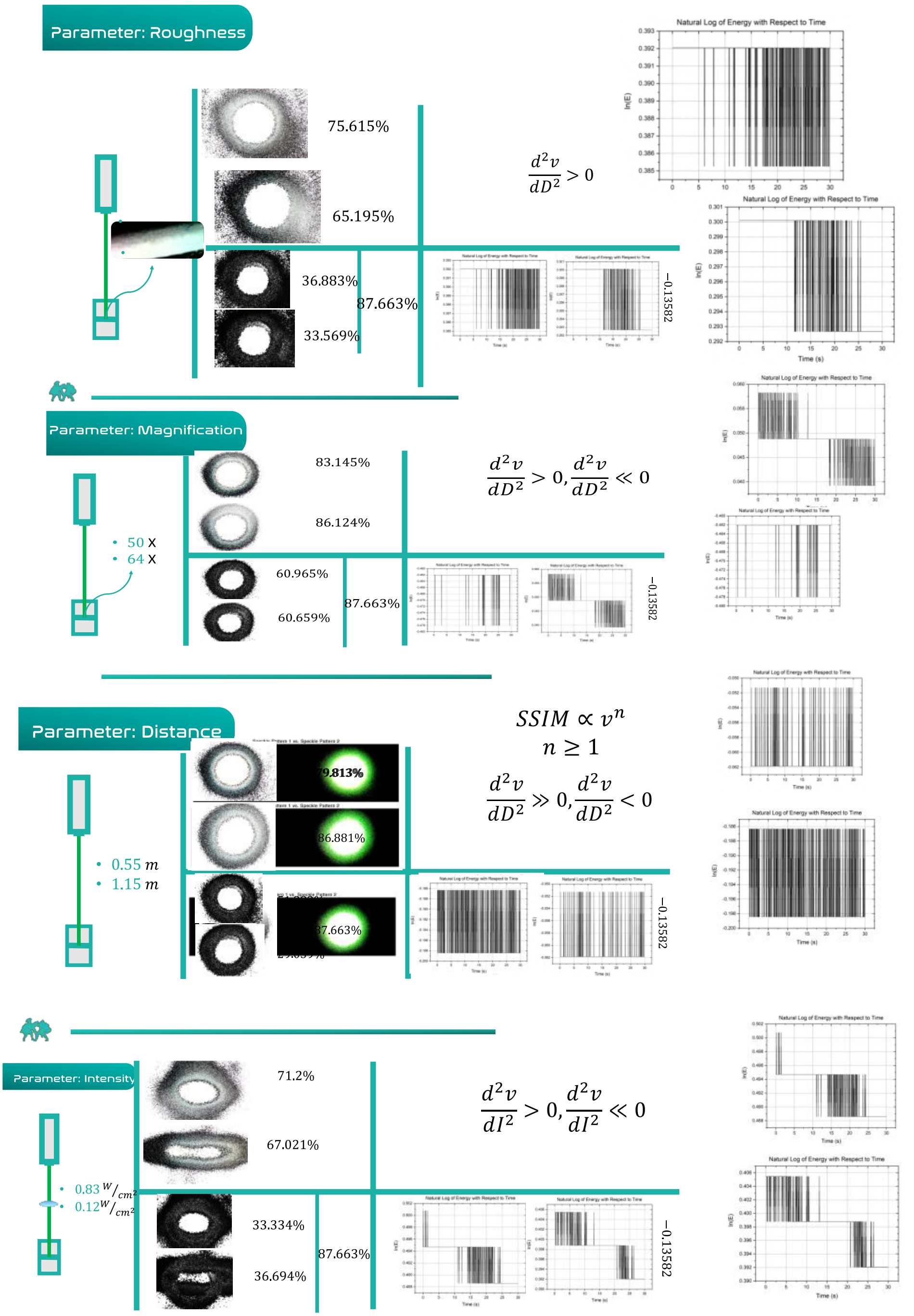
87.663%

$\frac{d^2v}{d\ell^2} > 0$





-0.13582


Parameter: Roughness
75.615%
65.195%
36.883%
33.569%
87.663%
$\frac{d^2v}{dD^2} > 0$
−0.13582
Natural Log of Energy with Respect to Time
Parameter: Magnification
50 x
64 x
83.145%
86.124%
60.965%
60.659%
87.663%
$\frac{d^2v}{dD^2} > 0, \frac{d^2v}{dD^2} \ll 0$
−0.13582
Parameter: Distance
0.55 m
1.15 m
79.813%
86.881%
87.663%
$SSIM \propto v^n$
$n \geq 1$
$\frac{d^2v}{dD^2} \gg 0, \frac{d^2v}{dD^2} < 0$
−0.13582
Parameter: Intensity
$0.83\ ^{W}/_{cm^2}$
$0.12\ ^{W}/_{cm^2}$
71.2%
67.021%
33.334%
36.694%
87.663%
$\frac{d^2v}{dI^2} > 0, \frac{d^2v}{dI^2} \ll 0$
−0.13582

