How are roller coasters designed mathematically to provide thrill whilst ensuring physiological safety?

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Abstract

This paper explores the thread between maximising physiological thrill and ensuring safety in roller coaster design. By looking at the breakdown of mathematics behind the rides, I analysed how properties such as acceleration, g-forces, and jerk affect the dynamics of the ride. Using Hyperia at Thorpe Park as a prime example of creativity and engineering, I intended to investigate both human endurance as well as the physics used to maintain safety whilst providing thrill. My findings highlight the crucial role of biomechanical safety and calculations in dangerous rides. Whilst roller coasters deliver thrill on a psychological level, I will be focusing on the physiological thrill created.

Introduction and Background Information

The first complete roller coaster, the Gravity Switchback Railway, debuted in 1885 with a full circuit and lift hill. Introduced by Phillip Hinkle and constructed by LaMarcus A. Thompson, it achieved international success, earning over £600 daily (American Coaster Enthusiasts, History). This success sparked the rise in popularity of roller coasters, with early innovations like figure of eight designs and small dips enhancing the thrill.

Roller coasters are designed to provide thrilling experiences while ensuring safety, with precise calculations based on physics and maths. Designers create rides based on specific parameters from amusement parks, including budget and available space, while enthusiasts will travel worldwide to experience the new most exciting rides. The American Coaster Enthusiasts (ACE), founded in 1978, promotes both new and original roller coasters (American Coaster Enthusiasts, AboutACE). Enthusiasts enjoy the combination of fear and safety, as physical forces are counterbalanced by secure restraints (Neil Sawford 2023, BransonAlpineMountainCoaster). The desire for the most thrilling rides fuels competition among designers. As an example, Formula Rossa, the world's fastest roller coaster, reaches speeds of 240 km/h in the UAE. (GaleAcademic Onefile, Scream Machines, 2012).

Intamin and Bolliger & Mabillard (B&M) are two leading roller coaster design companies. Founded in 1967, in Liechtenstein, Intamin is most known for their innovation; this includes but is not limited to designing the first river rapids with Six Flags in 1979. They also set the record for most inversions with Colossus at Thorpe Park, featuring 10 inversions. Intamin has continued to push the limits with hydraulic and vertical launching systems, allowing rides to accelerate against gravity for more intense thrills (Intamin). On the other hand, B&M, based in Switzerland, tailors designs to meet specific client needs. In the UK, their notable ride is Nemesis at Alton Towers, famous for its high G-forces, reaching up to 3.5G, despite only having four inversions and a speed of 81 km/h (Alton towers 2024, Nemesis Reborn). Initially focused on the US market, B&M expanded into Europe and pioneered floorless coasters. They are also known for their use of box-section spines, which produce their distinctive sound as the track passes overhead (Wikipedia 2024, Bolliger & Mabillard) .

As part of my research, I consulted roller coaster enthusiast DigitalDan to understand the key elements that contribute to the thrill of a roller coaster. Initially, he considered factors like height, speed, and coverage area, but later shifted focus to acceleration, airtime, and intensity. Using Stealth at Thorpe Park as an example, he highlighted the significance of acceleration in creating thrills. Stealth's launch propels riders from 0 to 80 mph in 1.8 seconds, generating an acceleration of 19.86 m/s^2, which is 1.78 times the acceleration of an F-1 car. This rapid acceleration produces linear g-forces, pushing riders back into their seats and opposing horizontal motion, intensifying the thrill. Unlike constant velocity, which has no acceleration, acceleration creates forces in both horizontal and vertical directions, amplifying the sensation of speed.

Dan Richards (2024) also discussed the role of airtime and intensity, which are related to vertical g-forces. For instance, Nemesis at Alton Towers generates 3.5Gs on its first helix, pushing riders downward into their seats. Negative g-forces, or airtime, create a sensation of weightlessness, lifting the rider momentarily out of the seat. These intense forces and changes in body weight contribute to the overall thrill, as the body alternates between feeling heavier and weightless.