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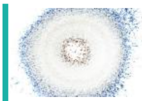
Parameter: Wavelength

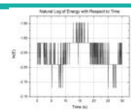
- 632.6 nm



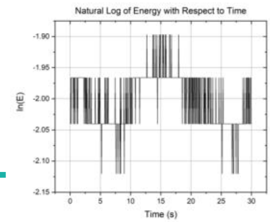
92.492%

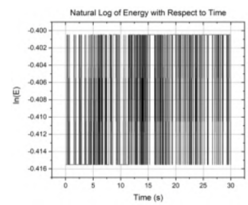
$$\frac{d^2v}{d\lambda^2} > 0$$

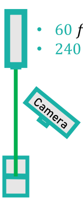
46.433% 87.663%



-0.13582







- 60 fps
- 240 fps

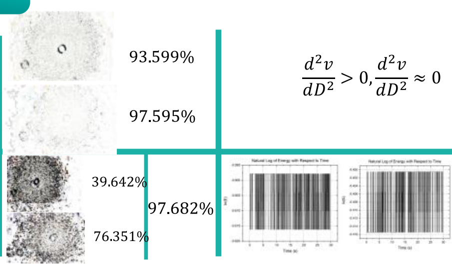
93.599%

97.595%

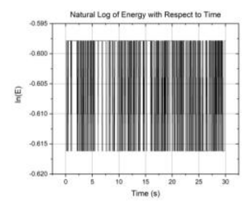
$$\frac{d^2v}{dD^2} > 0, \frac{d^2v}{dD^2} \approx 0$$

39.642% 97.682%

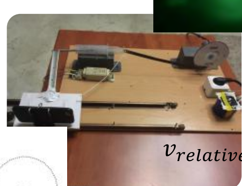
76.351%

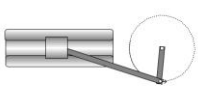


-0.13582



Effect of Relative Motion





$$v \propto \lambda$$

$$v \propto \ell_{external}$$

$$v_{relative} \propto n(0)_{black}$$

$d \rightarrow opt.$

$I \rightarrow opt.$

$M \rightarrow opt.$

$f \rightarrow cte.$

$$v_{relative} \propto n(0)_{black}$$

Theory vs. Experiments

$$\dot{d}_{r,centre} = v = -\sin(\theta + \phi)$$

v

$\lambda \propto v$	$CH \rightarrow opt.$	
$\ell \propto v$	✓	
$d \rightarrow opt.$	✓	$P(d)$
$I \rightarrow opt.$	✓	
$M \rightarrow opt.$	✓	
$f \rightarrow cte.$	✓	

References

[1] Superposition, D. A. Russel, State University of Pensylvania, (1996).

[2] Introduction to Polarization, Edmund Optics [Internet]

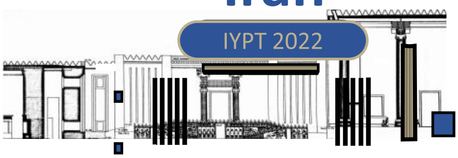
[3] Algorithms for simulation of speckle (laser and otherwise), Duncan, D & Kirkpatrick, Sean. (2008 DOI : 10.1117

[4] Laser speckle experiments for students, A. E. Ennos Physics, Education, Volume 31, (1996). DOI : 10.1088

[5] Ray Optics Module, COMSOL Multiphysics, A part of exemplary libraries Number 3,

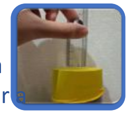
Problem No.14 **Ball on membrane**

Zahra Hosseini **Iran**

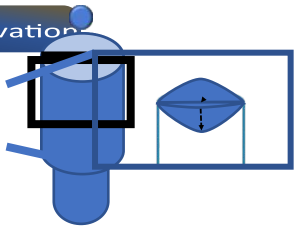


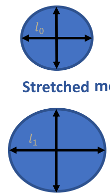
- It is stated that :

When dropping a metal ball on a rubber membrane stretched over a plastic cup, a sound can be heard. Explain the origin of this sound and explore how its characteristics depend on relevant parameters.