John Wardley and John Burton

John Burton, the senior creative lead at Merlin, was only 27 when he was entrusted with designing Thorpe Park's newest ride. With a budget of £18 million and the requirement for the ride to be at least 213 feet tall, Burton embarked on a global journey to find inspiration. He visited theme parks in California and Poland, as well as collaborating with other companies inside a storeroom at Madame Tussauds, another of Merlin's attractions (The Guardian)

Burton's childhood fear of roller coasters eventually transformed into a fascination with them. His curiosity led him to create a video game, *Roller Coaster Tycoon*, where he explored ride mechanics and design. Studying architecture at Birmingham University, Burton's love for both roller coasters and drama, especially West End musicals, influenced his approach to ride design. He appreciated the choreography and emotional flow in musicals, which he translated into creating *Hyperia* at Thorpe Park (The Guardian)

The ride, designed to be a thrilling combination of twists and turns on the edge of the park's island, aimed to be the UK's greatest. Burton also focused on an immersive experience with ethereal theming and a custom soundtrack. The name "Hyperia" was chosen to evoke the emotions Burton wanted riders to feel, derived from an earlier idea, "Hydra," reflecting the aquatic surroundings (The Guardian). After construction, the ride underwent rigorous testing with weighted plastic molds to simulate the forces experienced by riders. In 2023, Hyperia was ready for its first public tests.

John Wardley is a British roller coaster designer and engineer who is most known for revolutionising roller coaster design, mainly in the UK. His work at Alton Towers caused him to become a world recognised engineer . (John Wardley)

Wardley started working in the 1970s, inside the theater industry. His passion for theme parks and thrilling rides, similar to that of John Burtons', is what caused him to transition into roller coaster design and engineering. His first major success was the design of The Beast at Kings Island in Ohio, 1979. This ride was the beginning for one of the world's leading engineers in the field.

One of his most significant achievements was the creation of Nemesis, (Now Nemesis Reborn) at Alton Towers, which opened in 1994. The inverted coaster- A coaster where the carts are underneath the track- was groundbreaking, including four inversions and it was the world's first coaster built inside a hole in the ground. This was due to Alton Towers' rule from the council where no coasters can exceed the tree lines. Nemesis became one of the most commercially celebrated coasters worldwide and set a higher standard for innovative coaster design, highlighting Wardley's blend of technicality and creative vision.

Wardley was also involved in the development of Oblivion in Alton Towers, the world's first completely vertical drop roller coaster. Known for its 180-foot drop, Oblivion was considered a milestone in roller coaster design. During 2002, Alton Towers released the coaster, Air (now known as Galactica), to the public. This was the first ever flying coaster, where riders are positioned horizontally. This offered a new experience for enthusiasts. It was important as B&M has taken this idea and made it a worldwide staple for different parks.

The Smiler Accident

The Smiler accident at Alton Towers in 2015 was a tragic event that left several people injured. The incident was due to two trains on the roller coaster colliding; one train had stopped on the track due to a delay, however the second train was allowed to move forward without safety checks. This mistake led to severe injuries for four riders, where each of them needed medical attention. (The Guardian (2024))

The accident was caused by a mix of human and mechanical issues. One major problem was that Alton Towers staff didn't follow the proper procedures, as found by the Health and Safety Executive (HSE). They allowed the second train to move ahead even though the first one hadn't cleared the track, which goes against established safety protocols by the HSE, The ride’s mechanical problems also played a big role. The Smiler’s braking system, using magnetic brakes, had faced issues before but they were not completely addressed. In the six months leading up to the crash, the ride had stopped 14 times, indicating ongoing technical problems that had not been fixed.

On top of this, there were safety concerns about the lack of barriers to stop riders from moving between trains, and maintenance practices were criticized for being rushed or not thorough enough.

After the accident, the HSE came to the conclusion that both human mistakes and mechanical failures were to blame. Alton Towers was fined £5 million for failing to keep riders safe whilst there. This has indirectly caused safety regulations to be much stricter to prevent similar events happening again.

An expert had calculated the kinetic energy as being equivalent to a family car with a mass of over a ton colliding at 90mph. (R(HSE) v Merlin Attractions Operations LTD, 2015)

Biomechanical Safety & G-Force Limits

Human bodies have varying g-force tolerances when exposed for long periods of time:

Condition	Tolerance (g)
Comfortable	1–2 g
Safe limit	4–5 g
Risk of injury	6–9 g

Fatal	10+ g

Restraints apply force across large body areas to help prevent injuries. For example:

- Lap bars press against the thighs due to the strength of the femur.
- Over-the-shoulder restraints disperse force to the shoulders and chest.

With roller coasters which contain inversions, the ride has over the shoulder restraints typically as it provides greater safety due to more contact and surface area on the human body, decreasing pressure. This is typically cushioned to ensure comfort for the rider and to reduce the impact in the event of a collision.

Restraints are locked either hydraulically or magnetically normally. Typically, more than one set of locking mechanisms are used to ensure safety in case the other fails. Modern coasters are controlled from a different power source, preventing riders from controlling their own safety.

Over the shoulder restraints are U-shaped which keep riders sat upright in a safe posture to allow more force to be absorbed. The restraint between the riders legs prevents them swinging and hitting each other. However, injuries can happen more readily when the ride is rough, so lap bars are often preferred. (Coasterpedia, 2021)

Reach envelope

As defined by the American Psychological Association, the reach envelope is the area that an operator can reach from a seated or standing position. It is used in workspace design to determine the placement of displays and tools to be used by the operator. (APA, 2018)

In a lot of rides, such as Swarm at Thorpe Park, there are several near miss elements. Designers need to prevent riders from being able to reach these elements to prevent grievous harm and potential loss of limbs. This is done through a reach envelope, also known as a patron clearance envelope.

Rides often have a maximum height requirement as well as a minimum height requirement. This is due to people of that height and above being able to physically interact with the elements. The envelope accounts for the maximum extension in the human body around the train whilst seated, mainly from the arms. (Robert A. Di Domizio, Jr, P.E, 2015)

The reach envelope is part of the American National Standards Institute’s rules for roller coasters, covered in ASTM F2291-24: Standard Practice for Amusement Ride Design. This establishes the design criteria for all rides made, ensuring that the reach envelope is considered, as well as the general forces and restraints which affect the body in motion. (ASTI, 2024)

Safety considerations

The intensity of wind in the area is a consideration for structural stability. Often, elements are tested in a wind tunnel to ensure the force does not damage the structure. This is found using the drag coefficient equation:

$$F_w = \frac{1}{2} C_d A P v^2$$

Where:

- F is the Force acting on the object
- Cd is the drag coefficient
- A is the exposed area of contact
- P is the density of the air
- v is the wind speed

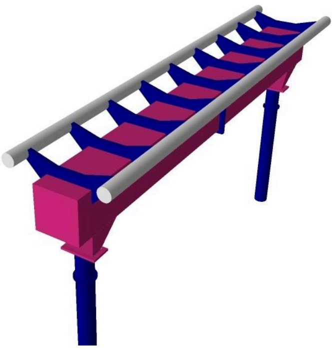
Typically, most coasters are made of steel or wood. On steel coasters, the most effective layout is a triangular truss due to its strength and flexibility. Wooden coasters typically have latticed layouts with extra binding to prevent the wood bending.

Coasters will generally try to keep their centre of mass low to the ground. This is why deep concrete foundations are used as anchors as it distributes forces around the ground instead of on the track.

Supports

Supports differ between companies, but all are calculated to withhold the load of the track and the objects on top as well. The most common is a steel circular hollow tube, as it is symmetrical and easy to manufacture. As roller coasters become taller, the supports grow exponentially as the supports also need to be reinforced at height to prevent toppling.

Less supports can be used if the spine of the track is made more stable and longer, often using cross-sections to reinforce the stability. However, if there are minimal supports, this can cause the coaster to oscillate in the wind or as the train passes over the element, causing danger to riders safety. (Master's thesis: design of roller coasters, 2018 , Available at URL: https://aaltodoc.aalto.fi/server/api/core/bitstreams/5e6a206c-13c5-41b6-87f6-da0bc4ec7aef/content, accessed at 13/3/25)



(CAD model of B&M's signature box-spined track)

The splines and the spine of this track piece are designed to create a distinctive sound associated with B&M. However these are modified according to the coaster. For example, a Wing Coaster has a wider load which means it requires a wider spline to account for it.