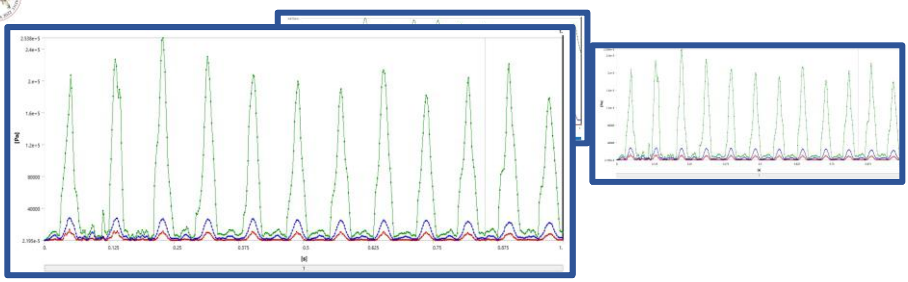


Initial observation





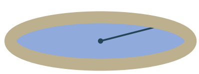
$$\varepsilon = \frac{l_1 - l_0}{l_0}$$



Theoretical Framework

$$\frac{\partial^2 T}{\partial t^2} = c^2 T k \rightarrow T = A\cos(cst) + B\sin(cst) \rightarrow f = \frac{cs}{2\pi}$$

Wave
- Frequency
- Propagation speed
- Amplitude
- Wavelength

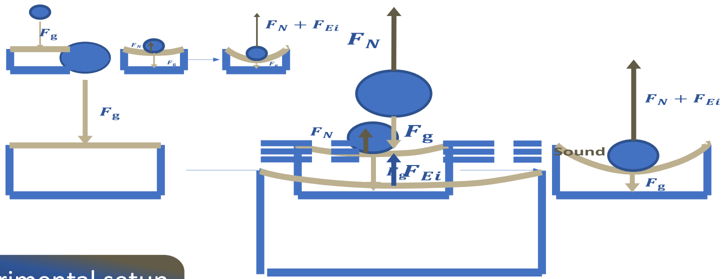


$$u(a,\theta,t) = 0 \rightarrow r = a \rightarrow u = RT\Theta \rightarrow R(a) = 0$$

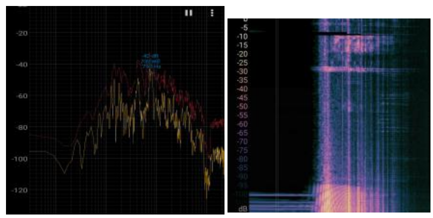
$$R = EJ_n(sr) \rightarrow J_n(as) = 0 \rightarrow as = \alpha_{nm} \rightarrow S = \frac{nm}{a}$$

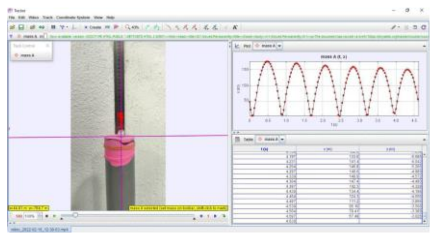
$$u = \sum_{n,m} E_{n,m} J_n\left(\frac{\alpha_{n,m}}{a} r\right)\left(A\cos\left(\frac{C\alpha_{n,m}}{a} t\right) + B\sin\left(\frac{C\alpha_{n,m}}{a} t\right)\right)(C_n\cos(n\theta) + C_n\sin(n\theta))$$