Materials

Most roller coasters are constructed out of wood or steel, but often have aluminium parts within them. The European nation in coordination with the UK has standards set for the construction of steel and aluminium structures. These structures must follow all regulations set within these standards. As an example, BS EN 1999-1-1 is on the design structure of aluminium. (The European Union 2007)

All elements of aluminium must undergo either elastic or plastic global analysis to find internal moments and forces. . Elastic assumes a linear stress-strain relationship and can be used in all cases, whereas plastic can only be used when a structure has sufficient rotation at a plastic hinge. A plastic hinge is a part of a structure which yields due to bending, allowing for rotation without increasing moments significantly.

Beams and other members of the structure are then classified upon their ability to form a plastic hinge- there are 4 classes. These are determined by their slenderness parameters, which is their base length to thickness ratio. If there is a stress gradient across the element, then that must be factored in as a coefficient for the ratio. These can then be compared to set values to determine the class of metal being used.

There are many other factors which must be considered for each individual element:

Name of factor	**What it means**
Bending	Bending occurs when a force causes a deformation in a curved shape in an element. It results in tension on one side and compression on the other.
Buckling	Buckling is a sideways failure of a structural element under compressive stress. Instead of compressing straight, it bends or bows outwards.
Moment of inertia	A measure of how a shape's area is distributed about an axis. It reflects how well a section resists bending and buckling.
Tension	A pulling force that stretches a material.
Compression	A pushing force that shortens a material.

Shear	A force that causes parts of a material to slide past each other, acting parallel to the surface.
Radii of gyration	It's the theoretical distance from the centre where the entire area could potentially be concentrated and still have the same moment of inertia.

(J.J Lugthart 2013)

Once everything has been calculated, a report must be made to display the calculations to ensure safety for each member of the structure. I found this through primary research in work experience, as I was able to make one alongside engineers.

Physics and Mathematics of Roller Coaster Design

Loops, Corkscrews, and Inversions

Roller coasters are built using an understanding of mechanics and physics to maximise both safety and thrill, whilst working within the previously stated parameters and standards. As an example, vertical loops are not circles but rather, clothoid shapes. These gradually increase and decrease in curvature. This design minimises excessive G-forces that could harm riders by distributing forces more evenly.

In a vertical loop:

- Centripetal force keeps riders pressed against their seats.
- At the top of the loop, potential energy is maximized, and velocity is lowest, ensuring manageable forces.

Corkscrews use helical structures to provide a smooth rotation whilst maintaining stability. Engineers calculate the angular velocity and centripetal acceleration to make sure the maneuver feels pleasurable but not disorienting.

Angular Velocity and Energy Conservation

Angular velocity (ω) is the average velocity of an object moving in a circular motion. Angular velocity is defined as:

$$\omega = v/r$$

Where:

- v is the linear velocity (ms^{-1})
- r is the radius of the circular path. (m)

By increasing or decreasing the radius of the curves, designers are able to control the angular velocity and the forces experienced by riders. Smaller radii increase angular velocity but also increase centripetal acceleration, which must be managed to ensure safety.

Energy conservation underpins the entire operation of a roller coaster. The train's potential energy (E_p) at the highest point is converted into kinetic energy (E_k) as it descends: $E_p = mgh$ $E_k = 1/2\ mv^2$ where:

- m is the mass of the train and riders,
- g is the acceleration caused by gravity ($9.8\ m/s^2$),
- h is the height of the train,
- v is its velocity.

Engineers make sure these energy transformations work as intended to allow the train to maintain enough speed to complete the course without breaking safety limits. Friction and air resistance are also considered, as they oppose motion, which cause the train to require additional boosts from chain lifts or launch systems.

Brakes and Chain Lifts

Braking systems are used to reduce the train's speed and ensure safe stops at times. Modern roller coasters use magnetic brakes, which are dependent on Eddy currents to generate resistive forces through electromagnetism. Eddy currents are produced when a permanent magnet passes through a magnetic material, where the kinetic energy converts to electrical energy inducing a current, which creates a force in the opposing direction to motion, therefore slowing it down. (ShawMagnets) Conductive fins on the coaster train pass through a magnetic field, which induces currents that oppose the train's motion as it passes through, creating smooth deceleration which doesn't include physical contact. This method minimises physical wear and tear, which in turn ensures long-term reliability.