$$f = \frac{c\alpha_{n,m}}{2\pi a}$$

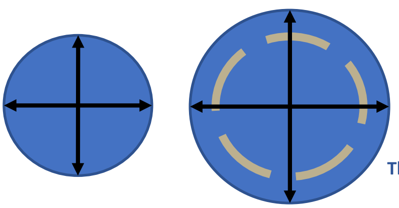


Experimental setup



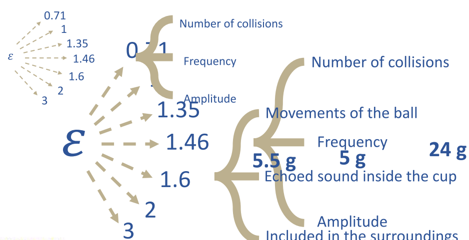


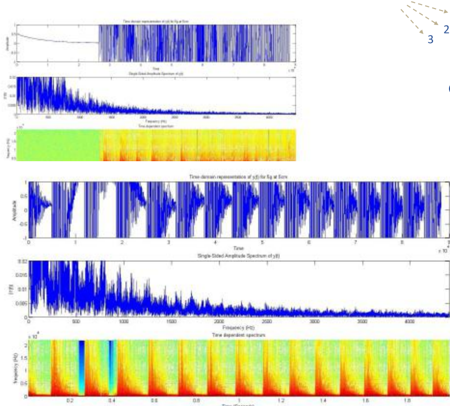
Normal membrane **Stretched mode**

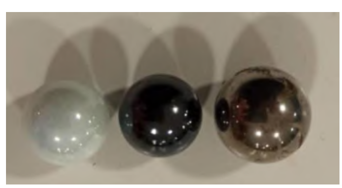


The Sound Can Be heard

Elastic membrane from top view







5 gram at 5 cm – 1 layer membrane and 5 gram at 5 cm – 2 layer membrane

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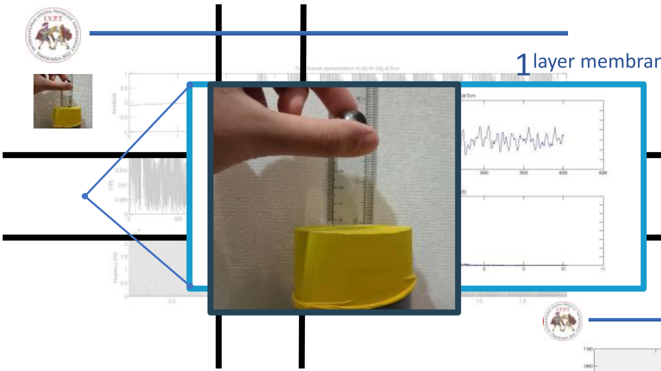
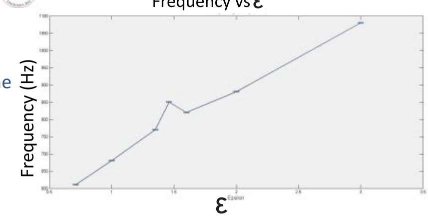
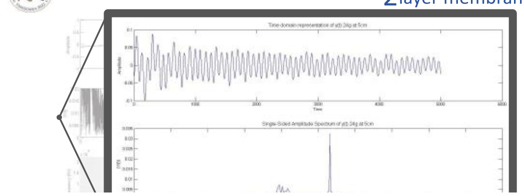
1 layer membrane

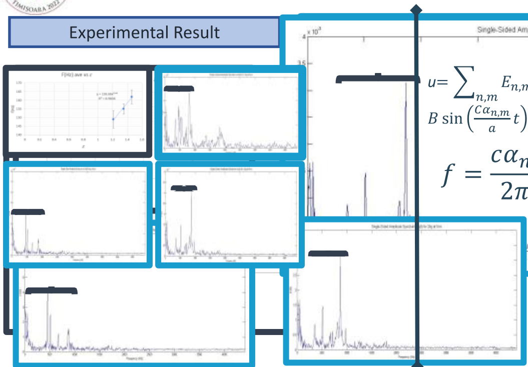




2 layer membrane



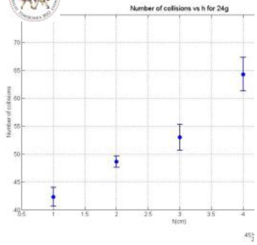
Experimental Result

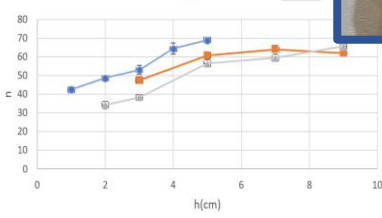


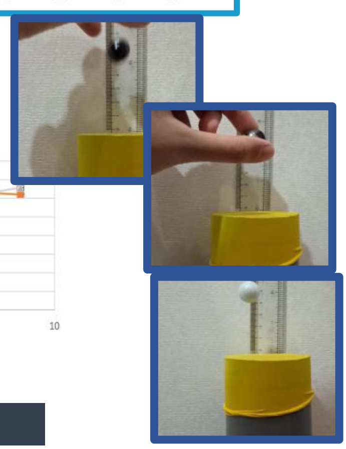
Theoretical Result

$$u = \sum_{n,m} E_{n,m} J_n\left(\frac{\alpha_{n,m}}{a} r\right)\left(A\cos\left(\frac{c\alpha_{n,m}}{a} t\right) + B\sin\left(\frac{c\alpha_{n,m}}{a} t\right)\right)\left(C_n\cos(n\theta) + C_n\sin(n\theta)\right)$$