Chain lifts are used to elevate trains to the initial drop, providing gravitational potential energy. A motorised chain, which contains evenly spaced links, connects with a hook or dog under the train. As the motor turns, it pulls the chain and trains on the track upwards. The energy provided is later converted into kinetic energy during the descent.

Powering Chain Lifts

Chain lifts are powered by electric motors that push the chain along sprockets. The motor's power is calculated through knowledge of the weight of the train, the height of the lift hill, and the speed of uphill travel. To ensure reliability, motors are equipped with feedback monitors that watch the chain's tension and movement.

Rollback Mechanisms

A vital safety feature of chain lifts is the rollback mechanism, which prevents the train from sliding backward if the chain fails. This is achieved using anti-rollback ratchets, also known as "dogs," which engage with a series of teeth on the track. As the train ascends, the ratchet clicks over the teeth, locking the train in place if the chain stops. This design ensures that even in the event of a mechanical failure, the train remains securely positioned on the lift hill. If power is removed and the train is allowed to roll back, the teeth will catch it and provide a force opposing the direction of downwards travel due to its weight which will hold it in place.

Why Loops Are Not Circular

Loops, as stated previously, are not a perfect circle. Instead, they are shaped as clothoids, also known as Euler spirals. This design minimizes the extreme forces experienced by riders, ensuring comfort and safety. (Vox, 2022)

In a circular loop, the centripetal force required to maintain the train's path is given by:

$F_c = (mv^2)/r$

Where:

- F_c: Centripetal force (N)
- m: Mass of the object (kg)
- v: velocity of the object (m/s)
- r: Radius of the loop (m)

At the top of a circular loop, where velocity is lowest, riders experience dangerously high G-forces. A clothoid loop gradually increases the radius as the train ascends, distributing the centripetal force more evenly and reducing G-forces.

Clothoid loops have a radius that varies with arc length, allowing for smoother transitions. This ensures the forces remain within tolerable limits, preventing discomfort or injury.

Centripetal acceleration

$a_c = 2gh/r$

Where:

- a_c is the centripetal acceleration
- g is the acceleration due to gravity ($9.81 ms^{-2}$)
- h is the vertical distance from rest
- r is the radius of pitch

This allows designers to manipulate g-forces in order to evoke physical responses from the rider. By increasing the radius, the acceleration is smaller but by increasing the height, the acceleration will increase subsequently. Rest position is typically the lowest point of the ride or also the position where the ride begins. (Master's thesis: design of roller coasters, 2018)

Euthanasia Coaster

The Euthanasia Coaster, a hypothetical concept designed by Julijonas Urbonas, is a display of the extreme application of physics. It is intended to induce a happy and painless death through constant G-forces.

The ride has seven consecutive loops, which decrease in size each time. The decreasing radius increases the centripetal acceleration experienced by riders, leading to forces of 10 Gs to affect them; this is enough to cause cerebral hypoxia (oxygen deprivation). The extended exposure to high G-forces could lead to a loss of consciousness and would result ultimately in death.

Barrel Rolls, Heartline Rolls, and Zero-G Rolls

Roller coaster inversions are highly important to the thrill which enthusiasts chase, with the different elements such as barrel rolls, heartline rolls, and zero-G rolls creating different physiological experiences. These elements to the average rider are indistinguishable but can be sensed through the physiological forces applied to the body.

Barrel Roll

A barrel roll is an inversion where the train rotates 360° around its horizontal axis, in a cylindrical manner. The key features of a barrel roll are its smooth rotation and a constant curvature.

Physics:

- The rotation occurs about the train's longitudinal axis, meaning riders would experience lateral forces combined with mild variations in vertical forces.

- Centripetal acceleration is significant, as the curved path requires continuous inward force towards the centre to maintain rotation:

  ac=v^2/r

Where:

- ac: Centripetal acceleration (m/s²)
- v: Velocity of the train (m/s)
- r: Radius of the circular arc (m)

Design:

- The radius of curvature in a barrel roll is consistent but relatively small compared to other inversions. This amplifies the rotational sensation, which creates a feeling of disorientation.
- Engineers ensure smooth transitions in and out of the roll by adjusting curvature at entry and exit points, reducing sharp force changes that could injure riders.

Heartline Roll

A heartline roll is an inversion where the train's center of rotation aligns with the riders' approximate center of mass—typically around chest level or the level of your heart. Hence, Heartline Roll. This element is designed to minimise lateral forces on riders, making it feel smooth.

The heartline roll minimises discomfort by aligning the axis of rotation close to the center of gravity of the rider's body. This design reduces the torque felt by the rider, which makes it feel safer.

- Unlike barrel rolls, where the rotational axis is parallel to the track's centerline, the heartline roll shifts the axis upward to match the heartline, effectively reducing the feeling of being pushed outward.

Zero-G Roll

A zero-G roll is a highly specialised inversion where the train rotates around its axis while simultaneously traversing a parabolic trajectory, creating the sensation of weightlessness. This element is a hallmark of modern coaster design, offering a unique physical and emotional experience.

Physics:

- During a zero-G roll, the train follows a parabolic flight path where the forces acting on riders balance to approximate zero effective gravity. This is achieved by carefully shaping the element to simulate freefall conditions.
- The sensation of weightlessness occurs because the downward gravitational force is counteracted by the upward normal force from the train's motion. Mathematically: Force of gravity−Force of normal≈0N
- Angular velocity during the roll is matched with the train's velocity (v) and the radius of rotation to ensure synchronized motion:

Design:

- The curvature of the track and the rotational speed are precisely tuned to maintain the weightless effect throughout the inversion.
- The apex of the parabola is where the train's velocity provides just enough centripetal force to counteract gravity, creating the zero-G sensation.

Key differences

Feature	Barrel Roll	Heartline Roll	Zero-G Roll
Axis of Rotation	Track's longitudinal centerline	Rider's heartline (center of mass)	Varies
Forces Felt	Lateral and moderate vertical forces	Minimal lateral forces	Near-zero effective gravity (weightlessness)

Path Shape	Helical	Cylindrical with elevated axis	Parabolic with synchronized rotation
Sensation	Spinning or flipping	Smooth rotation with minimal torque	Floating or flying

How to determine minimum and maximum speeds?

Calculating minimum and maximum speeds is vital for ensuring safety of a ride, as it allows engineers to determine what the conditions must be for the ride to operate softly as well as the reinforcements and energy needed to provide a train to allow it to smoothly and safely traverse the track. Minimum speed is found to show that the ride can complete the track regardless of conditions but maximum speed is found to prevent extreme g forces on riders.

Minimum speeds are found by setting the starting speed of the train to 0 at the top of a chain lift and simulating the gain in kinetic energy as the only input of speed for the train. Unfavourable conditions such as high speeds in the opposite direction of travel are simulated to showcase extreme scenarios. It must be able to complete the track in these conditions. This is called a gravity run.

Maximum speeds are found using the lowest possible coefficient of friction, launches and favourable conditions and simulating the train on tracks. Different manufacturers have different coefficients of friction. If it is too fast, elements must be added to slow it down or reduce g-forces.

The elements normally added are trim brakes. These slow down the train, which can lead to each ride being very similar to the last as it mitigates external factors affecting speed in different parts of the coaster. This differs to block brakes which stop the train as it passes over. (Master's thesis: design of roller coasters, 2018)

Motion in a parabolic hill

Below is the mechanics of why riders experience a feeling of weightlessness whilst on the parabolic airtime hill. This includes the acceleration vertically, the resultant force of zero newtons at the peak and the increasing horizontal acceleration as you decline down the track.

In a parabolic hill, the track follows a quadratic shape, which is modelled as a function. f(x)=ax^2+bc+c. In this case, y=f(x). x is a function of time, which is why you are able to differentiate with respect to time.

Horizontal velocity assuming negligible friction can be found by differentiating f(x) with respect to time. Therefore, Vx=$\frac{dy}{dt}$which gives you the initial horizontal velocity.