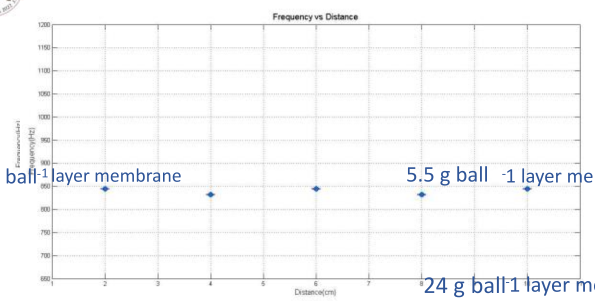
$$f = \frac{c\alpha_{n,m}}{2\pi a}$$



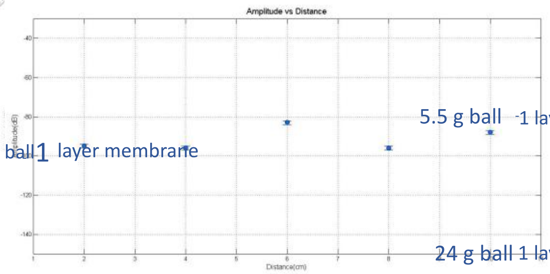




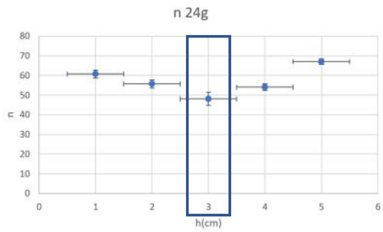
$\varepsilon = \frac{l_1 - l_0}{l_0}$ Number of collisions $\varepsilon = 1.46$



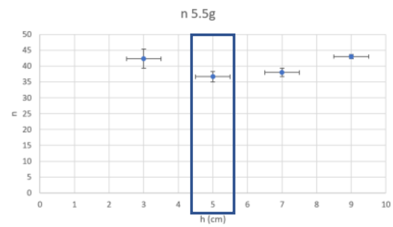
$\varepsilon = \dfrac{l_1 - l_0}{l_0}$ | Frequency $\varepsilon = 1.46$



$\varepsilon = \dfrac{l_1 - l_0}{l_0}$ | Amplitude $\varepsilon = 1.46$

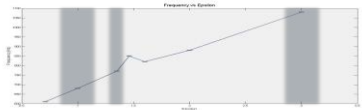


Number of collisions for ball 24 g $\varepsilon = 1.21$



Number of collisions for ball 5.5 g $\varepsilon = 1.21$

Experimental Result



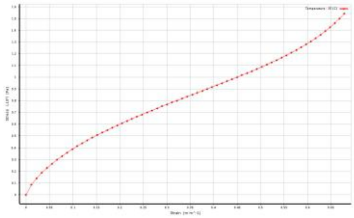
	ε						
Frequency	0.71	1	1.35	1.46	1.6	2	3
	610	680	770	850	820	880	1080

Theoretical Result

$$f = \frac{c\alpha_{n,m}}{2\pi a}$$

0.005
a = 0.015 m
0.007

790 Hz
f = 1113 Hz
= 660 Hz



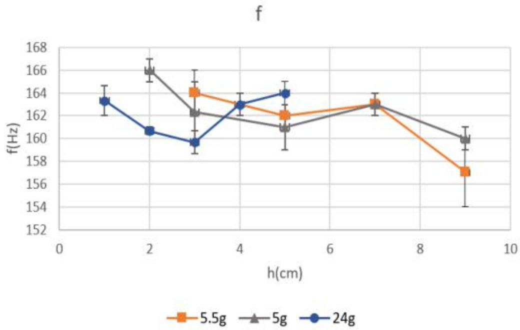
Stress-Strain relation of hyperplastic material

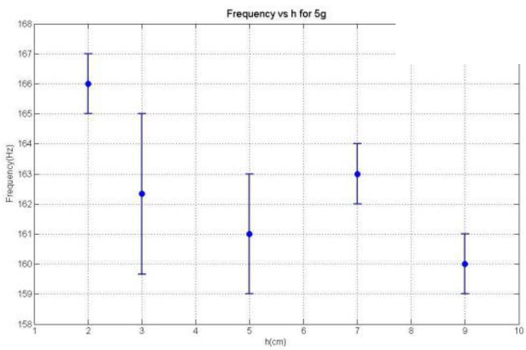
Young modulus for latex : 1.2 Mpa

stress = (elastic modulus) × strain

Almost negligible

$\varepsilon \sim \partial$

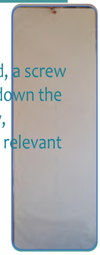




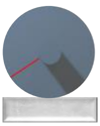
ART AN AMAZING FACT IN SCIENCE

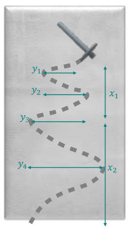
Problem Statement

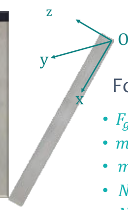
When placed on its side on a ramp and released, a screw may experience growing oscillations as well as it travels down the ramp. Investigate how the motion of the screw, the growth of these oscillations depend on the relevant parameters.

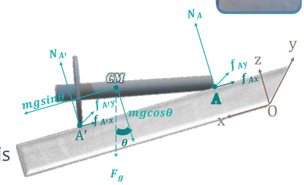


Motions







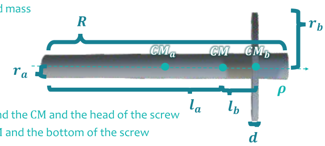


Force analysis

- F_g = Gravity force
- $mgsin\theta$ = Parallel force to ramp
- $mgcos\theta$ = Perpendicular weight force to ramp
- $N_{A'}$ = Normal force of incline of point A'
- N_A = Normal force of incline of point A
- $f_{A'x}$ = x-axis friction of point A'
- f_{Ax} = x-axis friction of point A
- $f_{A'y}$ = y-axis friction of point A'
- f_{Ay} = y-axis friction of point A

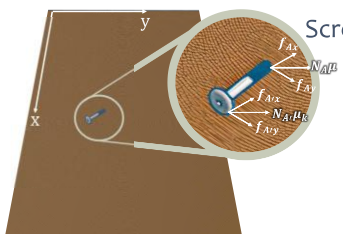
Parameters

CM_a =The center of the body mass
CM =The center of mass
CM_b =The center of the head mass
r_a =Body radius
R =Body length
r_b =Head radius
d =Head thickness
ρ =Density
l_b = Distance between the and the CM and the head of the screw
l_a =Distance between the CM and the bottom of the screw

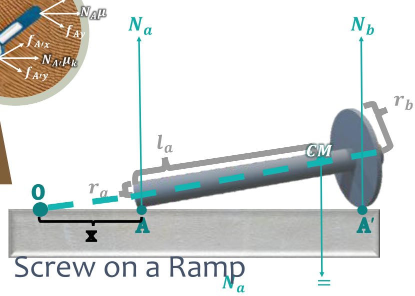


$$(N_{A'}\mu_K)^2 = f_{A'x}{}^2 + f_{A'y}{}^2$$
$$(N_A\mu_K)^2 = f_{Ax}{}^2 + f_{Ay}{}^2$$

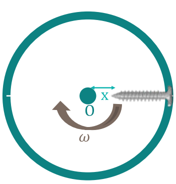
- $N_{A',A}$ = Normal forces
- μ_K = Coefficient of kinetic friction
- $f_{A',Ax}$ = x-axis frictions
- $f_{A',Ay}$ = y-axis frictions