Vertical velocity is found using the chain rule. Vy=$\frac{dy}{dt}$. $\frac{dy}{dt}$=$\frac{dy}{dx}$*$\frac{dx}{dt}$. Using our model f(x), $\frac{dy}{dx}$ is 2ax+b, therefore meaning Vy= (2ax+b)Vx.

The magnitude of velocity is found using pythagoras, where $\sqrt{v_x^2 + v_y^2}$ = v, where v is the magnitude of velocity.

$$v = \sqrt{v_x^2 + ((2ax + b)v_x)^2)}$$

$$v = v_x(\sqrt{1 + (2ax + b)^2}$$

Acceleration is found by differentiating velocity again. Vx is considered a constant, therefore giving 0 acceleration horizontally.

Vertical acceleration is found differentiating again.

$$a_y = \frac{dv_y}{dt}$$

$$\frac{dv_y}{dt} = \frac{d}{dt} v_x(2ax + b)$$

$$a_y = 2av_x^2$$

In order to use vertical acceleration in g forces, you must find the radius of curvature and the normal acceleration.

$$a_n = \frac{v^2}{R}$$

Where R is the radius of curvature.

R is found using:

$$R = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

Substituting values from differentiation back in:

$$R = \frac{\left(\left(1+(2ax+b)^2\right)^{\frac{3}{2}}}{|2a|}$$

Using the formula for normal acceleration:

$$a_n = \frac{v^2}{\frac{((1+(2ax+b)^2)^{\frac{3}{2}}}{|2a|}}$$

Substituting the value for v back in:

$$a_n = \frac{(v_x\sqrt{1+(2ax+b)^2})^2}{\frac{((1+(2ax+b)^2)^{\frac{3}{2}}}{|2a|}}$$

Simplifying:

$$a_n = \frac{2av_x^2}{(1+(2ax+b)^2)^{\frac{1}{2}}}$$

(R.C Coates, 1988)

G-force is the normal acceleration, divided by the acceleration due to gravity (9.81 ms^{-2})

This is used to ensure that the g-force exerted on the riders does not exceed limits as it can cause significant damage to the body, leading to potential lawsuits and loss of life.

Conclusion

Roller coasters are the ultimate cumulation of maths and engineering. Despite the negative stigma often associated with them, they provide immense thrill to a large proportion of the population. Behind all the fun perceived from the surface are some of the most creative engineers who ensure that roller coasters are not only safe, but comfortable.

Engineers work meticulously to design the rides to international standards, whilst making them thrilling. Each material has its own eurocode and at every point on the ride, reach envelopes are employed to prevent riders being injured. Incidents such as the smiler crash aren't due to machinery failures but human error, proving that humans are the main cause for fear on these rides.

Brakes, boosters, chain lifts, inversions, everything is calculated to ensure the rider's safety whilst maximising physiological thrill. Roller coasters have developed over the past few centuries, with modern influential people challenging the structure of coaster development, pushing the boundaries as pioneers outstanding in their field.

Whilst coaster development is still being pushed to the fastest, tallest and largest, corporations are still providing limits for the people to permit hazardous constructions from being released to the public. But is the world going too far in terms of safety that thrill is being limited?

It is evident that whilst safety is becoming an increasingly important issue in modern society, one can believe we have not even started to imagine what roller coasters could evolve into. In the future, there is speculation on what might be around.

The main thing we are currently progressing towards is Maglev coasters- which comes from magnetic levitation. There is no contact between the trains and the track, leading to no friction and no wear and tear. This is a massive step forward in terms of technology, which is currently considered to be more useful to high speed trains before they reach roller coasters.

Due to the growth of AI however, there is also talk of AI powered roller coasters which are able to change the ride based on your biometrics which it can sense. This could be integrated with VR to provide a fully customisable ride, unique for each rider. This includes scents, sounds and sight to fully immerse the rider- this is an idea best explored by Soarin' at Epcot.

Overall, the future of roller coasters, whilst uncertain, looks promising provided we are not bound by the limits of extreme safety. Freak accidents may very rarely occur, but should not cause limitations to human creativity and expressionism as a response.