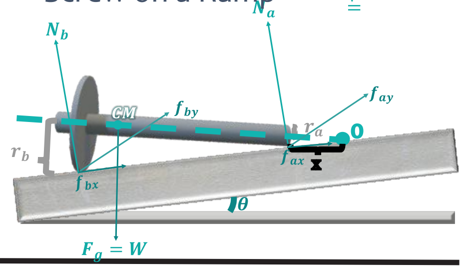
Screw on a Horizontal Surface



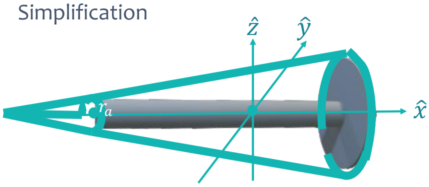
Perfect Rolling



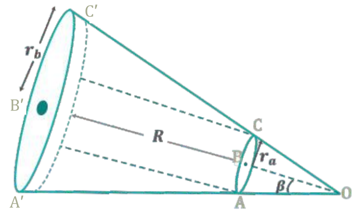
Screw on a Ramp



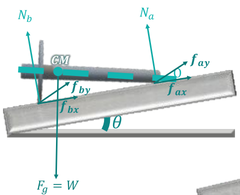
Simplification

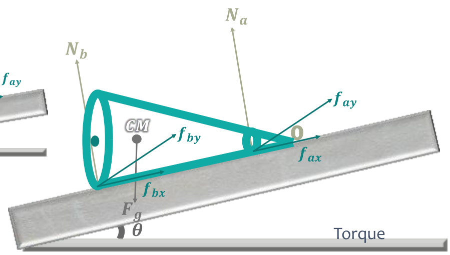


Cone on a Horizontal surface



Cone on a Ramp

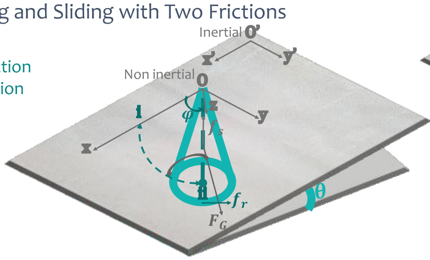


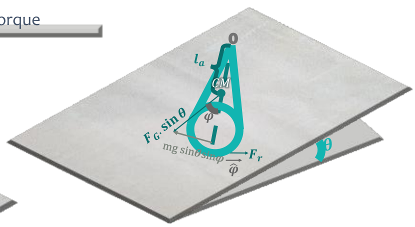


Cone Rotating and Sliding with Two Frictions

f_s =Slipping friction

f_r =Rolling friction





Rolling with Sliding

$\vec{f_r} = kN_r\hat{\varphi}$

$k \simeq k_o + k_1 v$

v =Tangential velocity⇒ $v = l\varphi$

According to I equation:

$-F_G \sin\theta \sin\varphi\, l_a + f_2 R = I_\circ \ddot{\varphi}$

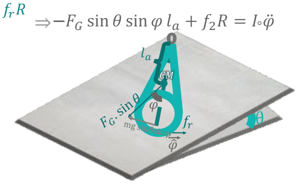
Constant multiplicative (A,B,C) :

$\ddot{\varphi} = -A\sin\varphi + B\dot{\varphi} + C$

Torque

$$\begin{cases} \tau_\varphi = -F_G \sin\theta \sin\varphi\, l_a + f_r R \\ \tau_\varphi = I_\circ \ddot{\varphi} \end{cases}$$

$\Rightarrow -F_G \sin\theta \sin\varphi\, l_a + f_2 R = I_\circ \ddot{\varphi}$



- Shosuke Sasaki ,Yohe Namba,Tadao Iwanari, Yasuyuki Kitano,2021 Analysis of the rotational motion based on rolling friction and torque article

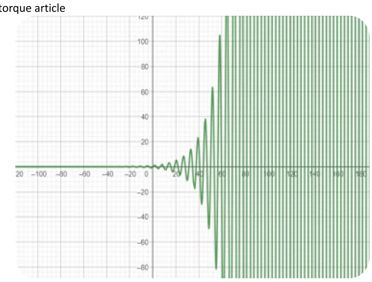
Considering the differential equation analysis

1. If φ be small enough:

$$\ddot{\varphi} = -A\sin\varphi + B\dot{\varphi} + C$$

$$\ddot{\varphi} \approx -A\varphi + B\dot{\varphi} + C$$

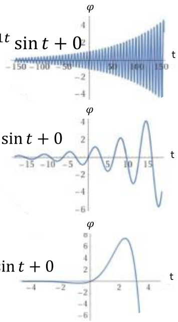
$$\Longrightarrow \varphi \approx C_1 e^{at} \sin t + C_2$$



$\varphi \approx e^{0.01t} \sin t + 0$

$\varphi \approx e^{0.1t} \sin t + 0$

$\varphi \approx e^{1t} \sin t + 0$

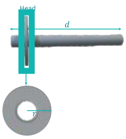


2. If φ get bigger that it can not anymore consider equals $\sin\varphi$:

$$\sin\varphi \not\approx \varphi$$

$$\ddot{\varphi} = -A\sin\varphi + B\dot{\varphi} + C$$

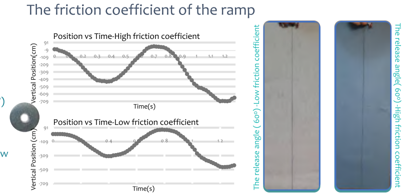
$$12° < \varphi < 90°$$



Parameters

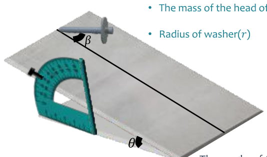
- The friction coefficient of ramp
- The angle of the ramp(θ)
- The release angle of the screw(β)
- The body length(d)
- The mass of the head of the screw
- Radius of washer(r)

The friction coefficient of the ramp

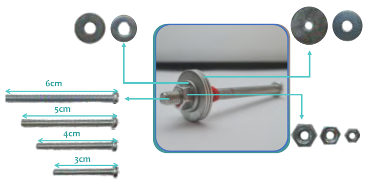


Parameters

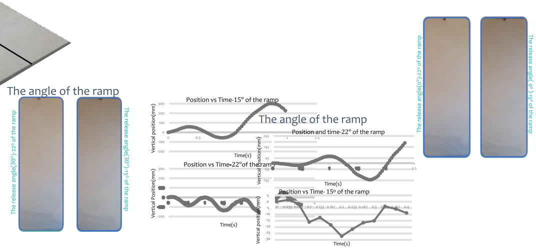
- The friction coefficient of ramp
- The angle of the ramp(θ)
- The release angle of the screw(β)



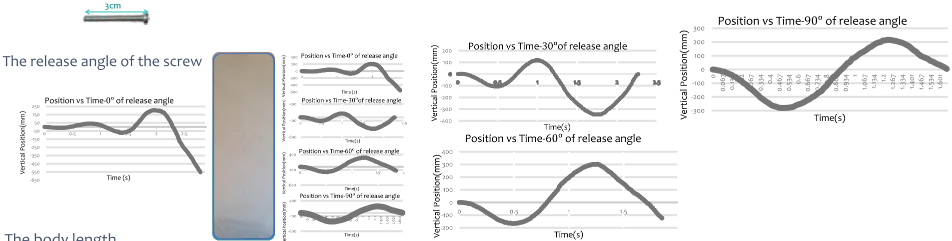
Experimental Setup



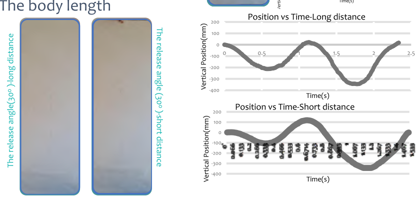
The angle of the ramp



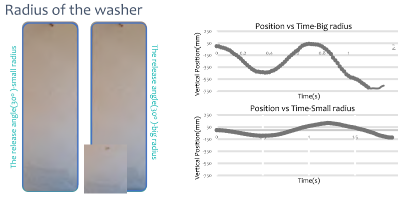
The release angle of the screw



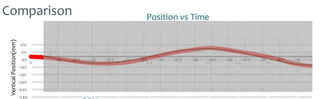
The body length



Radius of the washer

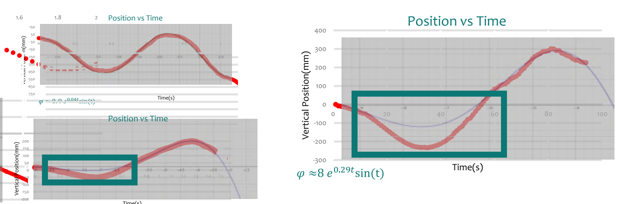


Comparison

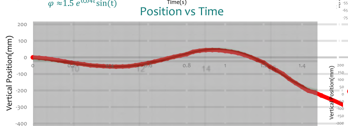


$\varphi \approx 1.5\, e^{0.04t} \sin(t)$

$\varphi \approx 10\, e^{0.59t} \sin(t)$



$\varphi \approx 8\, e^{0.29t} \sin(t)$



$\varphi \approx 0.008\, e \quad \sin(t)$