References

1. (American Coaster Enthusiasts, AboutACE. Available at URL: https://www.aceonline.org/page/AboutACE. Accessed at 7/11/24, 14/11/24)
2. (Neil Sawford 2023, BransonAlpineMountainCoaster. Available at https://www.bransonalpinemountaincoaster.com/. Accessed at 14/11/24)
3. (GaleAcademic Onefile, 2012, Scream Machines. Available at URL: https://go.gale.com/ps/i.do?id=GALE%7CA289215181&sid=googleScholar&v=2.1&it=r&linkaccess=abs&issn=10411410&p=AONE&sw=w&userGroupName=anon%7E6495128a&aty=open-web-entry. Accessed at 14/11/24)
4. (Intamin, Available at URL: https://www.intamin.com/company/about-us/ Accessed at 28/11/24)
5. (Alton Towers 2024, Nemesis Reborn Available at URL: https://www.altontowers.com/explore/theme-park/rides-attractions/nemesis-reborn/. Accessed at 28/11/24)
6. (Wikipedia 2024, Bolliger & Mabillard, Available at URL: https://en.wikipedia.org/wiki/Bolliger_%26_Mabillard. Accessed at 28/11/24)
7. (Dan Richards (2024) Email to Jack Fox, 14th November).
8. (The Guardian, Available at URL: https://www.theguardian.com/travel/2024/oct/24/rollercoaster-designer-john-burton-thorpe-park-hyperia, Accessed on: 9/1/25)
9. (John Wardley, Available at URL: http://www.john-wardley.co.uk/about.html, Accessed on 23/1/25)
10. (The Guardian 2024, Available at URL: https://www.theguardian.com/travel/2016/sep/26/alton-towers-smiler-crash-was-like-car-crashing-at-90mph, Accessed at: 23/1/25)

11. (R(HSE) v Merlin Attractions Operations LTD, 2015, Available at URL: https://www.judiciary.uk/wp-content/uploads/2016/09/r-hse-v-merlin-attractions-operations-ltd-sentencing-remarks.pdf, Accessed 6/2/25)
12. (Coasterpedia, 2021. URL available at: https://coasterpedia.net/wiki/Over-the-shoulder_harness, Accessed at 6/3/25)
13. (APA, 2018, URL available at https://dictionary.apa.org/reach-envelope, Accessed on 6/3/25)
14. (Robert A. Di Domizio, Jr, P.E, 2015, URL available at https://www.philadelphiatoboggancoastersinc.com/wp-content/uploads/2019/12/Patron-Clearance-Envelope.pdf, Accessed at 6/3/25)
15. (ASTI, 2024, URL available at https://blog.ansi.org/astm-f2291-24-standard-practice-amusement-rides/, Accessed at 6/3/25)
16. (CAD model of B&M's signature box-spined track. Available at:https://en.wikipedia.org/wiki/Bolliger_%26_Mabillard#/media/File:BandMTrack.jp Downloaded: 13/3/25).
17. (The European Union 2007, BS EN 1999-1-1, URL available at https://www.phd.eng.br/wp-content/uploads/2014/11/en.1999.1.1.2007.pdf, Accessed at 6/4/25)
18. (J.J Lugthart 2013, Buckling of a beam, URL available at https://www.universiteitleiden.nl/binaries/content/assets/science/mi/scripties/bachlugthart.pdf, Accessed 5/4/25)
19. (ShawMagnets, Lenz's Law Apparatus, Accessed at 6/2/25)
20. (Vox, 2022, URL available at: https://www.vox.com/videos/2022/6/29/23188504/roller-coaster-loops-circular-g-force#:~:text=Between%20the%201840s%20and%20early,rattled%20passengers%20to%20their%20core., Accessed on 23/1/25)
21. (Master's thesis: design of roller coasters, 2018 , Available at: https://aaltodoc.aalto.fi/server/api/core/bitstreams/5e6a206c-13c5-41b6-87f6-da0bc4ec7aef/content, Accessed at 23/1/25)
22. (R.C Coates (1988) Structural Mechanics, Third Edition. Place of publication: London. Accessed at 3/4/25)